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# Excited Baryons Phenomenology from Large-$N_c$ QCD

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## Abstract

We present a phenomenological analysis of the strong couplings of the negative-parity $L = 1$ baryons from the perspective of the large-$N_c$ expansion. In the large-$N_c$ limit the mass spectrum and mixing pattern of these states are constrained in a very specific way. The mixing angles are completely determined in this limit, with predictions in good agreement with experiment. In the combined large-$N_c$ and SU(3) limits the pion couplings of the five negative-parity octets to the ground state baryons are given in terms of only 3 independent couplings. The large-$N_c$ predictions for the ratios of strong couplings are tested against experimental data.

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## I. INTRODUCTION

The large-$N_c$ expansion [1] proved to be a valuable guide for a qualitative and even quantitative understanding of gauge theories. In the past few years its application to baryons in QCD pioneered by Witten [2,3] has been substantiated and greatly expanded in a series of papers by Dashen, Jenkins and Manohar (DJM) and others [4,5] (and references cited therein).

In a recent paper [6] we studied the strong couplings of the orbitally excited baryons in the framework of the large-$N_c$ expansion, extending the results obtained by DJM in the s-wave sector. The general structure of the pion couplings to these states has been derived from a set of consistency conditions which follow from requiring the total scattering amplitude to satisfy the Witten scaling rules. The analysis presented in [6] assumed only isospin symmetry and was for the most part limited to baryons containing only $u$ and $d$ quarks. The present paper is a continuation to [6] and its aim is two-fold: first, to extend the results of [6] by incorporating SU(3) symmetry and second, to present a phenomenological analysis of the existing experimental data from the perspective of the large-$N_c$ expansion.

In Section II we demonstrate that the combined large-$N_c$ and SU(3) limits of QCD provide very strong constraints on the structure of the mass spectrum and mixing pattern of the $L = 1$ light baryons. A set of relations are derived among strong transition amplitudes between p-wave and s-wave baryons in Sec.III which are then compared against available experimental data. These relations are shown explicitly to agree with those derived in the quark model with arbitrary number of colors in the limit $N_c \to \infty$. For $N_c = 3$ they reduce to the usual SU(6) predictions of the quark model [7,8]. However, the large-$N_c$ approach turns out to be both less and more predictive than the SU(6)-based. On the one hand it predicts well-defined values for the mixing angles (which are left completely arbitrary in the quark model) but on the other hand, due to the small value of the number of colors in the real world, its applicability to the decuplet states is limited. One of the large-$N_c$ relations among $S$-wave pion couplings appears to be badly violated and we discuss a few possible explanations, one of which involves a different quark model assignment for the observed $S_{11}$ states. We summarize our conclusions in Sec.IV.

## II. SU(3) SPIN-FLAVOR STRUCTURE OF THE EXCITED BARYONS

The structure of the baryon spectrum in the large-$N_c$ limit can be obtained by examining the symmetry properties of the states under permutations of two quarks. The ground state s-wave baryons transform according to the completely symmetric representation of the permutation group shown in Eq.(2.1). For baryons containing two flavors this means that their spin-flavor wavefunction must transform like the totally symmetric representation of SU(4), which is decomposed into representations of $SU(2)_{isospin} \times SU(2)_{spin}$ with $I = J$. The analogous decomposition of the totally symmetric representation of SU(6) into representations of $SU(3)_{flavor} \times SU(2)_{spin}$, relevant for the baryons containing 3 light flavors is shown in Eq.(2.1). For $N_c = 3$ this representation contains the familiar spin-1/2 octet and the spin-3/2 decuplet baryons.

$$
\overbrace{\Box\Box\cdots\Box}^{N_c} = \left(\begin{smallmatrix}\Box\Box\cdots\Box\Box\\ \Box\Box\cdots\Box\end{smallmatrix}, J=\frac{1}{2}\right) + \left(\begin{smallmatrix}\Box\Box\cdots\Box\Box\Box\Box\\ \Box\Box\cdots\Box\end{smallmatrix}, J=\frac{3}{2}\right) \tag{2.1}
$$

$$
+\cdots+\left(\Box\Box\cdots\Box, J=\frac{N_c}{2}\right) .
$$

The spectrum of the p-wave baryons can be obtained in a similar way from symmetry considerations. In the real world with $N_c = 3$ the spin-flavor wavefunction of the $L = 1$ light baryons transforms according to the mixed symmetry representation **70** of SU(6). Its decomposition into spin-flavor multiplets takes the form [9]

$$
\begin{smallmatrix}\Box\Box\\ \Box\end{smallmatrix} = (\mathbf{1}\,, S=\frac{1}{2}) \oplus (\mathbf{10}\,, S=\frac{1}{2}) \oplus (\mathbf{8}\,, S=\frac{1}{2}) \oplus (\mathbf{8}\,, S=\frac{3}{2}) . \tag{2.2}
$$

After adding the orbital angular momentum $L = 1$ the resulting states reproduce the observed spectrum of the p-wave light baryons [13].

We would like in the following to construct the generalization of this procedure to the case of arbitrary $N_c$. The corresponding representation of SU(6) is obtained by adding additional boxes to the first line of the Young diagram. Its decomposition under the flavor-spin SU(3)$\times$SU(2) subgroup can be obtained as described in [6] for the corresponding SU(4) representation. One starts with the product of SU(6) representations

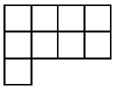
$$
\overbrace{\Box\Box\cdots\Box}^{N_c-1} \otimes \Box = \overbrace{\Box\Box\cdots\Box}^{N_c} \oplus \begin{smallmatrix}\overbrace{\Box\Box\cdots\Box}^{N_c-1}\\ \Box\end{smallmatrix} \tag{2.3}
$$

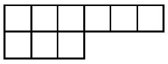
The decomposition of the symmetric representation on the left-hand side is known from Eq.(2.1). Subtracting from the product on the left-hand side the representations of SU(3)$\times$SU(2) corresponding to the symmetric representation on the right-hand side we obtain

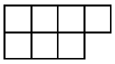
$$
\begin{aligned}
\begin{smallmatrix}\overbrace{\Box\Box\cdots\Box}^{N_c-1}\\ \Box\end{smallmatrix} &= \left(\begin{smallmatrix}\Box\Box\Box\cdots\Box\\ \Box\Box\Box\cdots\Box\\ \Box\end{smallmatrix}, S=\frac{1}{2}\right) + \left(\begin{smallmatrix}\Box\Box\cdots\Box\Box\Box\Box\\ \Box\Box\cdots\Box\end{smallmatrix}, S=\frac{1}{2},\frac{3}{2},\frac{5}{2}\right) \\
&+ \left(\begin{smallmatrix}\Box\Box\cdots\Box\Box\\ \Box\Box\cdots\Box\end{smallmatrix}, S=\frac{1}{2},\frac{3}{2}\right) + \left(\begin{smallmatrix}\Box\Box\Box\cdots\Box\Box\Box\\ \Box\Box\Box\cdots\Box\\ \Box\end{smallmatrix}, S=\frac{1}{2},\frac{3}{2}\right) \\
&+\cdots+\left(\overbrace{\Box\Box\cdots\Box}^{N_c}, S=\frac{N_c}{2}-1\right) + \left(\begin{smallmatrix}\overbrace{\Box\Box\cdots\Box}^{N_c-1}\\ \Box\end{smallmatrix}, S=\frac{N_c}{2}-1,\frac{N_c}{2}\right) .
\end{aligned} \tag{2.4}
$$

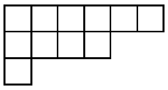
The physical multiplets with well-defined spin $J$ are obtained by adding the orbital angular momentum $\vec{J} = \vec{S} + \vec{L}$ with $L = 1$.

The first three SU(3) representations on the right-hand side of (2.4) correspond for $N_c = 3$ to **1**, **10** and **8** respectively. The others are new and appear only for $N_c > 3$. Their isospin content for each value of the strangeness number $K = n_s/2$ can be read off from the corresponding weight diagrams and is given below for the first few representations.

 $\rightarrow (K = \frac{1}{2}\,, I = 0) + (K = 1\,, I = \frac{1}{2}) + \cdots$ (2.5)

 $\rightarrow (K = 0\,, I = \frac{3}{2}) + (K = \frac{1}{2}\,, I = 1, 2) + (K = 1\,, I = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}) + \cdots$ (2.6)

 $\rightarrow (K = 0\,, I = \frac{1}{2}) + (K = \frac{1}{2}\,, I = 0, 1) + (K = 1\,, I = \frac{1}{2}, \frac{3}{2}) + \cdots$ (2.7)

 $\rightarrow (K = \frac{1}{2}\,, I = 1) + (K = 1\,, I = \frac{1}{2}, \frac{3}{2}) + \cdots\,.$ (2.8)

All the other SU(3) multiplets in (2.4) contain, for $K = 0$, isospin multiplets with $I \geq 2$. Let us consider now in turn the sectors with different values of the strangeness number $K = n_s/2$.

## A. K=0

We list in Table 1 the lowest-lying $K = 0$ p-wave light baryons containing only $u, d$ quarks. They are contained in the SU(3) representations (2.5,2.6,2.7) which will be called in the following **1**, **10** and **8** respectively, corresponding to their dimension for $N_c = 3$.

| State | $(I, J^P)$ | $\Delta$ | $(I, S)$ | $(SU(3), SU(2))$ |
|---|---|---|---|---|
| N(1535) | $(\frac{1}{2}, \frac{1}{2}^-)$ | 1 | $(\frac{1}{2}, \frac{1}{2})$ | $(\mathbf{8}, \mathbf{2})$ |
| N(1520) | $(\frac{1}{2}, \frac{3}{2}^-)$ | | | |
| N(1650) | $(\frac{1}{2}, \frac{1}{2}^-)$ | 0 | | |
| N(1700) | $(\frac{1}{2}, \frac{3}{2}^-)$ | 2 | $(\frac{1}{2}, \frac{3}{2})$ | $(\mathbf{8}, \mathbf{4})$ |
| N(1675) | $(\frac{1}{2}, \frac{5}{2}^-)$ | | | |
| $\Delta(1620)$ | $(\frac{3}{2}, \frac{1}{2}^-)$ | – | $(\frac{3}{2}, \frac{1}{2})$ | $(\mathbf{10}, \mathbf{2})$ |
| $\Delta(1700)$ | $(\frac{3}{2}, \frac{3}{2}^-)$ | – | | |

**Table 1.** The p-wave light baryons containing only $u, d$ quarks and their quantum numbers.

The entries in the last three columns of this table require some explanation. Usually these states are labeled by the quark model quantum numbers $(I, S)$, the total isospin and spin of the quarks. The assignments shown in Table 1 for this quantum number are the conventional ones [13]. Of course, in Nature $S$ is not a good quantum numbers and the physical eigenstates of $(I, J)$ are linear combinations of states with different values of $S$. This mixing is usually considered to have a dynamical origin and is treated in a phenomenological way.

The large-$N_c$ treatment of these states discussed in [6] suggests a different picture. In this approach the physical states are classified into towers of states, each labelled by a spin vector $\Delta$. The members of a given tower have quantum numbers $(I, J)$ which are constrained by the condition $|I - J| \leq \Delta$ and are degenerate in the large-$N_c$ limit. $1/N_c$ corrections will in general remove this degeneracy and will split the states of the tower.

The connection between the tower states and the $(I, S)$ quark model states has been given in [6] for states containing only $u$ and $d$ quarks (Eq.(3.23) in [6])

$$
|I,(PL)\Delta;Jm\alpha\rangle = \tag{2.9}
$$
$$
(-)^{I+P+L+J}\sum_S \sqrt{(2S+1)(2\Delta+1)}\left\{\begin{array}{ccc} I & P & S \\ L & J & \Delta \end{array}\right\}|(IP)S,L;Jm\alpha\rangle\,.
$$

Here $P=1$ is the so-called $P$-spin introduced in [6] to relate $I$ and $S$ for quark model states transforming under the mixed symmetry representation of SU(4). For the p-wave states in Table 1 one has $L=1$. One can see that in general the tower states do not have well-defined values of $S$ and the relation (2.9) yields the following mixing matrices.

The sector $(I,J)=(\frac12,\frac12)$.

$$
|I=\frac12,\Delta=0;J=\frac12\rangle = -\frac{1}{\sqrt3}|I=\frac12,S=\frac12;J=\frac12\rangle + \sqrt{\frac23}|I=\frac12,S=\frac32;J=\frac12\rangle \tag{2.10}
$$
$$
|I=\frac12,\Delta=1;J=\frac12\rangle = \sqrt{\frac23}|I=\frac12,S=\frac12;J=\frac12\rangle + \frac{1}{\sqrt3}|I=\frac12,S=\frac32;J=\frac12\rangle \tag{2.11}
$$

The sector $(I,J)=(\frac12,\frac32)$.

$$
|I=\frac12,\Delta=1;J=\frac32\rangle = -\frac{1}{\sqrt6}|I=\frac12,S=\frac12;J=\frac32\rangle + \sqrt{\frac56}|I=\frac12,S=\frac32;J=\frac32\rangle \tag{2.12}
$$
$$
|I=\frac12,\Delta=2;J=\frac32\rangle = \sqrt{\frac56}|I=\frac12,S=\frac12;J=\frac32\rangle + \frac{1}{\sqrt6}|I=\frac12,S=\frac32;J=\frac32\rangle \tag{2.13}
$$

An examination of the mass spectrum of the $I=\frac12$ states in Table 1 suggests their association into towers of states with the shown values of $\Delta$. The relations (2.10-2.13) give then a prediction for the mixing matrices of these states, which can be compared with experimental data. Adopting the definitions of [12] the mixing of the $N$ states is parametrized as

$$
N(1650) = \cos\theta_{N_1}|S=\frac32\rangle - \sin\theta_{N_1}|S=\frac12\rangle \tag{2.14}
$$
$$
N(1535) = \cos\theta_{N_1}|S=\frac12\rangle + \sin\theta_{N_1}|S=\frac32\rangle \tag{2.15}
$$

and

$$
N(1520) = \cos\theta_{N_3}|S=\frac12\rangle + \sin\theta_{N_3}|S=\frac32\rangle \tag{2.16}
$$
$$
N(1700) = -\sin\theta_{N_3}|S=\frac12\rangle + \cos\theta_{N_3}|S=\frac32\rangle\,. \tag{2.17}
$$

We obtain from (2.10-2.13) the following predictions for the mixing angles $\theta_{N_1}=0.615\,,\theta_{N_3}=1.991$. The fit of [12] to the strong decays of these states gave the results $\theta_{N_1}=0.61\pm0.09$ and $(\theta_{N_3})_{fit\,1}=3.04\pm0.15$, $(\theta_{N_3})_{fit\,2}=2.60\pm0.16$. The result for $\theta_{N_1}$ is in excellent agreement with the data. The disagreement on $\theta_{N_3}$ can probably be ascribed to finite-$N_c$ corrections. Indeed, due to the fictitious nature of the $P$-spin (which becomes apparent in the fact that the states $S=I=N_c/2$ are forbidden), one expects the deviations from the large-$N_c$ mixing (2.9) to be largest for $S$, $I$ approaching their maximal values $N_c/2$.

The $I = 1/2$ states of the towers belong to SU(3) "octets" whose Young diagram is shown in (2.7). There are five such octets, two with $J = 1/2$, two with $J = 3/2$ and one with $J = 5/2$. The large-$N_c$ mass spectrum of the $K = 0$ towers constrains therefore the mass spectrum of these octets, which are predicted to be degenerate in pairs with $J = (1/2, 3/2)$ and $J = (3/2, 5/2)$, corresponding to $\Delta = 1$ and 2 in the $K = 0$ sector respectively. This is very different from the picture suggested by the quark model, where one expects these octets to fall into two groups with $J = (1/2, 3/2)$ and $J = (1/2, 3/2, 5/2)$, corresponding to the two values taken by the total quark spin $S = 1/2, 3/2$. One problem with the quark model picture is the inversion of the two levels with $J = 3/2$ and $J = 5/2$, which is difficult to understand by assuming a spin-orbit interaction alone [9,10].

The mixing of the octets with identical values of $J$ can be predicted from the mixings in the $K = 0$ sector (2.10-2.13). These relations can be extended to all the states in these multiplets as

$$|\mathbf{8}, J = \frac{1}{2}\rangle_{\Delta=0} = -\frac{1}{\sqrt{3}}|\mathbf{8}, J = \frac{1}{2}\rangle_{S=1/2} + \sqrt{\frac{2}{3}}|\mathbf{8}, J = \frac{1}{2}\rangle_{S=3/2} \tag{2.18}$$

$$|\mathbf{8}, J = \frac{1}{2}\rangle_{\Delta=1} = \sqrt{\frac{2}{3}}|\mathbf{8}, J = \frac{1}{2}\rangle_{S=1/2} + \frac{1}{\sqrt{3}}|\mathbf{8}, J = \frac{1}{2}\rangle_{S=3/2} \tag{2.19}$$

and

$$|\mathbf{8}, J = \frac{3}{2}\rangle_{\Delta=1} = -\frac{1}{\sqrt{6}}|\mathbf{8}, J = \frac{3}{2}\rangle_{S=1/2} + \sqrt{\frac{5}{6}}|\mathbf{8}, J = \frac{3}{2}\rangle_{S=3/2} \tag{2.20}$$

$$|\mathbf{8}, J = \frac{3}{2}\rangle_{\Delta=2} = \sqrt{\frac{5}{6}}|\mathbf{8}, J = \frac{3}{2}\rangle_{S=1/2} + \frac{1}{\sqrt{6}}|\mathbf{8}, J = \frac{3}{2}\rangle_{S=3/2} . \tag{2.21}$$

The notation $|\mathbf{8}, J\rangle_\Delta$ does not imply that all the states of the $\mathbf{8}$ belong to a $\Delta$-tower but only labels the SU(3) representation in terms of its $K = 0$ members.

Unfortunately, no unambiguous tower assignments can be made for the excited $I = \frac{3}{2}$ $\Delta$ baryons. Because of the small value of $N_c$ in the real world the tower structure for $I = \frac{3}{2}$ is incomplete. For example, instead of a total number of two states with $(I, J) = (\frac{3}{2}, \frac{1}{2})$ expected in the large-$N_c$ limit, there is only one such state. To fill up all the $I = \frac{3}{2}$ members of the towers with $\Delta = 0, 1, 2$, additional states would be required with $(I, S) = (\frac{3}{2}, \frac{3}{2}), (\frac{3}{2}, \frac{5}{2})$, which however do not appear for $N_c = 3$. This problem did not exist for $s$-wave baryons and has as consequence an unfortunate loss of predictive power for the large-$N_c$ expansion when applied to the excited baryons.

### B. K=1/2

The observed and expected p-wave baryons with one strange quark are listed in Table 2, together with their quantum numbers. In the quark model these states are labelled by $(I, S)$ with $S$ the total spin of the quarks in the baryon. As discussed above, physical states are in general linear combinations of quark model states with different values of $S$. The large-$N_c$ expansion combined with SU(3) symmetry can be used to predict this mixing.

From the point of view of large-$N_c$ QCD the observed $K = 1/2$ states fall into 7 towers of states, three towers with $\Delta = 1/2$, three towers with $\Delta = 3/2$ and one tower with $\Delta = 5/2$.

Although the tower structure is complete only for the lowest value of the isospin $I = 0$, we can use SU(3) symmetry to assign the $\Sigma$ states in the octets well-defined values of $\Delta$. However, just as in the case of the $I = 3/2$ states in the $K = 0$ sector, this cannot be done in an unambiguous way for the decuplet baryons. Therefore we cannot make predictions for the couplings of these states.

States with the same quantum numbers will mix in the general case. We parametrize this mixing in the $I = 0$ sector as in [12] in terms of six angles. For the $J = 1/2$ states we introduce three angles $\theta_{1i}$ with $i = 1, 2, 3$ as

$$\begin{pmatrix} \Lambda(1670) \\ \Lambda(1800) \\ \Lambda(1405) \end{pmatrix} = \begin{pmatrix} c_{11}c_{12} & s_{11}c_{12} & s_{12} \\ -s_{11}c_{13} - c_{11}s_{13}s_{12} & c_{11}c_{13} - s_{11}s_{12}s_{13} & s_{13}c_{12} \\ s_{11}s_{13} - c_{11}c_{13}s_{12} & -c_{11}s_{13} - s_{11}c_{13}s_{12} & c_{13}c_{12} \end{pmatrix} \begin{pmatrix} \Lambda_{11} \\ \Lambda_{31} \\ \mathrm{Singlet}_{11} \end{pmatrix}, \qquad (2.22)$$

with $c_{11} = \cos\theta_{11}\,, s_{11} = \sin\theta_{11}$, etc.

The quark model states on the RHS are denoted as $\Lambda_{2S,2J}$. In the SU(3) limit two of the angles vanish $\theta_{12} = \theta_{13} = 0$, as there is no mixing between the singlet and octet. The third angle $\theta_{11}$ can be determined by noting that some of the $I = 0$ states $\Lambda$ belong to the same SU(3) "octets" as the $K = 0$ states. Therefore (2.18-2.21) can be used to obtain their relation to the quark model states with well-defined $S$ and we find $\theta_{11} = 0.615$.

| State | $(I, J^P)$ | $\Delta$ | $(I, S)$ | $(SU(3), SU(2))$ |
|---|---|---|---|---|
| $\Lambda(1405)$ | $(0, \frac{1}{2}^-)$ | $\frac{1}{2}$ | $(0, \frac{1}{2})$ | $(\mathbf{1}, \mathbf{2})$ |
| $\Lambda(1520)$ | $(0, \frac{3}{2}^-)$ | $\frac{3}{2}$ | | |
| $\Lambda(1670)$ | $(0, \frac{1}{2}^-)$ | $\frac{1}{2}$ | $(0, \frac{1}{2})$ | $(\mathbf{8}, \mathbf{2})$ |
| $\Sigma(1620)$ | $(1, \frac{1}{2}^-)$ | | $(1, \frac{1}{2})$ | |
| $\Lambda(1690)$ | $(0, \frac{3}{2}^-)$ | $\frac{3}{2}$ | $(0, \frac{1}{2})$ | |
| $\Sigma(1670)$ | $(1, \frac{3}{2}^-)$ | | $(1, \frac{1}{2})$ | |
| $\Lambda(1800)$ | $(0, \frac{1}{2}^-)$ | $\frac{1}{2}$ | $(0, \frac{3}{2})$ | $(\mathbf{8}, \mathbf{4})$ |
| $\Sigma(1750)$ | $(1, \frac{1}{2}^-)$ | | $(1, \frac{3}{2})$ | |
| $\Lambda(?)$ | $(0, \frac{3}{2}^-)$ | $\frac{3}{2}$ | $(0, \frac{3}{2})$ | |
| $\Sigma(?)$ | $(1, \frac{3}{2}^-)$ | | $(1, \frac{3}{2})$ | |
| $\Lambda(1830)$ | $(0, \frac{5}{2}^-)$ | $\frac{5}{2}$ | $(0, \frac{3}{2})$ | |
| $\Sigma(1775)$ | $(1, \frac{5}{2}^-)$ | | $(1, \frac{3}{2})$ | |
| $\Sigma(?)$ | $(1, \frac{1}{2}^-)$ | – | $(1, \frac{1}{2})$ | $(\mathbf{10}, \mathbf{2})$ |
| $\Sigma(?)$ | $(1, \frac{3}{2}^-)$ | – | | |

**Table 2.** The p-wave hyperons containing one strange quark and their quantum numbers. $(I, S)$ denote the usual quark model assignments of the states and $\Delta$ gives their large-$N_c$ tower assignment.

The sector $J = 3/2$ can be treated in an analogous way. The mixing of these states is parametrized in terms of three angles $\theta_{3i}$ defined as

$$\begin{pmatrix} \Lambda(1690) \\ \Lambda(?) \\ \Lambda(1520) \end{pmatrix} = R(\theta_{31}, \theta_{32}, \theta_{33}) \begin{pmatrix} \Lambda_{13} \\ \Lambda_{33} \\ \mathrm{Singlet}_{13} \end{pmatrix} \qquad (2.23)$$

where the unitary matrix $R$ is defined in analogy to the one in (2.22). We find for this case, in the limit of SU(3) symmetry, $\theta_{31} = 1.991\,, \theta_{32} = \theta_{33} = 0$. Similar predictions can be made in the limit of SU(3) symmetry for the mixing matrix of the $\Sigma$ states.

The experimental situation with these angles is not very clear. The fit of [12] gave six different possible solutions for the $\theta_{1i}$ and four solutions for $\theta_{3i}$. The values taken by the angles $\theta_{i2}$, $\theta_{i3}$ in these solutions do not come close to the SU(3) value (0), which can be explained by a sizable violation of SU(3) symmetry. This implies in turn the existence of similar large deviations from the SU(3)-based prediction for $\theta_{i1}$. However, the large-$N_c$ predictions for decays of tower states to be presented in the next Section do not depend on a precise knowledge of the mixing matrix.

## III. STRONG DECAYS

Let us first recapitulate the results obtained in [6] for strong decays of excited baryons in the large-$N_c$ limit by assuming only isospin symmetry. Excited baryons can decay to s-wave baryons through pion emission in $S$-wave and $D$-wave. The respective couplings are related to matrix elements of the axial current taken between tower states ($\Delta \to \Delta'$)

$$\langle J'I'; m', \alpha' | \bar{q}\gamma^0\gamma_5 \frac{1}{2}\tau^a q | JI; m, \alpha\rangle = N_c^\kappa \langle J'I'; m', \alpha' | Y^a | JI; m, \alpha\rangle \tag{3.1}$$

$$\begin{aligned}\langle J'I'; m', \alpha' | \bar{q}\gamma^i\gamma_5 \frac{1}{2}\tau^a q | JI; m, \alpha\rangle &= N_c^\kappa q^j \langle J'I'; m', \alpha' | Q^{ij,a} | JI; m, \alpha\rangle \\ &\quad + N_c^\kappa \epsilon_{ijk} q^j \langle J'I'; m', \alpha' | R^{k,a} | JI; m, \alpha\rangle\end{aligned} \tag{3.2}$$

with $q^\mu$ the momentum of the current. $\kappa$=0 for a decaying state transforming under the mixed symmetry representation of SU(4). The operators $Y^a$ and $Q^{ij,a}$ parametrize the $S$-wave and $D$-wave pion couplings respectively. Their matrix elements are determined, at leading order in $N_c$, by four reduced matrix elements $c(\Delta', \Delta), c_{1-3}(\Delta', \Delta)$

$$\langle J'I'; m', \alpha' | Y^a | JI; m, \alpha\rangle = c(\Delta', \Delta)\sqrt{2I+1}(-)^{I-J-\Delta'}\delta_{JJ'}\delta_{mm'} \left\{ \begin{array}{ccc} I' & 1 & I \\ \Delta & J & \Delta' \end{array} \right\} \langle I'\alpha' | I1; \alpha a\rangle \tag{3.3}$$

$$\begin{aligned}\langle J'I'; m', \alpha' | Q^{ka} | JI; m, \alpha\rangle &= (-)^{J+I+J'+I'}\sqrt{(2J+1)(2I+1)} \\ &\quad \times \sum_{y=1,2,3} c_y(\Delta', \Delta) \left\{ \begin{array}{ccc} \Delta' & I' & J' \\ \Delta & I & J \\ y & 1 & 2 \end{array} \right\} \langle J'm' | J2; mk\rangle \langle I'\alpha' | I1; \alpha a\rangle\,.\end{aligned} \tag{3.4}$$

In the following we will extend these results to the case of SU(3) symmetry. As explained above, we will restrict our considerations to octet and singlet states. There are five octets and two singlets, which will be represented by SU(3) tensors constructed as in [4].

The spin-1/2 octet whose $K = 0$ members belong to the $\Delta = 0$ tower will be represented by the tensor $(\mathcal{B}_1)^i_{j_1 j_2 \cdots j_\nu}$ with one upper and $\nu = (N_c - 1)/2$ lower indices. The two spin-1/2 and 3/2 octets whose $K = 0$ members belong to the $\Delta = 1$ tower are represented by the tensors $(\mathcal{B}_2)^i_{j_1 j_2 \cdots j_\nu}$ and $(\mathcal{B}_3)^i_{j_1 j_2 \cdots j_\nu}$ respectively. Finally, the two spin-3/2 and 5/2 octets whose $K = 0$ members belong to the $\Delta = 2$ tower will be assigned the tensors $(\mathcal{B}_4)^i_{j_1 j_2 \cdots j_\nu}$ and $(\mathcal{B}_5)^i_{j_1 j_2 \cdots j_\nu}$ respectively.

The spin-1/2 and 3/2 singlet baryons are each represented by a SU(3) tensor with $\nu-1$ lower indices $(\mathcal{S}_1)_{j_1 j_2 \cdots j_{\nu-1}}$ and $(\mathcal{S}_2)_{j_1 j_2 \cdots j_{\nu-1}}$. The nonvanishing components of these tensors for the $\Lambda$ states are $\mathcal{S}_{33\ldots 3} = 1$. For $N_c = 3$ these tensors go over into SU(3) scalars, as they should.

The s-wave baryons are represented by the usual octet tensor $\mathcal{B}^i_{j_1 j_2 \cdots j_\nu}$ (for the spin-1/2 baryons) and the decuplet tensor $\mathcal{T}^{i_1 i_2 i_3}_{j_1 j_2 \cdots j_{\nu-1}}$ (for the spin-3/2 baryons).

The couplings of the Goldstone bosons are described by interaction Lagrangians built out of the SU(3) tensors introduced above. The part containing the $S$-wave couplings is written in terms of seven SU(3) invariants $\mathcal{M}_{1,2}\,, \mathcal{N}_{1,2}\,, \mathcal{L}_{1,2}\,, \mathcal{P}_1$ as

$$\begin{aligned}
\mathcal{L}_S = {} & \mathcal{M}_1 \mathrm{tr}\ (\bar{\mathcal{B}}\gamma_\mu A^\mu \mathcal{B}_1) + \mathcal{N}_1 \mathrm{tr}\ (\bar{\mathcal{B}}\gamma_\mu \mathcal{B}_1 A^\mu) \\
& + \mathcal{M}_2 \mathrm{tr}\ (\bar{\mathcal{B}}\gamma_\mu A^\mu \mathcal{B}_2) + \mathcal{N}_2 \mathrm{tr}\ (\bar{\mathcal{B}}\gamma_\mu \mathcal{B}_2 A^\mu) \\
& + \mathcal{L}_1 \mathrm{tr}\ (\bar{\mathcal{T}}_\mu \gamma_\nu A^\nu \mathcal{B}_3^\mu) + \mathcal{L}_2 \mathrm{tr}\ (\bar{\mathcal{T}}_\mu \gamma_\nu A^\nu \mathcal{B}_4^\mu) \\
& + \mathcal{P}_1 \mathrm{tr}\ (\bar{\mathcal{B}}\gamma_\mu A^\mu \mathcal{S}_1)\,.
\end{aligned} \tag{3.5}$$

The nonlinear axial current field $A_\mu$ is defined by $A_\mu = i/2(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger)$ with $\xi = \exp(iM/f_\pi)$ and $f_\pi = 132$ MeV. The matrix $M$ contains the Goldstone boson fields and is given by $M = \frac{1}{\sqrt{2}}\pi^a \lambda^a$.

The $D$-wave couplings of the Goldstone bosons are described by an analogous Lagrangian containing twelve SU(3) invariants

$$\begin{aligned}
\mathcal{L}_D = {} & m_B \mathcal{M}_3 \mathrm{tr}\ (\bar{\mathcal{B}} A^\mu \gamma_5 \mathcal{B}_3^\mu) + m_B \mathcal{N}_3 \mathrm{tr}\ (\bar{\mathcal{B}}\gamma_5 \mathcal{B}_3^\mu A_\mu) \\
& + m_B \mathcal{M}_4 \mathrm{tr}\ (\bar{\mathcal{B}} A^\mu \gamma_5 \mathcal{B}_4^\mu) + m_B \mathcal{N}_4 \mathrm{tr}\ (\bar{\mathcal{B}}\gamma_5 \mathcal{B}_4^\mu A_\mu) \\
& + \mathcal{M}_5 \mathrm{tr}\ (\bar{\mathcal{B}}(D_\mu A_\nu + D_\nu A_\mu)\mathcal{B}_5^{\mu\nu}) + \mathcal{N}_5 \mathrm{tr}\ (\bar{\mathcal{B}}\mathcal{B}_5^{\mu\nu}(D_\mu A_\nu + D_\nu A_\mu)) \\
& + m_T \mathcal{L}_3 \mathrm{tr}\ (\bar{\mathcal{T}}_\mu A^\mu \gamma_5 \mathcal{B}_1) + m_T \mathcal{L}_4 \mathrm{tr}\ (\bar{\mathcal{T}}_\mu A^\mu \gamma_5 \mathcal{B}_2) \\
& + i\mathcal{L}_5 \mathrm{tr}\ (\bar{\mathcal{T}}^\mu (D_\mu A_\nu + D_\nu A_\mu + \text{ t.t.})\mathcal{B}_3^\nu) + i\mathcal{L}_6 \mathrm{tr}\ (\bar{\mathcal{T}}^\mu (D_\mu A_\nu + D_\nu A_\mu + \text{ t.t.})\mathcal{B}_4^\nu) \\
& + \mathcal{L}_7 i\varepsilon_{\alpha\beta\gamma\delta} \mathrm{tr}\ (\bar{\mathcal{T}}^\alpha (D^\rho A^\beta + D^\beta A^\rho) v^\gamma B_5^{\delta\rho}) + m_B \mathcal{P}_2 \mathrm{tr}\ (\bar{\mathcal{B}}\gamma_5 A_\mu \mathcal{S}_2^\mu)\,.
\end{aligned} \tag{3.6}$$

We extracted factors of $m_B, m_T$ in the definition of some couplings such that their expansion in powers of $1/N_c$ starts with a term of O(1). The form of the trace terms "t.t.", needed to project out a pure $D$-wave, is given in the Appendix. In these expressions only the Lorentz indices are written explicitly. The traces over the SU(3) indices have the following form:

a) octet-octet coupling

$$\mathrm{tr}\ (\bar{\mathcal{B}} A \mathcal{B}_1) = \bar{\mathcal{B}}_a^{b_1 b_2 \cdots b_\nu} A_c^a (\mathcal{B}_1)^c_{b_1 b_2 \cdots b_\nu}\,, \qquad \mathrm{tr}\ (\bar{\mathcal{B}} \mathcal{B}_1 A) = \bar{\mathcal{B}}_a^{c b_2 \cdots b_\nu} (\mathcal{B}_1)^a_{d b_2 \cdots b_\nu} A_c^d$$

b) octet-decuplet coupling

$$\mathrm{tr}\ (\bar{\mathcal{T}} A \mathcal{B}_1) = \varepsilon^{\alpha\beta\gamma} \bar{\mathcal{T}}_{\alpha\mu\nu}^{b_1 b_2 \cdots b_{\nu-1}} A_\beta^\mu (\mathcal{B}_1)^\nu_{\gamma b_1 b_2 \cdots b_{\nu-1}}$$

c) octet-singlet coupling

$$\mathrm{tr}\ (\bar{\mathcal{B}} A \mathcal{S}) = \bar{\mathcal{B}}_a^{b_1 b_2 \cdots b_\nu} A_{b_1}^a \mathcal{S}_{b_2 \cdots b_\nu}\,.$$

The interplay of the large-$N_c$ predictions (3.3,3.4) with the SU(3) symmetry leads to significant simplifications in the structure of the Lagrangian (3.5,3.6). Thus, the $S$-wave pion couplings of the excited baryon octets to ground state baryons are described in this limit by just one common reduced matrix element (instead of five, assuming only isospin invariance) and in the $D$-wave sector only two independent couplings are required (instead of seven).

These additional relations can be derived by writing representative transition amplitudes in two alternative ways, using the SU(3) and SU(2) relations respectively. We obtain in this way the following model-independent predictions for the $S$-wave couplings

$$\mathcal{M}_1 = \mathcal{O}(1/N_c) \tag{3.7}$$
$$\frac{\mathcal{M}_2}{\mathcal{L}_1} = -\frac{2}{\sqrt{3}} + \mathcal{O}(1/N_c) \tag{3.8}$$
$$\mathcal{L}_2 = \mathcal{O}(1/N_c) \tag{3.9}$$

and for the $D$-wave couplings

$$\mathcal{L}_3 = \mathcal{O}(1/N_c) \tag{3.10}$$
$$\frac{\mathcal{M}_3}{\mathcal{L}_5} = -\frac{8}{3} + \mathcal{O}(1/N_c) \tag{3.11}$$
$$\frac{\mathcal{M}_3}{\mathcal{L}_4} = -\frac{2}{\sqrt{3}} + \mathcal{O}(1/N_c) \tag{3.12}$$
$$\frac{\mathcal{M}_4}{\mathcal{L}_6} = -\frac{4}{3} + \mathcal{O}(1/N_c) \tag{3.13}$$
$$\frac{\mathcal{M}_4}{\mathcal{L}_7} = 4\sqrt{\frac{2}{5}} + \mathcal{O}(1/N_c) \tag{3.14}$$
$$\frac{\mathcal{M}_5}{\mathcal{L}_7} = -\frac{2}{3} + \mathcal{O}(1/N_c)\,. \tag{3.15}$$

The $\mathcal{N}$ parameters in the Lagrangians (3.5,3.6) contribute to the pion couplings only to subleading order (although they contribute to the same order as $\mathcal{M}$ to the kaon couplings). Therefore, in order to obtain information about them, knowledge of the pion couplings to next-to-leading order in $1/N_c$ is required. This will have to be obtained from model calculations.

In practice the $1/N_c$ corrections to the predictions (3.7-3.9), (3.10-3.15) can be sizable. In the following we compare these predictions against available experimental data on strong decays of these states. To avoid additional complications related to SU(3) breaking effects and a more complex mixing structure, we will restrict ourselves to pion decays of nonstrange excited baryons.

The relation (3.8) between $S$-wave amplitudes can be tested by examining the ratio of decay widths

$$(R_1)_{th} = \frac{\Gamma(N(1535) \to [N\pi])}{\Gamma(N(1520) \to [\Delta\pi]_S)} = 5.227\frac{\mathcal{M}_2^2}{\mathcal{L}_1^2} = 6.969\,. \tag{3.16}$$

We used on the RHS the theoretical expression for the widths together with the coupling ratio (3.8). The experimental value of this ratio is [13]

$$(R_1)_{exp} = 6.625^{+18.35}_{-4.46}\,. \tag{3.17}$$

Not all relations for $S$-wave couplings work as well. For example, one expects from (3.7) the coupling $\mathcal{M}_1$ to be suppressed by $1/N_c$ relative to $\mathcal{M}_2$. However, the corresponding ratio of decay widths

$$(R_2)_{th} = \frac{\Gamma(N(1650) \to [N\pi])}{\Gamma(N(1535) \to [N\pi])} = 1.58 \frac{\mathcal{M}_1^2}{\mathcal{M}_2^2} \tag{3.18}$$

takes the experimental value $(R_2)_{exp} = 0.58 - 4.88$, which is at least a factor of 4 larger than the one obtained with the naive estimate $\mathcal{M}_1^2/\mathcal{M}_2^2 \simeq 0.1$.

The situation with the prediction (3.9) is less clear, as the PDG does not quote branching ratios for the decay mode $N(1700) \to [\Delta\pi]_{S,D}$. The $S$-wave mode appears however to be suppressed in comparison to the $D$-wave one [11], in agreement with the large-$N_c$ expectation from (3.9).

This analysis can be extended to the $D$-wave couplings. The following ratios of decay widths can be used to test (3.11), (3.12), (3.14) and (3.15).

$$(R_3)_{th} = \frac{\Gamma(N(1520) \to [N\pi]_D)}{\Gamma(N(1520) \to [\Delta\pi]_D)} = 2.151 \frac{\mathcal{M}_3^2}{\mathcal{L}_5^2} = 15.30\,, \quad (R_3)_{exp} = 3.57 - 6.01 \tag{3.19}$$

$$(R_4)_{th} = \frac{\Gamma(N(1520) \to [N\pi]_D)}{\Gamma(N(1535) \to [\Delta\pi]_D)} = 4.216 \frac{\mathcal{M}_3^2}{\mathcal{L}_4^2} = 5.62\,, \quad (R_4)_{exp} \geq 4.4 \tag{3.20}$$

$$(R_5)_{th} = \frac{\Gamma(N(1675) \to [N\pi]_D)}{\Gamma(N(1675) \to [\Delta\pi]_D)} = 4.595 \frac{\mathcal{M}_5^2}{\mathcal{L}_7^2} = 2.042\,, \quad (R_5)_{exp} = 0.66 - 1.00 \tag{3.21}$$

$$(R_6)_{th} = \frac{\Gamma(N(1700) \to [N\pi]_D)}{\Gamma(N(1675) \to [\Delta\pi]_D)} = 0.883 \frac{\mathcal{M}_4^2}{\mathcal{L}_7^2} = 5.651\,, \quad (R_6)_{exp} = 0.055 - 0.2\,. \tag{3.22}$$

We do not present a comparison with data for the ratio (3.13) because of the lack of data on $N(1700) \to [\Delta\pi]_D$.

The deviations of these ratios from the large-$N_c$ predictions can be understood partly as a consequence of the finite value of $N_c$ and partly because of the sensitivity of these ratios to the precise value of the mixing angle $\theta_{N_3}$. We will use in the following the quark model with $N_c = 3$ to illustrate the importance of the $1/N_c$ corrections. The couplings of the $N^*$ states are related in the quark model to the reduced matrix elements $\mathcal{T}(I', SI)$ introduced in [6]. Their explicit formulas for arbitrary $N_c$ are (normalized to (3.50) of [6] in the large-$N_c$ limit)

$$\mathcal{T}(\frac{1}{2}, \frac{1}{2}\frac{1}{2}) = -\frac{2\sqrt{2}}{3}\sqrt{\frac{(N_c-1)(N_c+3)}{N_c(N_c+2)}}\mathcal{I}\,, \qquad \mathcal{T}(\frac{1}{2}, \frac{3}{2}\frac{1}{2}) = -\frac{2}{3}\sqrt{\frac{N_c-1}{N_c+2}}\mathcal{I}\,, \tag{3.23}$$

$$\mathcal{T}(\frac{3}{2}, \frac{1}{2}\frac{1}{2}) = \frac{2}{3}\sqrt{\frac{(N_c+3)(N_c+5)}{N_c(N_c+2)}}\mathcal{I}\,, \qquad \mathcal{T}(\frac{3}{2}, \frac{3}{2}\frac{1}{2}) = -\frac{2}{3}\sqrt{5}\sqrt{\frac{N_c+5}{N_c+2}}\mathcal{I}$$

with $\mathcal{I}$ a common overlap integral. We obtain for example for the ratio (3.8) of the $S$-wave couplings

$$\frac{\mathcal{M}_2}{\mathcal{L}_1} = -2\sqrt{\frac{2}{3}}\frac{\mathcal{T}(\frac{1}{2}, \frac{1}{2}\frac{1}{2})\cos\theta_{N_1} + \mathcal{T}(\frac{1}{2}, \frac{3}{2}\frac{1}{2})\sin\theta_{N_1}}{\mathcal{T}(\frac{3}{2}, \frac{1}{2}\frac{1}{2})\cos\theta_{N_3} + \mathcal{T}(\frac{3}{2}, \frac{3}{2}\frac{1}{2})\sin\theta_{N_3}}\,. \tag{3.24}$$

In the large-$N_c$ limit and for the mixing angles given in Sec.II.A the value of this ratio reduces to (3.8). For $N_c = 3$ it gives [7,8]

$$\begin{aligned}\frac{\mathcal{M}_2}{\mathcal{L}_1} &= \sqrt{\frac{2}{3}}\frac{2\cos\theta_{N_1} + \sin\theta_{N_1}}{\sqrt{2}\cos\theta_{N_3} - \sqrt{5}\sin\theta_{N_3}} \\ &= -0.689(\theta_{N_3} = 1.991)\,, \quad -0.763(\theta_{N_3} = 2.6)\,, \quad -1.105(\theta_{N_3} = 3.04)\,.\end{aligned} \tag{3.25}$$

The numerical values shown are computed with the large-$N_c$ value for $\theta_{N_1} = 0.615$ which was seen to agree well with the experimental one. For the largest value of $\theta_{N_3} = 3.04$, the ratio (3.25) predicts $(R_1)_{th} = 6.382$ which is in good agreement with the experimental value (3.17).

The ratio $\mathcal{M}_1/\mathcal{M}_2$ depends only on the angle $\theta_{N_1}$ and is given by

$$\begin{aligned}\frac{\mathcal{M}_1}{\mathcal{M}_2} &= \frac{-\mathcal{T}(\frac{1}{2},\frac{1}{2}\frac{1}{2})\sin\theta_{N_1} + \mathcal{T}(\frac{1}{2},\frac{3}{2}\frac{1}{2})\cos\theta_{N_1}}{\mathcal{T}(\frac{1}{2},\frac{1}{2}\frac{1}{2})\cos\theta_{N_1} + \mathcal{T}(\frac{1}{2},\frac{3}{2}\frac{1}{2})\sin\theta_{N_1}} \\ &\to -\frac{2\sin\theta_{N_1} - \cos\theta_{N_1}}{2\cos\theta_{N_1} + \sin\theta_{N_1}} = (-0.056) - (-0.241)\,, \qquad (N_c = 3)\,.\end{aligned} \tag{3.26}$$

In the last line we used the experimental value $\theta_{N_1} = 0.61 \pm 0.09$ [12]. This yields in turn a result for the ratio (3.18) $(R_2)_{th} = 0.005 - 0.092$, which is still smaller than the experimental value $(R_2)_{exp} = 0.58 - 4.88$. We will return later to a discussion of this discrepancy.

Similar results are obtained for the ratios of $D$-wave couplings. For example, we get

$$\frac{\mathcal{M}_3}{\mathcal{L}_5} = -\frac{4}{3}\frac{2\sqrt{5}\mathcal{T}(\frac{1}{2},\frac{1}{2}\frac{1}{2})\cos\theta_{N_3} - \sqrt{2}\mathcal{T}(\frac{1}{2},\frac{3}{2}\frac{1}{2})\sin\theta_{N_3}}{\sqrt{\frac{5}{2}}\mathcal{T}(\frac{3}{2},\frac{1}{2}\frac{1}{2})\cos\theta_{N_3} - 2\sqrt{\frac{2}{5}}\mathcal{T}(\frac{3}{2},\frac{3}{2}\frac{1}{2})\sin\theta_{N_3}}\,. \tag{3.27}$$

Taking in this expression $N_c = 3$ gives

$$\begin{aligned}\frac{\mathcal{M}_3}{\mathcal{L}_5} &= -\frac{4}{3}\frac{-2\sqrt{10}\cos\theta_{N_3} + \sin\theta_{N_3}}{\sqrt{10}\cos\theta_{N_3} + 4\sin\theta_{N_3}} \\ &= -1.972(\theta_{N_3} = 1.991)\,, \quad 12.217(\theta_{N_3} = 2.6)\,, \quad 2.493 - 4.112(\theta_{N_3} = 3.04 \pm 0.15)\,.\end{aligned} \tag{3.28}$$

This ratio is particularly sensitive to the mixing angle $\theta_{N_3}$ as the physical value of this angle lies in the vecinity of 2.47, where the denominator vanishes. The ratio $R_3$ corresponding to $\theta_{N_3} = 3.04 \pm 0.15$ is still larger by about a factor of 2 than the experimental value (3.19). Similar large values for $R_3$ appear to be predicted also in other quark model calculations [12].

The ratio (3.15) of the couplings of the $J^P = 5/2^-$ state is given in the quark model by

$$\frac{\mathcal{M}_5}{\mathcal{L}_7} = -\frac{2}{3}\sqrt{5}\frac{\mathcal{T}(\frac{1}{2},\frac{3}{2}\frac{1}{2})}{\mathcal{T}(\frac{3}{2},\frac{3}{2}\frac{1}{2})} = -\frac{2}{3}\sqrt{\frac{N_c - 1}{N_c + 5}}\,. \tag{3.29}$$

For $N_c = 3$ this implies $(R_5)_{th} = 0.510$ which is in reasonable agreement (although somewhat smaller) with the experimental result (3.21).

Finally, the ratio (3.14) is given by

$$\frac{\mathcal{M}_4}{\mathcal{L}_7} = \frac{8}{3\sqrt{3}} \frac{\sqrt{10}\mathcal{T}(\frac{1}{2},\frac{1}{2}\frac{1}{2})\sin\theta_{N_3} + \mathcal{T}(\frac{1}{2},\frac{3}{2}\frac{1}{2})\cos\theta_{N_3}}{\mathcal{T}(\frac{3}{2},\frac{3}{2}\frac{1}{2})} \tag{3.30}$$

which for $N_c = 3$ reduces to

$$\frac{\mathcal{M}_4}{\mathcal{L}_7} = \frac{4}{3\sqrt{15}}(\cos\theta_{N_3} + 2\sqrt{10}\sin\theta_{N_3}) = 1.847(\theta_{N_3} = 1.991)\,, \tag{3.31}$$
$$0.491 - 1.142(\theta_{N_3} = 2.6 \pm 0.16)\,, \qquad (-0.449) - (0.209)(\theta_{N_3} = 3.04 \pm 0.15)\,.$$

For $\theta_{N_3} = 3.04 \pm 0.15$ this gives $(R_6)_{th} = 0.038 - 0.178$ which is in agreement with the experimental value (3.22).

Perhaps the most puzzling disagreement between the large-$N_c$ predictions and experiment concerns the large experimental value of the ratio $R_2$ (3.18). Among the possible explanations for this disagreement, we can mention: a) wrong assignments of the $\Delta$ quantum numbers for the $S_{11}$ states; b) a large deviation of the mixing angle $\theta_{N_1}$ from its predicted value $\theta_{N_1} = 0.615$; c) the presence of a third $S_{11}$ state in the region around 1.6 GeV. The first possibility entails assigning $\Delta = 1$ to $N(1650)$ and $\Delta = 0$ to $N(1535)$, which results into the prediction $\theta_{N_1} = -0.955$. This would give in turn a value for the ratio (3.18) $(R_2)_{th} = 67.47$ which is almost a factor of 5 larger than the one obtained with the dimensional estimate $\mathcal{M}_1/\mathcal{M}_2 \simeq N_c = 3$. The second alternative b) requires the angle $\theta_{N_1}$ to be of the order of $-0.08$ or $1.04$. Furthermore, the large splitting between the members of the $\Delta = 1$ tower in the case a) together with the large disagreement in the value of $\theta_{N_1}$ with other determinations [12] combine to make these two possible explanations rather unattractive.

Recent analyses of the $\pi N$ scattering data [14] show evidence for a new $J^P = 1/2^-$ state with a mass of 1712 MeV. Since its mass is very close to that of $N(1650)$, it is possible that the data quoted by the PDG [13] referring to the latter in fact cummulates over the decays of both states. It is interesting to note that the new state has a small branching ratio for decays into the $N\pi$ mode, of about 20% [14], which fits the large-$N_c$ prediction for the $\Delta = 0$ state. It is tempting therefore to identify this state with the $J = 1/2$ member of the $\Delta = 0$ tower. It is not yet clear what the quark model interpretation of each of the three $S_{11}$ states is (for example, in [15] it is proposed to interpret one of them as a bound state $\Sigma K$, see also [16]). Further investigation of these states is required to help settle this apparent puzzle of the large-$N_c$ expansion.

## CONCLUSIONS

We have analyzed in this paper the phenomenological consequences of the large-$N_c$ expansion for the $L = 1$ orbitally excited baryons, following from the formalism described in [6]. These states are organized into towers of states, whose couplings to the ground state baryons are related in a simple way. In the large-$N_c$ limit the members of a given tower are degenerate, which yields constraints on the masses of these states which are distinct from those of the quark model with SU(6) symmetry. Quite remarkably, the mixing angles of the five octets of $L = 1$ excited baryons are completely predicted in the combined large-$N_c$ and SU(3) limits. Unfortunately, because of the small value of the $N_c$ parameter in the real world, we cannot accomodate the decuplet states into the picture suggested by large-$N_c$ QCD. Despite these shortcomings, we believe that this approach could be used (much in the same way as done in [17] for the ground state baryons) as the starting point for a systematic study of the $1/N_c$ and SU(3) breaking corrections for these states.

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## APPENDIX A

We present in this Appendix the partial wave decomposition for the decay $3/2^- \to (3/2^+, 0^-)$ which can proceed through both $S$- and $D$-wave. The invariant transition matrix element is decomposed as

$$\mathcal{M} = \bar{u}^\mu(p')\left\{c_D\left(q_\mu q_\nu + 2\vec{q}^{\,2}\frac{m_P^2}{m_P^2+4m_Pm_S+m_S^2-q^2}g_{\mu\nu}\right)\right. \\ \left. +c_S\left(g_{\mu\nu}+\frac{2}{(m_S+m_P)^2-q^2}q_\mu q_\nu\right)\right\}u^\nu(p)\,, \tag{A1}$$

with $\vec{q}$ the pion 3-momentum in the rest frame of the decaying particle. The masses of the initial and final particles are denoted as $m_P$ and $m_S$ respectively. The partial decay widths are given by

$$\Gamma_S = \frac{1}{8\pi}c_S^2\frac{(m_S+m_P)^2-q^2}{m_P^2}|\vec{q}\,| \tag{A2}$$

$$\Gamma_D = \frac{1}{2\pi}c_D^2\frac{m_P^2[(m_S+m_P)^2-q^2]}{(m_P^2+4m_Pm_S+m_S^2-q^2)^2}|\vec{q}\,|^5\,. \tag{A3}$$

## REFERENCES


[1] G. t'Hooft, *Nucl.Phys.* **B72** 461 (1974); **B75** 461 (1974).
[2] E. Witten, *Nucl.Phys.* **B160** 57 (1979).
[3] S. Coleman, *1/N* in *Aspects of Symmetry: Selected Erice Lectures*, Cambridge University Press, Cambridge 1985.
[4] R. Dashen, E. Jenkins and A.V. Manohar, *Phys.Rev.* **D49** 4713 (1994); **D51** 3697 (1995).
[5] J. Dai, R. Dashen, E. Jenkins and A.V. Manohar, *Phys.Rev.* **D53** 273 (1996).
[6] D. Pirjol and T.M. Yan, CLNS-97/1500, hep-ph/9707485.
[7] D. Faiman and D. Plane, *Nucl.Phys.* **B50** 379 (1972).
[8] A. Hey, P. Litchfield and R. Cashmore, *Nucl.Phys.* **B95** 516 (1975).
[9] F.E. Close, *An Introduction to Quarks and Partons*, Academic Press, 1979.
[10] N. Isgur and G. Karl, *Phys.Lett.* **B72** 109 (1977); *Phys.Rev.* **D18** 4187 (1978).
[11] D.M. Manley and E.M. Saleski, *Phys.Rev.* **D45** 4002 (1992).
[12] C.D. Carone, H. Georgi, L. Kaplan and D. Morin, *Phys.Rev.* **D50** 5793 (1994).
[13] Particle Data Group, R.M. Barnett *et al.*, *Phys.Rev.* **D54** 1 (1996).
[14] R.A. Arndt, I.I. Strakovsky, R.L. Workman and M.M. Pavan, *Phys.Rev.* **C52** 2120 (1995)
[15] N. Kaiser, P.B. Siegel and W.Weise, *Phys.Lett.* **B362** 23 (1995).
[16] Z. Li and R. Workman, *Phys.Rev.* **C53** 549 (1996).
[17] E. Jenkins, *Phys.Rev.* **D53** 2625 (1996).