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A Review of Three-Family Grand Unified String Models

Zurab Kakushadze[1,2*], Gary Shiu[3†], S.-H. Henry Tye[3‡], Yan Vtorov-Karevsky[3§]
[1] *Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138*
[2] *Department of Physics, Northeastern University, Boston, MA 02115*
[3] *Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, NY 14853*
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Abstract

We review the construction and classification of three-family grand unified models within the framework of asymmetric orbifolds in perturbative heterotic superstring. We give a detailed survey of all such models which is organized to aid analysis of their phenomenological properties. We compute tree-level superpotentials for these models. These superpotentials are used to analyze the issues of proton stability (doublet-triplet splitting and R-parity violating terms) and Yukawa mass matrices. To have agreement with phenomenological data all these models seem to require certain degree of fine-tuning. We also analyze the possible patterns of supersymmetry breaking in these models. We find that the supersymmetry breaking scale comes out either too high to explain the electroweak hierarchy problem, or below the electroweak scale unless some degree of fine-tuning is involved. Thus, none of the models at hand seem to be phenomenologically flawless.

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*E-mail: zurab@string.harvard.edu

†E-mail: shiu@mail.lns.cornell.edu

‡E-mail: tye@mail.lns.cornell.edu

§E-mail: gilburd@mail.lns.cornell.edu

I. INTRODUCTION

If string theory is relevant to nature, it must possess a vacuum whose low energy effective field theory describes the Standard Model of strong and electroweak interactions. The question whether such a string vacuum exists is difficult to answer as the space of classical string vacua has a very large degeneracy, and there lack objective criteria that would select a particular string vacuum among the numerous possibilities. One might expect non-perturbative string dynamics to lift, partially or completely, this degeneracy in the moduli space. If this lifting is complete, then a thorough understanding of string dynamics may be sufficient to find a complete description of our universe. *A priori*, however, it is reasonable to suspect that non-perturbative dynamics may *not* select the unique vacuum, but rather pick out a number of consistent vacua, some in four dimensions with completely broken supersymmetry; and our universe would be realized as one of the consistent vacua in this (probably large) set. If so, we would need to impose some additional, namely, phenomenological constraints to find the string vacuum in which we live. This approach has been known as "superstring phenomenology". The latter must still be augmented with an understanding of non-perturbative dynamics as superstring is believed not to break supersymmetry perturbatively.

It might seem, at least naively, that there is more than enough phenomenological data to identify the superstring vacuum corresponding to our universe. It is, however, not known how to fully embed the Standard Model into string theory with all of its complexity, so one is bound to try to incorporate only a few phenomenologically desirable features at a time (such as, say, the gauge group, number of families, *etc.*). Since such constraints are not very stringent, this ultimately leads to numerous possibilities for embedding the Standard Model in superstring that *a priori* seem reasonable [1]. Thus, to make progress in superstring phenomenology, one needs as restrictive phenomenological constraints as possible. It might be advantageous to impose such phenomenological constraints taking into account specifics of a partcular framework for superstring model building.

In the past decade, the main arena for model building within the context of superstring phenomenology has been perturbative heterotic superstring. The reason is that such model building is greatly facilitated by existence of relatively simple rules (such as free-fermionic [2] and orbifold [3,4] constructions). Moreover, many calculational tools (such as, say, scattering amplitudes and rules for computing superpotentials [5]) are either readily available, or can be developed for certain cases of interest. Despite enormous progress made in the past few years in understanding non-perturbative superstring vacua, the state of the art there is still far from being competitive with perturbative heterotic superstring. The tools available in the latter framework must be first generalized to include the non-perturbative vacua before superstring phenomenology can step into this new terrain.

To be specific, let us concentrate on perturbative heterotic superstring. Within this framework the total rank of the gauge group (for $N = 1$ space-time supersymmetric models) is 22 or less. After accommodating the Standard Model of strong and electroweak interactions (with gauge group $SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$ whose rank is 4), the left-over rank for the hidden and/or horizontal gauge symmetry is 18 or less. The possible choices here are myriad [6] and largely unexplored. The situation is similar for embedding unification within a *semi-simple* [7] gauge group $G \supset SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$ (*e.g.*, $SU(5) \otimes U(1)$).

The state of affairs is quite different if one tries to embed grand unification within a *simple* gauge group $G \supset SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$. Thus, an adjoint (or some other appropriate higher dimensional) Higgs field must be present among the light degrees of freedom in effective field theory to break the grand unified gauge group G down to that of the Standard Model. In perturbative heterotic superstring such states in the massless spectrum are compatible with $N = 1$ supersymmetry and chiral fermions only if the grand unified gauge group is realized via a current algebra at level $k > 1$ (see, *e.g.*, Ref [8]). This ultimately leads to reduction of total rank of the gauge group, and, therefore, to smaller room for hidden/horizontal symmetry, which greatly limits the number of possible models.

The limited number of possibilities is not the only distinguishing feature of grand unified models in superstring theory. Grand unified theories (GUTs) possess a number of properties not shared by superstring models with either the Standard Model or a semi-simple gauge group. One such property concerns the gauge coupling unification problem in superstring theory [9]. Thus, the strong and electroweak couplings α_3, α_2 and α_1 of $SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$ in the minimal supersymmetric Standard Model (MSSM) unify at the GUT scale $M_{GUT} \sim 10^{16}$ GeV [10] at the value of $\alpha_{GUT} \sim 1/24$. Running of these couplings is schematically shown in Fig.1a as a function of the energy scale E. For comparative purposes, a dimensionless gravitational coupling $\alpha_G = G_N E^2$ is introduced, where G_N is the Newton's constant. In string theory the unification demands that all couplings meet at a single scale. Note that the gravitational coupling becomes equal α_{GUT} at a scale roughly two orders of magnitude higher than M_{GUT} (Fig.1a). Several possible approaches have been proposed to reconcile this apparent discrepancy [9], some of which are listed below:

- The subgroups of $SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$ unify into a single GUT gauge group G, and the gauge coupling of G meets with α_G as shown in Fig.1b.
- The gauge group remains that of the MSSM all the way up, but some extra (compared with the MSSM spectrum) matter multiplets are present which change the running of the couplings so that their unification scale is pushed up to meet α_G. Such a scenario requires a judicious choice of the extra fields, as well as their masses [11]. The situation is similar for the models with other semi-simple gauge groups.
- Another possibility is suggested by M-theory [12], where the graviton can propagate in a 5-dimensional spacetime (the bulk), while the gauge and matter fields live on 4-dimensional "walls". Below the energy scale of the order of the inverse "thickness" of the bulk, all fields propagate in 4 space-time dimensions [12]. Above this scale, however, the gauge and matter fields propagate in 4 dimensions, while the graviton propagates in 5 dimensions. As a result, α_G runs faster and catches up with the strong and electroweak couplings as shown in Fig.1c.

The simplest way to obtain higher-level current algebras (required for GUT embeddings) in perturbative heterotic superstring is via the following construction. Start from a k-fold product $G \otimes G \otimes \cdots \otimes G$ of the grand unified gauge group G (of rank r) realized via a level-1 current algebra. The diagonal subgroup $G_{diag} \subset G \otimes G \otimes \cdots \otimes G$ is then realized via level k current algebra. (Note that in carrying out this procedure the rank of the gauge group is reduced from kr to r.) As far as the Hilbert space is concerned, here we are identifying the states under the $\mathbf{Z}_k$ cyclic symmetry of the k-fold product $G \otimes G \otimes \cdots \otimes G$. This is nothing but $\mathbf{Z}_k$ orbifold action, namely, modding out by the outer automorphism.

FIGURES

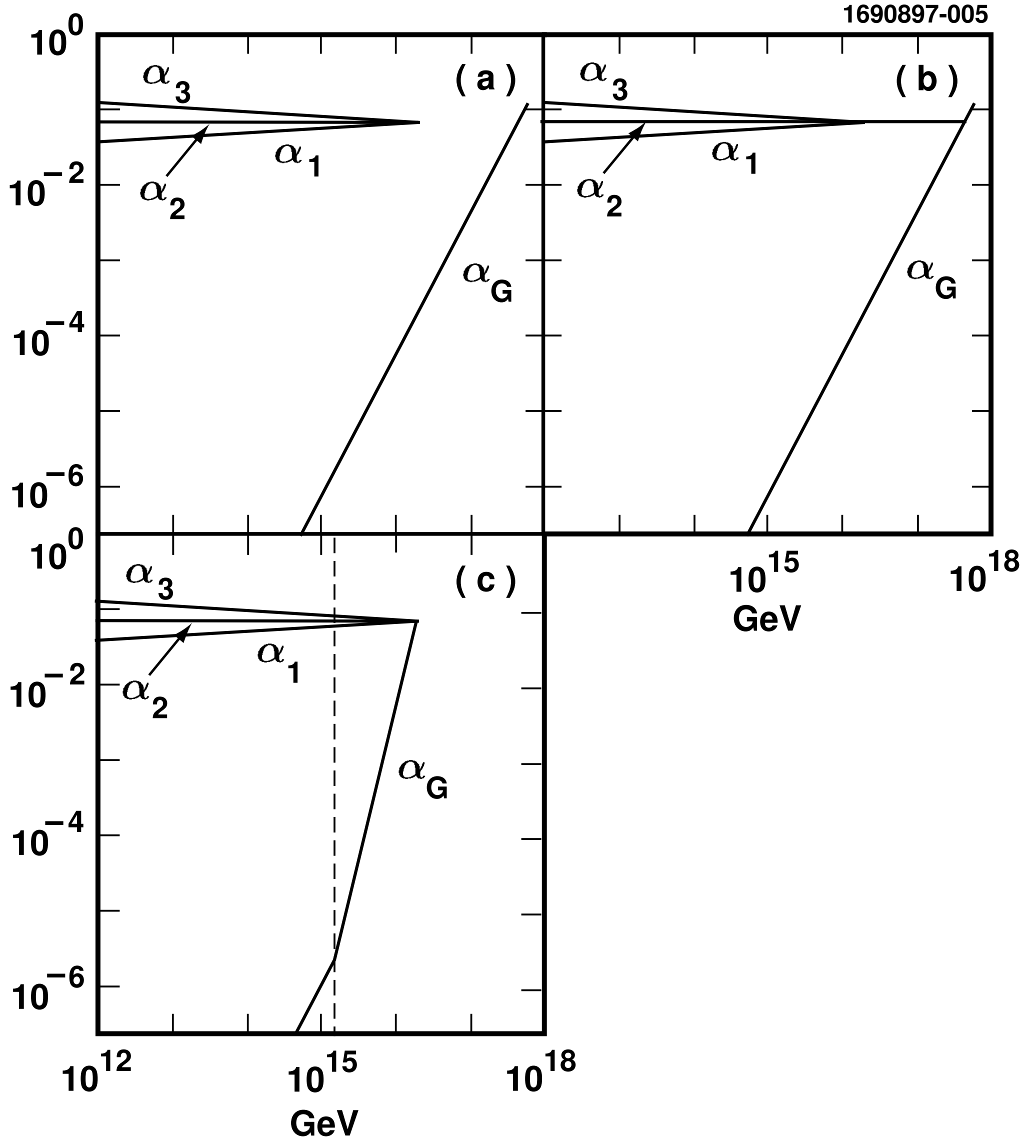

FIG. 1. Running of the MSSM couplings vs. dimensionless gravitational coupling

An immediate implication of the above construction is a rather limited number of possibilities. For example, for a grand unified gauge group $G = SO(10)$ with, say, $k = 3$, the left-over rank (for the hidden and/or horizontal gauge symmetry) is at most 7 ($= 22 - 3 \times 5$). This is to be compared with the left-over rank 18 in the case of the Standard Model embedding. Taking into account that the number of models grows rapidly as a function of the left-over rank, it becomes clear that grand unified model building is much more restricted than other embeddings.

Since desired adjoint (or higher dimensional) Higgs fields are allowed already at level $k = 2$, multiple attempts have been made in the past several years to construct level-2 grand unified string models. None of them, however, have yielded 3-family models. The first $SO(10)$ string GUT realized via a level-2 current algebra was obtained by Lewellen [8] within the framework of free-fermionic construction [2]. Soon after Schwartz [13] obtained an $SU(5)$ level-2 string GUT within the same framework. Both of these models have four chiral families. Multiple attempts have been made ever since to construct three-family string GUTs realized via level-two current algebras within free-fermionic construction [14] and within the framework of symmetric [15] as well as asymmetric [16] Abelian orbifolds [3]. Finally, three of us tried non-Abelian orbifolds [17] within both the free-fermionic and the bosonic formulations [4]. There is no formal proof that 3-family models cannot be obtained from level-2 constructions, but one can intuitively understand why attempts to find such models have failed. In the $k = 2$ construction the orbifold group is $\mathbf{Z}_2$. So the numbers of fixed points in the twisted sectors, which are related to the number of chiral families, are always even in this case.

Thus, it is natural to consider $k = 3$ models. The orbifold action in this case is $\mathbf{Z}_3$, and one might hope to obtain models with 3 families as the numbers of fixed points in the twisted sectors are some powers of 3. The level-3 model building appears to be more involved than that for level-2 constructions. The latter are facilitated by existence of the $E_8 \otimes E_8$ heterotic superstring in 10 dimensions which explicitly possesses a $\mathbf{Z}_2$ outer automorphism symmetry of the two E_8's. Constructing a level-2 model then can be carried out in two steps: first one embeds the grand unified gauge group G in each of the E_8's, and then performs the outer automorphism $\mathbf{Z}_2$ twist. In contrast to the $k = 2$ construction, $k = 3$ model building requires explicitly realizing $\mathbf{Z}_3$ outer automorphism symmetry which is not present in 10 dimensions. The implication of the above discussion is that one needs relatively simple rules to facilitate model building. Such rules have been derived [18] within the framework of *asymmetric orbifolds* [19].

With the appropriate model building tools available, it became possible to construct [18,20,21] and classify [22,23] 3-family grand unified string models within the framework of asymmetric orbifolds in perturbative heterotic string theory. Here we briefly discuss the results of this classification. (A detailed survey of three-family grand unified string models is given in section II.) For each model we list here, there are additional models connected to it via classically flat directions [22,23]:

• One E_6 model with 5 left-handed and 2 right-handed families, and asymptotically free $SU(2)$ hidden sector with 1 "flavor".

• One $SO(10)$ model with 4 left-handed and 1 right-handed families, and $SU(2) \otimes SU(2) \otimes SU(2)$ hidden sector which is *not* asymptotically free at the string scale.

• Three $SU(6)$ models:

(*i*) The first model has 6 left-handed and 3 right-handed families, and asymptotically free $SU(3)$ hidden sector with 3 "flavors".

(*ii*) The second model has 3 left-handed and no right-handed families, and asymptotically free $SU(2) \otimes SU(2)$ hidden sector with matter content consisting of doublets of each $SU(2)$ subgroup as well as bi-fundamentals.

(*iii*) The third model has 3 left-handed and no right-handed families, and asymptotically free $SU(4)$ hidden sector with 3 "flavors". (This model has not been explicitly given in Ref [23], and will be presented in Appendix G.)

• Finally, there are some additional $SU(5)$ models which do not seem to be phenomenologically appealing (see section IV for details).

All of the above models share some common phenomenological features. Thus, there is only one adjoint and no other higher dimensional Higgs fields in all of these models. The E_6

and $SO(10)$ models (and other related models) do *not* possess anomalous $U(1)$. All three $SU(6)$ models listed above *do* have anomalous $U(1)$ (which in string theory is broken via the Green-Schwarz mechanism [24]). The above models all possess non-Abelian hidden sector. There, however, exist models where the hidden sector is completely broken.

To study phenomenological properties of these models it is first necessary to deduce their tree-level superpotentials. This turns out to be a rather non-trivial task as it involves understanding scattering in asymmetric orbifolds. There, however, are certain simplifying circumstances here due to the fact that asymmetric orbifold models possess enhanced discrete and continuous gauge symmetries. Making use of these symmetries, the tools for computing tree-level superpotentials for a class of asymmetric orbifold models (which includes the models of interest for us here) have been developed in Ref [25]. The perturbative superpotentials for some of the three-family grand unified string models were computed in Ref [25]. We compute perturbative superpotentials for other relevant models in this paper following the rules of Ref [25].

The knowledge of tree-level superpotentials allows one to analyze certain phenomenological issues such as proton stability (doublet-triplet splitting and R-parity violating terms) and Yukawa mass matrices. The question of supersymmetry breaking can also be addressed by augmenting the tree-level superpotentials with non-perturbative contributions which are under control in $N = 1$ supersymmetric field theories [26].

Thus, in Ref [27] doublet-triplet splitting problem and Yukawa mass matrices were studied for the $SO(10)$ models. It was found that certain degree of fine-tuning is required to solve the doublet-triplet splitting problem, suppress dangerous R-parity violating terms and achieve realistic Yukawa mass matrices. All $SU(5)$ models suffer from the severe fine-tuning problem steming from the doublet-triplet splitting as there are no “exotic” higher dimensional Higgs fields among the light degrees of freedom. The latter are required by all known field theory solutions to the problem.

In this paper we study similar issues for the $SU(6)$ models. The results of our analysis indicate that the doublet-triplet splitting does not seem to be as big of a problem for the $SU(6)$ models as it is for their $SO(10)$ and $SU(5)$ counterparts. However, the troubles with R-parity violating terms and Yukawa mass matrices still persist for these models.

In this paper we also analyze the possible patterns of supersymmetry breaking in the three-family grand unified string models. We find that the supersymmetry breaking scale in these models comes out either too high to explain the electroweak hierarchy problem, or below the electroweak scale unless some degree of fine-tuning is involved.

Since none of the three-family grand unified string models constructed to date appear to be phenomenologically flawless, one naturally wonders whether there may exist (even within perturbative heterotic superstring vacua) other such models with improved phenomenological characteristics. Thus, all *a priori* possible free-field embeddings of higher-level current algebras within perturbative heterotic superstring framework have been classified [28]. This, however, does not guarantee that any given embedding can be incorporated in a consistent string model, and even if this is indeed possible, there need not exist three-family models within such an embeding. The three-family grand unified string models descussed in this review are concrete realizations of the *diagonal* level-3 embedding $G_{diag} \supset G \otimes G \otimes G$. However, even if our models do exhaust all three-family grand unified string models within free-field realized perturbative heterotic superstring, there may exist non-free-field grand unified string models with three families. Tools for constructing such models are not available at this moment, so that for years to come the asymmetric orbifold models we discuss in this review might be the only ones available. Regardless of their phenomenological viability they provide the *proof of existence* for three-family grand unified string models; these models could also serve as a stringy paradigm for such model building in general, and also give insight to the "bottom-up" approach.

This review is organized as follows. In section II we give a detailed survey of three-family grand unified string models classified in Refs [22,23]. In section III we discuss supersymmetry (SUSY) breaking in these models. In section IV we discuss some aspects of SUSY GUT phenomenology in these models. In section V we give our conclusions and outlook. All the details are relegated to appendices.

II. SURVEY OF THREE-FAMILY GRAND UNIFIED STRING MODELS

In this section we give a systematic survey of all the (relevant) three-family grand unified string models. This survey is organized by the grand unified gauge group, number of generations and anti-generations, and hidden sector gauge group and matter. We will not list all the models (as most of them can be either found in Refs [22,23], or deduced from those listed there), but give only the models that are representative of classes of models with similar features, or stand by their own (for they possess special features). Some of the latter models, namely, with $SU(6)$ grand unified gauge group, have not been presented elsewhere, so we briefly outline their construction in Appendix G.

A. The E_6 Model

There is one E_6 model (referred to as the $E1$ or $E2$ model in Ref [22]; $E1$ and $E2$ stand for two different ways of constructing the same model [22,25]). It has 5 left-handed and 2 right-handed families, and $SU(2)_H$ hidden sector with 2 chiral doublets (corresponding to one "flavor"). The massless spectrum of the E_6 model is summarized in Table I.

The E_6 model is a special (enhanced gauge symmetry) point in a certain moduli space, which we will refer to as $\mathcal{M}$, that also contains other three-family models with grand unified gauge groups $SO(10)$, $SU(5)$ and $SU(6)$ [22,23]. All of these models are connected to each other (and to the E_6 model) via a web of (classically) flat directions. They all have 5 left-handed and 2 right-handed families, and $SU(2)_H$ hidden sector. There are two features that distinguish them: the hidden sector matter content (number of "flavors" of $SU(2)_H$) and the horizontal gauge symmetry. These two features are not completely uncorrelated. Thus, breaking the horizontal symmetry (via Higgsing) sometimes leads to some of the hidden sector doublets acquiring mass (via tree-level superpotential). On the other hand, sometimes hidden sector doublets can get heavy without breaking the horizontal symmetry, *i.e.*, via coupling to gauge singlets (present in some models) acquiring vevs. Depending on the number of hidden sector doublets, the $SU(2)_H$ group may or may not be asymptotically free. Here we are interested only in models with asymptotically free hidden sector (for the phenomenological reasons discussed earlier). If the number of hidden sector "flavors" is too large in a given model so that $SU(2)_H$ is not asymptotically free, it can always be connected to another model with smaller matter content of $SU(2)_H$ such that the latter is asymptotically free. Thus, we can confine our attention to the models with asymptotically free $SU(2)_H$ without loss of generality.

The moduli space $\mathcal{M}$ is multiply connected, *i.e.*, it has different branches. Thus, there are points where more than one branch meet, yet these branches have no other intersection points. Some of the branches lead to models with no hidden sector. We will not consider them here.

Let us summarize the possibilities for the $SU(2)_H$ matter content. There are models with one and three "flavors" (these are some of the $SO(10)$, $SU(5)$ and $SU(6)$ models [22,23]), and this feature is generic in these models, *i.e.*, there are branches in the moduli space with this matter content. Note that there are no models with 4 chiral doublets (two "flavors") of $SU(2)_H$. (There is also one $SU(5)$ model with five "flavors" [23]; this model exists only at a special point in the moduli space, and generically, the number of "flavors" is one on the branch where it lives because the other four become heavy via a coupling to a gauge singlet whose vev is a flat direction. The model with one "flavor" is the $F11(1,0)$ model given in Table IV; see subsection D of this section and subsection 4 of Appendix G for details.)

The $SO(10)$, $SU(5)$ and $SU(6)$ models with one "flavor" can be obtained from the E_6 model by adjoint breaking (the vev of the adjoint is a flat direction). There are four such breakings: $E_6 \supset SO(10) \otimes U(1)$, $E_6 \supset SU(6) \otimes U(1)$, $E_6 \supset SU(5) \otimes SU(2) \otimes U(1)$ and $E_6 \supset SU(5) \otimes U(1)^2$. The models with three "flavors" are connected to the models with one "flavor" via points with enhanced horizontal gauge symmetry where the branches with three "flavors" and one "flavor" intersect.

We note that the generation and anti-generation structure of the models with different numbers of the $SU(2)_H$ "flavors" is the same. In particular, the couplings involving the corresponding (grand unified) matter fields are similar. Thus, to study the SUSY GUT phenomenological aspects of these models it suffices to consider the models with one $SU(2)_H$ "flavor". The conclusions generically hold for all the other models. This implies that the E_6 model does not require a separate study as long as we analyze the $SO(10)$, $SU(5)$ and $SU(6)$ models obtained from the former by the adjoint breakings listed above. This is because with just one adjoint ($\mathbf{78}$) and pairs of fundamentals ($\mathbf{27}$) and anti-fundamentals ($\overline{\mathbf{27}}$) of E_6, the breaking to the standard model gauge group is bound to be via $SO(10)$, $SU(5)$ or $SU(6)$ branchings. (Thus, say, $E_6 \supset SU(3)_c \otimes SU(3)_w \otimes SU(3)$ breaking would require at least one $\mathbf{650}$ of E_6, which is not present amongst the massless fields of the E_6 model.)

Finally, we note that all the models in the moduli space $\mathcal{M}$ are $U(1)$ anomaly free.

B. The $SO(10)$ Models

One class of $SO(10)$ models are those living in the moduli space $\mathcal{M}$ discussed in the previous subsection. (All of these models have been explicitly constructed [22].) In the context of our discussion there, we need only consider the model that is obtained from the E_6 model via the adjoint breaking $E_6 \supset SO(10) \otimes U(1)$. The massless spectrum of this model (referred to as the $T1(1,1)$ or $T2(1,1)$ model in Ref [22,25]) is given in Table II.

There is one other $SO(10)$ model that does not live in the moduli space $\mathcal{M}$ (referred to as the $T5(1,0)$ or $T6(1,0)$ model in Ref [22]; $T5(1,0)$ and $T6(1,0)$ stand for two different ways of constructing the same model [22]). The massless spectrum of this model is summarized in

Table III. It has 4 left-handed and 1 right-handed families, and $SU(2)\otimes SU(2)\otimes SU(2)$ hidden sector. The latter is not asymptotically free at the string scale as each $SU(2)$ subgroup has 16 chiral doublets (eight "flavors"). It is possible that some of these doublets acquire masses once the horizontal gauge symmetry is (partially) Higgsed. Examining the model reveals that the resulting model would have six "flavors" in each $SU(2)$ subgroup. The one-loop β-function coefficient for this matter content is zero, and although the group is asymptotically free in the two-loop order, the scale of non-perturbative dynamics comes out too low (see section III for details). We note that this model is $U(1)$ anomaly free.

C. The $SU(6)$ Models

Just as in the case of $SO(10)$ models, there are $SU(6)$ models that live in the moduli space $\mathcal{M}$. We need only consider the model that is obtained from the E_6 model via the adjoint breaking $E_6 \supset SU(6) \otimes U(1)$. The massless spectrum of this model (referred to as the $S1(1,1)$ or $S2(1,1)$ model in Ref [23]) is given in Table IV.

Note that the spectrum of the $S1(1,1)$ model is similar to that of the $T1(1,1)$ model. In particular, the former can be obtained from the latter via replacing $SO(10) \otimes U(1)$ (the last $U(1)$ in the first column of Table II) by $SU(6) \otimes U(1)$ (the last $U(1)$ in the first column of Table IV). Under this substitution, $\mathbf{45}(0)$ of $SO(10) \otimes U(1)$ is replaced by $\mathbf{35}(0)$ of $SU(6) \otimes U(1)$, and $\mathbf{16}(-1) + \mathbf{10}(+2) + \mathbf{1}(-4)$ is replaced by $\mathbf{15}(0) + \overline{\mathbf{6}}(+1) + \overline{\mathbf{6}}(-1)$. (The complex conjugates of these states are substituted similarly.) This replacement procedure can be applied to all the $SO(10)$ models in the moduli space $\mathcal{M}$ to obtain the corresponding $SU(6)$ models. (Some of these models have been explicitly constructed previously [23]; others should also exist, albeit their explicit constructions have not been presented elsewhere.) In any case, for our purposes it suffices to consider the $S1(1,1)$ model. (Here we should mention that starting from the $T5(1,0)$ model that does not live in $\mathcal{M}$, we could obtain the corresponding $SU(6)$ model $S5(1,0)$. This model has not been explicitly constructed, but it should exist just as other $SU(6)$ models that do live in the moduli space $\mathcal{M}$. The hidden sector of this model is the same as that of the $T5(1,0)$ model, so that the discussion presented for the latter in the previous subsection also applies to the former.)

There are five additional $SU(6)$ models that do not live in the moduli space $\mathcal{M}$. Two of them have been previously constructed, whereas the explicit construction of the other three models appears in Appendix G for the first time. Next, we turn to describing these five models.

One model, referred to as $S1$ (or $S3$) in Ref [23], has 6 left-handed and 3 right-handed families, and $SU(3)_H$ hidden sector with 3 chiral triplets and antitriplets (corresponding to three "flavors"). The model has anomalous $U(1)$. Examining the superpotential of the model reveals that generically one of the "flavors" of $SU(3)_H$ is heavy, and the other two are light (see section III for details). The massless spectrum of this model is given in Table V.

Another model referred to as $S2$ (or $S4$) in Ref [23], has 3 left-handed and no right-handed families (the number of families in the $SU(6)$ case is determined by the number of $\mathbf{15}$'s), plus additional six $\mathbf{6} + \overline{\mathbf{6}}$ pairs. The hidden sector of the model is $SU(2) \otimes SU(2)$, where the first subgroup has 6 chiral doublets (3 "flavors"), whereas the second one has 10 chiral doublets (5 "flavors"). Some of these come in the bi-fundamental representation of $SU(2) \otimes SU(2)$. The model has anomalous $U(1)$. Examining the superpotential of the model reveals that generically two of the "flavors" of the second $SU(2)$ subgroup are heavy, and one is left with 3 "flavors" in each of the two subgroups (see section III for details). The massless spectrum of this model is given in Table V.

One of the models constructed in Appendix G, referred to as $\Sigma 1$, has 5 left-handed and 2 right-handed families, and $SU(3)_H$ hidden sector with no matter. The massless spectrum of this model is given in Table VI. Its part charged under the grand unified gauge group is very similar to that of the model $S1(1,1)$. The only difference is in the Higgs sector of $\mathbf{6} + \overline{\mathbf{6}}$'s. Note that this model does not have anomalous $U(1)$.

Another model, referred to as $\Sigma 2$, has 3 left-handed and no right-handed families, and the $SU(2)_H$ hidden sector with two chiral doublets (one "flavor"). It is also $U(1)$ anomaly free. Its massless spectrum is given in Table VI. In Appendix B we show that this model is connected to the $S2$ model via a classically flat direction.

Finally, the third model of Appendix G, referred to as $\Sigma 3$, has 3 left-handed and no right-handed families, and $SU(4)_H$ hidden sector with 3 chiral quartets and antiquartets (corresponding to three "flavors"). The model has anomalous $U(1)$. Examining the superpotential of the model reveals that generically two of the "flavors" of $SU(4)_H$ are heavy, and the other one is light (see section III for details). The massless spectrum of this model is given in Table VII. Note that unlike the other two $SU(6)$ models ($S2$ and $\Sigma 2$) with 3 left-handed and no right-handed families, the $\Sigma 3$ model is minimal in the sense that it is a minimal $SU(6)$ extension of the minimal $SU(5)$ model [29]. That is, the $\Sigma 3$ model possesses no $\mathbf{6} + \overline{\mathbf{6}}$ Higgs sector. Phenomenologically this is unappealing since the doublet-triplet splitting cannot be achieved without fine-tuning (just as in the minimal $SU(5)$ model).

In Appendix G we also construct four other models referred to as $\Sigma 1A$, $\Sigma 1B$, $\Sigma 2A$ and $\Sigma 2B$. The $\Sigma 1A$, $\Sigma 1B$ models are connected to the $\Sigma 1$ model by classically flat directions. Similarly, $\Sigma 2A$ and $\Sigma 2B$ live in the same moduli space as the $\Sigma 2$ model. The massless spectra of these models are given in Table VIII and Table IX. Note that these models have enhanced horizontal gauge symmetry ($U(1)$ is enhanced to $SU(2)_3$), and at the enhanced symmetry point they possess 4 chiral supermultiplets in $\mathbf{20}$ of $SU(6)$. (Also note that the $\Sigma 2A$ and $\Sigma 2B$ models have 3 "flavors" of $SU(2)_H$ at the enhanced symmetry point.)

The appearance of $\mathbf{20}$'s of $SU(6)$ is amusing as they have been considered in certain scenarios where the doublet-triplet splitting is achieved via the pseudo-Goldstone mechanism [30]. That is, electroweak Higgs doublets arise as pseudo-Goldstones in breaking $SU(6)$ grand unified gauge symmetry down to that of the Standard Model. These doublets then do not couple to matter that easily (to produce, say, top Yukawa coupling) unless some special arrangements are made. One of the solutions proposed in the literature [30] involves (an odd number of) $\mathbf{20}$'s of $SU(6)$ (so that they do not pair up and get Dirac mass). However, in the models $\Sigma 1A$, $\Sigma 1B$, $\Sigma 2A$ and $\Sigma 2B$ the $\mathbf{20}$'s (whose number is even) are vector-like and are massless only at the special point (of enhanced horizontal gauge symmetry) in the moduli space discussed above. Generically they are heavy and thus decouple from the massless sector. We will, therefore, regard the $\Sigma 1A$, $\Sigma 1B$, $\Sigma 2A$ and $\Sigma 2B$ as some special points in the respective moduli spaces of models $\Sigma 1$ and $\Sigma 2$, and consider only the latter for phenomenological purposes.

D. The $SU(5)$ models

As we already mentioned, there are various $SU(5)$ models most of which are connected (via flat directions) to the E_6, $SO(10)$ and $SU(6)$ models discussed above, and some of which are isolated points [18,21,23]. All of these models have only one adjoint but no higher dimensional Higgs fields in their massless spectra. Thus, they do not possess ingredients necessary for solving the doublet-triplet splitting without fine-tuning (as all the known solutions are based on existence of massless higher-dimensional Higgs fields). We will, therefore, not consider any of these models in detail. For illustrative purposes we give one of these models (referred to as $F11(1,0)$ in Ref [23]) in Table IV. (Note that this model lives in the moduli space $\mathcal{M}$, and can be obtained from the E_6 model via the adjoint breaking $E_6 \supset SU(5) \otimes SU(2) \otimes U(1)$.)

We note that the $SO(10)$ and $SU(6)$ models *a priori* do not look as hopeless (from the doublet-triplet splitting point of view) as the $SU(5)$ models, and this is why we investigate them in more detail.

E. Summary

Let us summarize the above discussion, and list the models we should focus upon. As far as $SO(10)$ models are concerned, we will concentrate on the $T1(1,1)$ model. We will also show in section III that the hidden sector in the $T5(1,0)$ model does not get strong at desired energy scale. The $SU(6)$ models that we need to consider are $S1(1,1)$, $S1$, $S2$, $\Sigma1$ and $\Sigma2$. We will also point out some of the phenomenological problems in the $\Sigma3$ model. Finally, as we explained before, we need not consider the E_6 model separately.

Here we would like to emphasize one feature that is common to all of the three-family grand unified models: they have one adjoint but no higher dimensional Higgs fields in their massless spectra.

III. HIDDEN SECTOR DYNAMICS

In this section we discuss some issues relevant for strong coupling dynamics in the hidden sector. The latter is important for possible supersymmetry breaking and stabilization of moduli. Although the current state of the art in this subject does not always allow one to carry out detailed quantitative analysis, in models with relatively simple hidden sectors (and all of the models we are studying in this paper are of this type) it is oftentimes possible to see whether supersymmetry is broken at all, and to estimate the scale of supersymmetry breaking. If in a given model this scale comes out too high compared with the electroweak scale $M_{EW} \sim 100$ GeV, then it is difficult to imagine how the electroweak gauge hierarchy would be explained in such a model without fine-tuning. On the other hand, if this scale comes out way below M_{EW}, then supersymmetry breaking in such a model is not compatible with experiment.

A. Hidden Sector Scale

Thus, consider a hidden sector with a simple gauge group G_H and some matter content (some number of "flavors") with the β-function coefficient b_0 at the string scale M_{st}. In some cases there could be a scale M_X below the string scale at which some of the matter charged under G_H becomes heavy and decouples. Such a scale, for instance, could be an anomalous $U(1)_A$ breaking scale, and if some of the matter couples (say, via three-point Yukawa couplings) to singlets (charged under $U(1)_A$) that acquire vevs to break $U(1)_A$, then it could develop Dirac mass. Let the β-function coefficient below the scale M_X be $\tilde{b}_0$. Let $\tilde{b}_0 > 0$, so that the hidden sector is asymptotically free below the scale M_X. Above the scale M_X it may (in which case $b_0 \geq 0$) or may not (*i.e.*, $b_0 < 0$ be asymptotically free. It is simple to see that the gauge coupling of G_H blows up at the scale M_H given by

$$M_H = \left(\frac{M_X}{M_{st}}\right)^{1-b_0/\tilde{b}_0} M_{st} \exp\left(-\frac{2\pi}{\alpha_{st}\tilde{b}_0}\right) . \tag{1}$$

Here α_{st} is the gauge coupling of G_H at the string scale. Note that in all of our models the hidden sector gauge group is realized via a level-1 current algebra, while the grand unified gauge group is realized via a level-3 current algebra. Thus, the relation between the gauge couplings of G_H and the observable sector (let the latter gauge coupling be α'_{st}) at the string scale is given by $\alpha_{st} = 3\alpha'_{st}$. Also note that if any of these models were to match the experimental data, we would ultimately have to assume that $\alpha'_{st} \geq \alpha_{GUT}$, where $\alpha_{GUT} \sim 1/24$ is the unification coupling obtained by extrapolating the LEP data with the assumption of minimal matter content and superpartner thresholds at ~ 1 TeV. Thus, we necessarily have $\alpha_{st} \geq 1/8$. For the value $\alpha_{st} = 1/8$, we can estimate the string scale [31] $M_{st} \sim 5 g_{st} \times 10^{17}$ GeV $= 6 \times 10^{17}$ GeV (note that $\alpha_{st} = g_{st}^2/4\pi$).

In models with no anomalous $U(1)$ the natural value for the scale M_X is $M_X \sim M_{Pl}$. In the models with anomalous $U(1)$ (precisely which are of interest for us), this scale (provided that X is, say, a singlet under G_H but carries Q_X anomalous $U(1)_A$ charge whose sign is opposite to that of the total $U(1)_A$ trace anomaly $\mathrm{Tr}\,(Q_A)$) can be estimated from the vanishing of the corresponding Fayet-Iliopoulos D-term.

B. Supersymmetry Breaking

Next, we turn to the supersymmetry breaking in the models we are studying. In Appendix A we provide all the necessary details, so our discussion here will be brief.

First consider the $\Sigma 3$ model. Its hidden sector is $SU(3)$ with no matter. There is a possibility that local supersymmetry is broken dynamically if suitable non-perturbative corrections are present in the Kähler potential [32], but the question remains open since computing these corrections is out of reach of our present technology. In any case, even if supersymmetry was broken dynamically in the hidden sector of the $\Sigma 3$ model, it would be transmitted (via the standard supergravity mediated scenario [33]) to the observable sector at a scale much higher than the one favored by phenomenology (see Appendix A).

Another case we need to consider is $SU(N_c)$ with $0 < N_f < N_c$, where N_f is the number of "flavors". Within the framework of global supersymmetry, this case is known to have no quantum vacuum as the meson fields (which are quark bi-linears) have a runaway behavior. That is, the there is a non-perturbative superpotential such that the meson F-terms (as well as the superpotential itself) are non-vanishing at any finite values of the meson vevs. Once we incorporate this theory within the local supergravity framework, however, the tree-level Kähler potential is enough to stabilize the meson vev [34], and local supersymmetry is broken. All the details can be found in Appendix A. There we also give the corresponding supersymmetry breaking scales for the models $S1$ ($SU(3)$ with two "flavors"), $T1(1,1)$, $S1(1,1)$ and $\Sigma 2$ ($SU(2)$ with one "flavor"), and $\Sigma 3$ ($SU(4)$ with one "flavor"). We find that the supersymmetry breaking scales in the models $S1$ and $\Sigma 3$ come out too high compared with M_{EW}, whereas for the models $T1(1,1)$, $S1(1,1)$ and $\Sigma 2$ they come out way below M_{EW} unless it is assumed that the supersymmetry breaking messenger scale is $\sim 10^{10} - 10^{11}$ GeV. In these models obtaining such low scales for the messenger $U(1)$'s seems to require some fine-tuning.

We are left with only one model to consider, namely, the $S2$ model. Below the anomalous $U(1)$ breaking scale this model has the following matter (chiral supermultiplets) in the hidden sector (the hidden sector gauge group is $SU(2) \otimes SU(2)$): $2(\Box, \Box)$, $2(\Box, \mathbf{1})$ and $2(\mathbf{1}, \Box)$ ($\Box$ stands for the fundamental, *i.e.*, doublet, of $SU(2)$) with different dynamically generated scales Λ_1 and Λ_2 for the two $SU(2)$'s, and the tree-level superpotential. In Appendix B we investigate the non-perturbative dynamics in this model. We show that there is no dynamically generated superpotential, and the quantum moduli space is the same as the classical one, unless one of the $SU(2)$ subgroups is completely Higgsed. In the latter case we simply flow into the $\Sigma 2$ model (which has $SU(2)$ with one "flavor" in its hidden sector), which we already discussed above.

C. Summary

Thus, the analysis in Appendix A seems to indicate that none of the models at hand are flawless as far as supersymmetry breaking is concerned. To make more definitive conclusions about phenomenological viability of these models, in the subsequent sections we will discuss issues that should be resolved at tree-level once the non-perturbative dynamics is assumed to work in favor of the models we are studying. If we can find some unsatisfactory features even at this level, then phenomenological viability of the models will most certainly be (at least) difficult to render.

IV. ASPECTS OF SUSY GUT PHENOMENOLOGY

In this section we address the following three SUSY GUT phenomenological issues of the models we are studying: (*i*) doublet-triplet splitting, (*ii*) R-parity violating terms, and (*iii*) Yukawa mass matrices. The first two issues are related to proton stability. In the third one we will confine our attention to figuring out if one can naturally get one of the three up-quarks to be heavy (top-like) while having the other two light. We note that all of these questions in a phenomenologically viable model should be resolved at the level of perturbative superpotential. Thus, we will assume that the non-perturbative dynamics in the hidden sector discussed in the previous section works out in favor of phenomenological viability of these models. Then we can see whether the three aspects mentioned above can work out without some unnatural fine-tuning of vevs, which in this approach are free parameters.

A. The $T1(1,1)$ Model

In Ref [27] we studied the $T1(1,1)$ model in detail. Let us first summarize the important features of this model (see Table II for details). There is one adjoint Higgs field (**45**), 5 generations (**16**'s), 2 antigenerations ($\overline{\mathbf{16}}$'s), 7 Higgs fields (**10**'s), and various singlets of $SO(10)$ grand unified gauge group. Note that these fields descend from the adjoint Higgs field (**78**), 5 generations (**27**'s), 2 antigenerations ($\overline{\mathbf{27}}$'s), and singlets of E_6 in the E_6 model. (Recall the branching $\mathbf{27} = \mathbf{16}(-1) + \mathbf{10}(+2) + \mathbf{1}(-4)$ under the breaking $E_6 \supset SO(10) \otimes U(1)$. Note that $\mathbf{78} = \mathbf{45}(0) + \mathbf{1}(0) + \mathbf{16}(+3) + \overline{\mathbf{16}}(-3)$. The $\mathbf{16}(+3) + \overline{\mathbf{16}}(-3)$ fields from the adjoint of E_6 are missing in the $SO(10)$ model as they are eaten in the super-Higgs mechanism upon breaking E_6 down to $SO(10) \otimes U(1)$.)

We note that due to the F-flatness conditions in the superpotential of the $T1(1,1)$ model (derived in [25]) one of the **10**'s does not couple to any of the other fields and can be dropped out of the analysis. Thus, we have only 6 **10**'s to consider. For the same reason of F-flatness conditions, only one $\mathbf{16} + \overline{\mathbf{16}}$ pair becomes heavy at the GUT scale. In particular, this implies that another $\mathbf{16} + \overline{\mathbf{16}}$ can only become heavy at a much lower scale (of the order of M_{SUSY} due to relaxation of the F-flatness conditions). Thus, we have 4 **16**'s and 1 $\overline{\mathbf{16}}$ to consider below the GUT scale.

Next, let us consider the possible scenarios for breaking $SO(10)$ down to $SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$. There is only one adjoint Higgs field in the massless spectrum of the model. The adjoint must break $SO(10)$ down to its subgroup that contains $SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$. This implies that the adjoint Φ must acquire a vev in the following direction: $\Phi = \epsilon \otimes \mathrm{diag}(a, a, a, b, b)$, where ϵ is a 2×2 antisymmetric matrix.

Now, let us discuss the Higgs sector (**10**'s). We will use the notation $\mathbf{10}_i$, $i = 1, \ldots, 6$, to refer to these Higgs fields. The mass matrices for the fields (*i.e.*, Higgs doublets and triplets) coming from $\mathbf{10}_i$ have the following form: $\lambda_{ij} \mathbf{10}_i \mathbf{10}_j + \lambda'_{ij} \mathbf{10}_i \mathbf{10}_j \mathbf{45}$. Here λ_{ij} are symmetric couplings (coming from the couplings of $\mathbf{10}_i$ to singlets), whereas λ'_{ij} are antisymmetric couplings (coming from the couplings of $\mathbf{10}_i$ to singlets and the adjoint). The latter couplings are antisymmetric since the adjoint **45** of $SO(10)$ is antisymmetric. In Ref [27] we showed that generically all six pairs of doublets and triplets coming from $\mathbf{10}_i$ are heavy due to the above couplings. To have a pair of light doublets one then needs one fine-tuning (of vevs that determine the above couplings).

Suppose now we have one pair of light doublets and all the other fields coming from $\mathbf{10}_i$ are heavy (in doing this we have to impose one fine-tuning, but let us do this as we are trying to make another point). There are terms in the superpotential that have the following form: $\lambda_{ABi} \mathbf{16}_A \mathbf{16}_B \mathbf{10}_i$. Here $\mathbf{16}_A$, $A = 1, \ldots, 5$, are the five generations. Note that in order to break $SO(10)$ all the way down to $SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$, one $\mathbf{16} + \overline{\mathbf{16}}$ pair has to acquire a vev (along with the adjoint). This pair is a linear combination of the five generations and two antigenerations. Thus, once this pair has a non-zero vev we ultimately get mixing between the fields in $\mathbf{10}_i$, *i.e.*, the Higgs sector, and $\mathbf{16}_A$, *i.e.*, the matter sector. Here one should be careful as *a priori* the mixing could be between states that become heavy and decouple. Thus, what one really needs to check is if there is mixing between the light pair of doublets from $\mathbf{10}_i$ and the three light generations that would have to correspond to the three families of quarks and leptons below the M_{SUSY} scale. Generically, such a mixing is present, and to avoid it one fine-tuning is required.

The reason why the above mixing should be avoided is the following: it generically generates R-parity violating terms which would lead to too rapid proton decay. Thus to avoid these R-parity violating terms we need another fine-tuning as described above. Note that the R-parity violating terms in this case are "vev induced", *i.e.*, they appear after certain fields acquire vevs. The reason is that there is no clear separation between the matter sector and the Higgs sector in this model at the string scale, *i.e.*, no well defined R-parity. This turns out to be a generic feature of all of the models we are studying.

Finally, we would like to address the issue of Yukawa mass matrices. Suppose we take the pair of light Higgs doublets coming from $\mathbf{10}_i$ (this requires one fine-tuning) and impose the requirement that the R-parity violating terms be absent (this requires another fine-tuning). Then we may ask a question of what the Yukawa coupling matrix of these Higgs fields to the three generations looks like. In Ref [27] we showed that generically this 3×3 Yukawa (mass) matrix has rank 3, and, therefore, two additional fine-tunings are required to get rank 1 *i.e.* to have only one heavy (top-like) quark with electric charge $+2/3$.

In conclusion we see that (according to the analysis of Ref [27]) the $T1(1,1)$, and all the other three-family $SO(10)$ grand unified string models of Ref [22], do not meet the above three phenomenological requirements at tree-level. To achieve desired phenomenological features they generically require multiple fine-tunings. The only chance for them to arise is via some very contrived non-perturbative dynamics (which by some "string miracle" would have to fix all the vevs at their fine-tuned values). This seems unnatural and improbable.

B. The $SU(6)$ Models

In this subsection we discuss some of the issues in $SU(6)$ SUSY GUT phenomenology. In Appendix D the reader can find more details of our analysis for particular models we are considering here.

As far as the doublet-triplet splitting is concerned, there is a crucial difference between $SO(10)$ and $SU(6)$ GUTs. It can be easily understood from the following point of view. In $SO(10)$ GUTs quarks and leptons come from $\mathbf{16}$ irreps, while doublet and triplet Higgses come from $\mathbf{10}$ irreps. Note that $\mathbf{10}$ of $SO(10)$ does *not* contain singlets of $SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$. Thus, they cannot participate in breaking the $SO(10)$ grand unified gauge group down to that of the Standard Model. In our models, therefore, the only fields that can take part in reducing $SO(10)$ to $SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$ are the $\mathbf{45}$ and a $\mathbf{16} + \overline{\mathbf{16}}$ pair. Then the doublets and triplets from the $\mathbf{10}$'s generically couple to these fields in much the same way, and one needs fine-tuning of vevs (as we discussed in the previous subsection) to achieve doublet-triplet splitting.

In $SU(6)$ GUTs the situations is quite different. Each "generation" arises from one $\mathbf{15}$ and a pair of $\overline{\mathbf{6}}$'s. There are some states within these irreps that are extra compared with the Standard Model quarks and leptons. They can play the role of Higgs doublets and triplets. In this case, however, the adjoint must acquire vev in the direction that breaks the $SU(6)$ to $SU(5) \otimes U(1)$. This can be easily seen from examining the quantum numbers of the states in

15 and $\overline{\mathbf{6}}$'s in terms of $SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$. The situation here is the same as in $SU(5)$ GUTs, and doublet-triplet splitting cannot be achieved without fine-tuning. (This is true taking into account that in our models there are no "exotic" massless higher-dimensional Higgs fields.) There is an alternative scenario, namely, to have extra $\mathbf{6} + \overline{\mathbf{6}}$ pairs. Then Higgs doublets and triplets could come from these additional fields. Naturally, the extra states in **15** and $\overline{\mathbf{6}}$'s must become heavy which can be achieved by having Yukawa couplings of the type $\mathbf{15} \cdot \overline{\mathbf{6}} \cdot \overline{\mathbf{6}}$ and giving vev to a $\mathbf{6} + \overline{\mathbf{6}}$. This is allowed since these states do contain a singlet of $SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$. Note that in this case the adjoint (**35**) of $SU(6)$ must acquire vev in a direction different from the one that breaks $SU(6)$ to $SU(5) \otimes U(1)$ (or else a $\mathbf{15} + \overline{\mathbf{15}}$ pair would have to acquire vev to break $SU(5) \otimes U(1)$ further down to $SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$, which is not possible since a $\mathbf{6} + \overline{\mathbf{6}}$ pair already has acquired a vev). There are two possibilities here for the adjoint breaking: $SU(6) \supset SU(4)_c \otimes SU(2)_w \otimes U(1)$ and $SU(6) \supset SU(3)_c \otimes SU(3)_w \otimes U(1)$. In the first case $\mathbf{6} + \overline{\mathbf{6}}$ vev breaks $SU(4)_c \otimes U(1)$ to $SU(3)_c \otimes U(1)_Y$, whereas in the second case $SU(3)_w \otimes U(1)$ is broken to $SU(2)_w \otimes U(1)_Y$. It is not difficult to see that doublet and triplet Higgses in these scenarios are not treated on the equal footing, and, therefore, doublet-triplet splitting may be achieved without fine-tuning [30].

One of the conclusions we can immediately draw from the above discussion is that the minimal three-family $SU(6)$ model, *i.e.*, the $\Sigma 3$ model, does not seem to be phenomenologically viable, and we will not consider it any further.

Although the doublet-triplet splitting problem can be relatively easily solved in $SU(6)$ GUTs, there are some additional difficulties. The most pressing of those is generating the top-quark mass. Note that to generate masses for up-quarks in general (assuming that massless **20**'s of $SU(6)$ are absent), one needs couplings of the form $\mathbf{15} \cdot \mathbf{15} \cdot \mathbf{6} \cdot \mathbf{6} \cdot f(\mathbf{35})$. Here one of the $\overline{\mathbf{6}}$'s acquires vev at a high scale ($\sim M_{st}$), whereas the other one contains the electroweak doublet responsible for generating the up-quark masses. The form of the polynomial function $f(\mathbf{35})$ depends upon details of the scenario. In any case, if couplings of this type are absent in a given model, it can be rendered phenomenologically inviable. In Appendix F we give superpotentilas for the $SU(6)$ models $S1(1,1)$, $S1$, $S2$, $\Sigma 1$ and $\Sigma 2$. From these superpotentials one can see that the $S1(1,1)$, $S1$, and $\Sigma 1$ models do possess the above type of couplings, and we will investigate them further. The models $S2$ and $\Sigma 2$, however, lack such couplings altogether, and are, therefore, phenomenologically inviable. We will not discuss them any further.

Thus, we are left with the $S1(1,1)$, $S1$ and $\Sigma 1$ models to consider. First, we would like to address the issue of Yukawa mass matrices. The $S1(1,1)$ and $\Sigma 1$ models are very similar. In particular, the terms relevant for generating Yukawa mass matrices are of the same form. In Appendix D we present the details, so here we simply state the results: there are two top-like quark generations in these models, and unlike, say, the $T1(1,1)$ model, even fine-tuning does not seem to help in obtaining rank-1 up-quark Yukawa mass matrix.

Note that since in the $S1(1,1)$ and $\Sigma 1$ models there is no well-defined R-parity at the string scale (*i.e.*, the Higgs sector is not clearly separated from the matter sector), there will generically be R-parity violating terms once a $\mathbf{6} + \overline{\mathbf{6}}$ pair acquires vev. These terms come from the Yukawa couplings $\mathbf{15} \cdot \mathbf{6} \cdot \mathbf{6}$. Some degree of fine-tuning will be needed to suppress these R-parity violating terms.

Finally, we comment on the $S1$ model without going into any detail. The problems with

Yukawa mass matrices and vev-induced R-parity violating terms that we have encountered in the $T1(1,1)$, $S1(1,1)$ and $\Sigma 1$ models are present in the $S1$ model as well. (In this model, however, it appears to be possible to get rank-1 up-quark Yukawa mass matrix in the leading approximation at the cost of fine-tuning.) This can be verified by analyzing the superpotential for this model given in Appendix F.

C. Summary

In this section we saw that fine-tuning seems to be required to achieve doublet-triplet splitting in the three-family $SO(10)$ grand unified string models we are considering. In our $SU(6)$ models the doublet-triplet splitting problem can be solved via mechanisms already developed in the field theory context [30]. All of the models seem to require fine-tuning to suppress vev generated R-parity violating terms. In some of the models it is possible to obtain rank-1 up-quark mass matrix (in the leading order) at the cost of fine-tuning, whereas in other models even this seems to be problematic.

V. CONCLUSIONS AND OUTLOOK

None of the three-family grand unified string models considered in this review agree with phenomenology unless some fine-tuning is involved. The possibility of such fine-tuning is due to the presence of moduli, or flat directions, in the classical supersymmetric vacuum. These flat directions could be lifted after dynamical SUSY breaking. Then such fine-tuning could *a priori* arise via some very contrived non-perturbative dynamics (which by some "string miracle" would have to fix all the vevs at their fine-tuned values). The latter possibility seems unlikely taking into account that multiple vevs have to be fine-tuned (in *a priori* uncorrelated fashion).

Here we would like to make a comparison between the field theory and string GUTs. Naively, one might expect a "string miracle" (such as a stringy discrete symmetry) that could solve some of the problems present in field theory GUTs. This, however, does not seem to be the case in the three-family grand unified string models considered in this review. Thus, the doublet-triplet splitting problem in the $SO(10)$ and $SU(5)$ string models was not any less severe than in their field theory counterparts. On the contrary, absence of "exotic" higher dimensional Higgs fields in the massless spectrum has led us to the conclusion that the doublet-triplet splitting would require unnatural fine-tuning in these models. On the other hand, the doublet-triplet splitting problem in the $SU(6)$ string models would have to be achieved via the already well-known field theory mechanism [30]. Even in this case the field theory approach turns out to have an advantage: the successful solution of the doublet-triplet splitting problem does *not* guarantee proton stability as R-parity violating terms might still mediate too–rapid proton decay. Within field theory one can simply require R-parity violating terms to be absent already at the string scale. However, in our models there is no well-defined R-parity: there is simply no clear separation between the matter and Higgs sectors at the string scale. This leads to "vev induced" R-parity violating terms. Such R-parity violating terms are a common feature for all of our models, and could be present even in more generic classes of models (which are not necessarily string GUTs).

Finally, we would like to comment on one of the "original motivations" to consider string GUTs. As discussed in Introduction, in string GUTs the gauge and gravitational coupling unification is automatic. Here one subtlety arises. In order to have agreement with the experimental data (such as $\sin^2\theta_W$), one must assume that there are *not* too many extra light states (compared with the MSSM spectrum) below the GUT scale. Such states (if in incomplete $SU(5)$ multiplets) generally tend to raise the unification scale (for the fixed $\sin^2\theta_W$ prediction) at one loop order, and sometimes could also spoil the unification of couplings. If they are too light, then the GUT scale can rise beyond the string scale which would not be compatible with the phenomenological data. Perhaps, some of the most troublesome of such light states are those in the adjoint of $SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$. They are the left-over states from the GUT gauge group G breaking down to that of the Standard Model. Since the adjoint Higgs in G is a flat direction in perturbative heterotic superstring, it is massless perturbatively. Non-perturbative effects responsible for supersymmetry breaking can typically generate masses of the order of 1 TeV for these fields. This, however, is not acceptable since the GUT scale is pushed up to the string scale already for the adjoints as heavy as $\sim 10^{12} - 10^{13}$ GeV [35]. There exist model-dependent mechanisms that can generate such high masses for the adjoints. However, unless these masses are actually much higher (of the order of $10^{16} - 10^{17}$ GeV), the GUT scale is basically the same as the string scale. In this case the original motivation for the automatic unification of the gauge and gravitational couplings disappears. One could turn this point around and argue that this could be another mechanism for solving the gauge coupling unification problem in string theory [35]. However, the price one has to pay (doublet-triplet splitting problem) seems to be too high.

In conclusion we see that the idea of string GUTs has its advantages, but it also faces many problems. Our particular models do not seem to provide satisfactory solutions to these problems, yet they give us a flavor of what a stringy paradigm for GUT model building might be.

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APPENDIX A: SUPERSYMMETRY BREAKING

In this section we discuss supersymmetry breaking in the models $T1(1,1)$, $S1(1,1)$, $S1$, $\Sigma 1$, $\Sigma 2$ and $\Sigma 3$. (The $S2$ model with be considered separately in Appendix B.) Note that all of these models have one simple non-Abelian gauge group in the hidden sector. All the models except for the $\Sigma 1$ model have matter.

Let us be general here and consider $SU(N_c)$ hidden sector with $0 < N_f < N_c$ "flavors". The non-perturbative superpotential for this theory is given by [26]:

$$W_{non-pert} = (N_c - N_f) \left(\frac{\Lambda^{3N_c - N_f}}{\det(M)} \right)^{1/(N_c - N_f)} . \tag{A1}$$

Here $M^i_{\bar{j}} \equiv Q^i \tilde{Q}_{\bar{j}}$ are the meson fields (Q^i and $\tilde{Q}_{\bar{j}}$ are the fundamental quarks and anti-quarks), and Λ is the dynamically generated scale. This superpotential has a "runaway" behavior. That is, for any finite values of the meson vevs $M^i_{\bar{j}}$, the F-flatness conditions cannot be simultaneously satisfied and the superpotential does not vanish. This implies local supersymmetry breaking due to Kähler potential contributions [34]. Intuitively this can be understood by noting that once such a theory is coupled to supergravity there is a natural shut-down scale for all the runaway directions, namely, the Plank scale. Generically, this results in local supersymmetry breakdown. The supersymmetry breaking scale M_{SUSY} in the observable sector is given by

$$M_{SUSY} \sim \frac{|F|}{M_M} . \tag{A2}$$

Here M_M is the messenger scale, and F is the supersymmetry breaking F-term. (Note that for a field z the F-term F_z is given by $F_z = \partial W/\partial z + M_{Pl}^{-2} W \partial K/\partial z$, where K is the Kähler potential.) Assuming that the meson vevs are stabilized at $\sim M_{Pl}$, and gravity is the messenger of supersymmetry breaking (*i.e.*, $M_M \sim M_{Pl}$), the supersymmetry breaking scale can be estimated to be $M_{SUSY} \sim |W_{non-pert}|/M^2_{Pl}$, where all the vevs of the canonically normalized fields in $W_{non-pert}$ are taken to be $\sim M_{Pl}$. This gives

$$M_{SUSY} \sim \frac{1}{M^2_{Pl}} \left(\frac{M_H^{3N_c - N_f}}{M_{Pl}^{2N_f}} \right)^{\frac{1}{N_c - N_f}} = M_{st} \left(\frac{M_{st}}{M_{Pl}} \right)^{\frac{2N_c}{N_c - N_f}} \exp\left(-\frac{2\pi}{\alpha_{st}(N_c - N_f)}\right) . \tag{A3}$$

This formula is in fact also correct for $N_f = 0$. In this case we have gaugino condensate $\langle \lambda\lambda \rangle \sim M^3_H$. Then the standard gravity mediated [33] scenario predicts the following supersymmetry breaking scale in the observable sector:

$$M_{SUSY} \sim \frac{M^3_H}{M^2_{Pl}} = \frac{M^3_{st}}{M^2_{Pl}} \exp\left(-\frac{2\pi}{\alpha_{st} N_c}\right) . \tag{A4}$$

1. The $\Sigma 1$ Model

Since there is no matter in the $SU(3)_H$ hidden sector in this model, supersymmetry breaking would have to occur via gaugino condensation. In our case of $SU(3)_H$ with no matter we have $b_0 = 9$, and according to Eq (1) we have $M_H \sim 2 \times 10^{15}$ GeV. Then from Eq (A4) the scale $M_{SUSY} \gtrsim 10^5$ TeV. This is too high to be compatible with electroweak gauge hierarchy.

2. The $S1$ Model

The tree-level superpotential relevant for the hidden sector dynamics in this model reads (see Appendix F for details and Table V for notation):

$$\begin{aligned} W_{tree} = \quad & \lambda_1 U_0 (U_{++} U_{--} + U_{+-} U_{-+}) + \lambda_2 T_0 (\tilde{T}_+ U_{-+} + \tilde{T}_- U_{++}) \\ & + \lambda_3 \tilde{T}_0 T_- U_0 + \lambda_4 \tilde{T}_0 T_0 \tilde{U}_- \ . \end{aligned} \tag{A5}$$

Note that T_0, T_+, T_- are chiral triplets of $SU(3)_H$, whereas $\tilde{T}_0, \tilde{T}_+, \tilde{T}_-$ are chiral anti-triplets. U's are $SU(3)_H$ singlets charged under various $U(1)$'s of the model. Thus, the mass matrix $m_{ij} T_i \tilde{T}_j$ $(i, j = 0, +, -)$ reads:

$$(m_{ij}) = \begin{pmatrix} \lambda_4 \tilde{U}_- & \lambda_2 U_{-+} & \lambda_2 U_{++} \\ 0 & 0 & 0 \\ \lambda_3 U_0 & 0 & 0 \end{pmatrix} \ . \tag{A6}$$

Next, we have the following F-flatness conditions:

$$0 = \partial W / \partial U_{--} = U_0 U_{++} \ , \quad 0 = \partial W / \partial U_{+-} = U_0 U_{-+} \ . \tag{A7}$$

From this we conclude that generically one of the "flavors" of $SU(3)_H$ is heavy with the mass of the order of $\lambda U \sim M_X$. Therefore, below the anomalous $U(1)$ breaking scale M_X we can treat the hidden sector as $SU(3)_H$ with two "flavors". For the $S1$ model ($N_c = 3$ and $N_f = 2$) M_{SUSY} then comes out way too low. Note that if we assume that all four vevs $U_0, U_{++}, U_{-+}, \tilde{U}_-$ are zero, then supersymmetry is not broken at all.

3. The $T1(1,1)$, $S1(1,1)$ and $\Sigma 2$ Models

The hidden sector gauge group and matter content are the same for the models $T1(1,1)$, $S1(1,1)$ and $\Sigma 2$. Here we have $SU(2)_H$ with one "flavor". Using Eq (A3) one can easily verify that the scale M_{SUSY} also comes out way too low.

4. The $\Sigma 3$ Model

The massless spectrum of this model is given in Table VIII. Following the rules for computing superpotentials given in Ref [25], one can easily deduce the tree-level superpotential relevant for hidden sector dynamics:

$$W_{tree} = U_0(Q_+\tilde{Q}_- + Q_-\tilde{Q}_+) \ . \tag{A8}$$

Here $U_0 \equiv (\mathbf{1},\mathbf{1})(+6,0)$, $Q_\pm \equiv 2(\mathbf{4},\mathbf{1})(-3,-3)$, and $\tilde{Q}_\pm \equiv 2(\overline{\mathbf{4}},\mathbf{1})(-3,+3)$. Note that the first $U(1)$ is anomalous (with total trace anomaly equal -72), so that the field U_0 acquires vev to break it. Thus, the fields $Q_\pm, \tilde{Q}_\pm$ get mass of the order of the anomalous $U(1)$ breaking scale M_X, and there is only one "flavor" (namely, $Q \equiv (\mathbf{4},\mathbf{1})(+3,+3)$ and $\tilde{Q}_\pm \equiv (\overline{\mathbf{4}},\mathbf{1})(+3,-3)$) below this scale. The supersymmetry breaking scale in this model can be estimated to be $M_{SUSY} \gtrsim 10^4$ TeV, which is too high.

5. Comments

In section II we pointed out that there are models connected to the E_6 model (and, therefore, also to the models $T1(1,1)$ and $S1(1,1)$) with $SU(2)_H$ hidden sector with numbers of flavors $N_f > 2$. In general, for $SU(N_c)$ with $N_f > N_c$ non-perturbative superpotential is not generated, and supersymmetry is not broken. So to break supersymmetry in these models all but one of the "flavors" (there are no models with two "flavors") would have to acquire masses. Then we are back to the $T1(1,1)$ and $S1(1,1)$ models. As we see, in none of the above models does the supersymmetry breaking scale come out right. It is either too high (as in the $\Sigma1$ and $\Sigma3$ models), or way too low (as in the models $S1$, $T1(1,1)$, $S1(1,1)$ and $\Sigma2$). In the latter models one could try to replace M_{Pl} by M_X (which could be, say, anomalous $U(1)$ breaking scale) and argue that all the runaway vevs are stabilized at M_X, and also that the mediation of supersymmetry breaking is due to a messenger (different from gravity) whose scale M_M is lower than M_{Pl} (and is, perhaps, related to M_X). This does not really help since (as can readily be verified from Eq (A3)) the main suppression comes from the exponential factor $\exp(-2\pi/\alpha_{st}(N_c - N_f))$, unless M_M is taken to be rather low, namely, $M_M \sim 10^{10} - 10^{11}$ GeV. The latter possibility seems to be very unlikely since it is unclear what would generate such a low scale for messenger $U(1)$'s in the models $S1$, $T1(1,1)$, $S1(1,1)$ and $\Sigma2$, unless some fine-tuning is involved. (Note that the messengers in these models could be either gravity or $U(1)$'s, and the natural scale for the latter is M_{Pl} in the models $T1(1,1)$, $S1(1,1)$ and $\Sigma2$, and M_{Pl} or the anomalous $U(1)$ breaking scale M_X in the $S1$ model.) To avoid such a large suppression, one could try to decrease the number of flavors N_f, which *a priori* is not impossible if one involves higher dimensional operators (such as, say, λ_5 and λ_6 couplings in the E_6 superpotential given in Appendix E). A more careful examination of the corresponding superpotentials, however, reveals that in the $SO(10)$ models this would require giving **10**'s of $SO(10)$ large vevs which would break the Standard Model gauge group at a high scale. In the $SU(6)$ models one would ultimately have to turn on large vevs for **15**'s of $SU(6)$. This necessarily leads to breaking $SU(6)$ via $SU(5)\otimes U(1)$, and doublet-triplet splitting then cannot be achieved without fine-tuning (see section IV and Appendix D for details). We, therefore, conclude that the models at hand do not seem to give rise to realistic supersymmetry breaking patterns.

APPENDIX B: HIDDEN SECTOR DYNAMICS IN THE $S2$ MODEL

In this section we study the hidden sector dynamics in the $S2$ model. The tree-level superpotential relevant for the hidden sector dynamics in this model reads (see Appendix F for details and Table V for notation):

$$W_{tree} = \lambda_1 U_0 (d_{++}d_{--} + d_{+-}d_{-+}) + \lambda_2 (D_+ \tilde{d}_- \Delta_- + D_- \tilde{d}_+ \Delta_+) \ . \tag{B1}$$

Note that the third $U(1)$ in this model is anomalous. In order to break the anomalous $U(1)$, the field U_0 must acquire a vev. Let the corresponding scale be M_X. Then the doublets $d_{+\pm}$ and $d_{-\pm}$ of the second $SU(2)$ subgroup decouple at this scale. At the end (*i.e.*, below the scale M_X) we are left with the $SU(2) \otimes SU(2)$ hidden sector with the following matter content: $2(\Box, \Box)$ (from $\Delta_\pm$), $2(\Box, \mathbf{1})$ (from $D_\pm$), and $2(\mathbf{1}, \Box)$ (from $\tilde{d}_\pm$). Here $\Box$ stands for the fundamental, *i.e.*, doublet, of $SU(2)$. The couplings of the two $SU(2)$'s are the same at the string scale, but they are *a priori* different at the M_X scale for the their runnings between M_{st} and M_X are different. Thus, we have different dynamically generated scales Λ_1 and Λ_2 for the two $SU(2)$'s. The tree-level superpotential below the scale M_X then reads:

$$W_{tree} = \lambda_2 (D_+ \tilde{d}_- \Delta_- + D_- \tilde{d}_+ \Delta_+) \ . \tag{B2}$$

1. Exact Superpotential

Next, we would like to understand the non-perturbative dynamics in this model. For later convenience, let us introduce some more appropriate notations. Consider $SU(2)_1 \otimes SU(2)_2$ theory with l fields $Q_{a\alpha\dot{\alpha}}$ transforming as fundamentals under both groups, $a = 1, \ldots, l$; $2n$ $SU(2)_1$ fundamentals $L_{i\alpha}$, $i = 1, \ldots, 2n$; and $2m$ $SU(2)_2$ fundamentals $R_{p\dot{\alpha}}$, $p = 1, \ldots, 2m$. Here $\alpha = 1, 2$ and $\dot{\alpha} = 1, 2$ are the gauge indices of $SU(2)_1$ and $SU(2)_2$, respectively. We refer to this theory as $[l, n, m]$ theory. This theory is non-chiral thus mass terms are allowed for the matter fields. (In the $S2$ model we are interested in, we have the $[2, 1, 1]$ theory with $\Delta_\pm$ being $Q_{a\alpha\dot{\alpha}}$, $D_\pm$ being $L_{i\alpha}$ and $\tilde{d}_\pm$ being $R_{p\dot{\alpha}}$.)

We refer to a pair of fundamentals as one "flavor". The field content of the $[l, n, m]$ theory is summarized in the following table. The $[l, n, m]$ theory has global symmetry $SU(2l) \otimes SU(2n) \otimes SU(2m) \otimes U(1)_A \otimes U(1)_R$. Note that the $U(1)_R$ charge assignment is *not* unique.

	$SU(2)_1$	$SU(2)_2$	$SU(2l)$	$SU(2n)$	$SU(2m)$	$U(1)_A$	$U(1)_R$
$Q_{a\alpha\dot{\alpha}}$	$\Box$	$\Box$	$\Box$	1	1	1	0
$L_{i\alpha}$	$\Box$	1	1	$\Box$	1	1	0
$R_{p\dot{\alpha}}$	1	$\Box$	1	1	$\Box$	1	0
Λ_1^{6-l-n}	1	1	1	1	1	$2l+2n$	$4-2l-2n$
Λ_2^{6-l-m}	1	1	1	1	1	$2l+2m$	$4-2l-2m$

Let us concentrate on the $[2, 1, 1]$ theory. The classical D-flat directions are given by the following operators:

$$X_{ab} \equiv \frac{1}{2}\epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}Q_{a\alpha\dot\alpha}Q_{b\beta\dot\beta} \ , \tag{B3}$$

$$Y_{aip} \equiv \frac{1}{2}\epsilon^{\alpha\beta}\epsilon^{\dot\alpha\dot\beta}Q_{a\alpha\dot\alpha}L_{i\beta}R_{p\dot\beta} \ , \tag{B4}$$

$$\mathcal{L} \equiv \frac{1}{2}\epsilon^{ij}\epsilon^{\alpha\beta}L_{i\alpha}L_{j\beta} \ , \tag{B5}$$

$$\mathcal{R} \equiv \frac{1}{2}\epsilon^{pq}\epsilon^{\dot\alpha\dot\beta}R_{p\dot\alpha}R_{q\dot\beta} \ . \tag{B6}$$

There is the following classical constrain that the above flat directions satisfy:

$$\mathcal{L}\mathcal{R}X_{ab} = \mathcal{Y}_{ab} \ , \tag{B7}$$

where

$$\mathcal{Y}_{ab} \equiv \frac{1}{2}\epsilon^{ij}\epsilon^{pq}Y_{aip}Y_{bjq} \ . \tag{B8}$$

We would like to understand whether the classical moduli space is modified quantum mechanically, *i.e.*, to determine the exact superpotential. Our strategy will be to start from a known case and "integrate in" [36]. Then we can check our result in appropriate limits. The most convenient known case where we can "integrate in" was considered in Ref [36]. This is the $[1,1,1]$ theory. To get from the $[1,1,1]$ theory to the $[2,1,1]$ theory, we have to "integrate in" one bi-fundamental (*i.e.*, one Q field).

The exact non-perturbative superpotential for the $[1,1,1]$ theory reads [36]:

$$\tilde{\mathcal{W}} = A(X_{11}\mathcal{L}\mathcal{R} - \tilde\Lambda_1^4\mathcal{R} - \tilde\Lambda_2^4\mathcal{L} - \mathcal{Y}_{11}) \ . \tag{B9}$$

Here A is a Lagrange multiplier, and $\tilde\Lambda_1$ and $\tilde\Lambda_2$ are the dynamical scales of the two $SU(2)$'s. Next, consider the following superpotential (which is obtained by adding Yukawa perturbations and mass terms to Eq (B9)):

$$\begin{aligned}\tilde{\mathcal{W}}' = & A(X_{11}\mathcal{L}\mathcal{R} - m_0\Lambda_1^3\mathcal{R} - m_0\Lambda_2^3\mathcal{L} - \mathcal{Y}_{11}) \\ & -(m_0X_{22} + 2m_1X_{12} + \frac{m_1^2}{m_0}X_{11}) \\ & -\frac{1}{2}\epsilon^{ij}\epsilon^{pq}\lambda_{ip}(Y_{2jq} + \frac{m_1}{m_0}Y_{1jq}) \\ & -\frac{1}{8m_0}\epsilon^{ij}\epsilon^{pq}\lambda_{ip}\lambda_{jq}\mathcal{L}\mathcal{R} \ .\end{aligned} \tag{B10}$$

Here Λ_1 and Λ_2 are the dynamical scales of the two $SU(2)$'s in the $[2,1,1]$ theory. The scale matching is given by:

$$\tilde\Lambda_1^4 = m_0\Lambda_1^3 \ , \quad \tilde\Lambda_2^4 = m_0\Lambda_2^3 \ . \tag{B11}$$

Next, we integrate out the chiral superfields m_0, m_1 and λ_{ip} from the superpotential (B10). The result reads:

$$\mathcal{W} = -\frac{\det(\mathcal{L}\mathcal{R}X_{ab} - \mathcal{Y}_{ab})}{\mathcal{L}\mathcal{R}(\Lambda_1^3\mathcal{R} + \Lambda_2^3\mathcal{L})} \ . \tag{B12}$$

Here the determinant in the numerator is understood for the 2×2 matrix with elements labeled by the indices a, b.

From the superpotential (B12) we see that the classical moduli space is not modified quantum mechanically. The non-perturbative superpotential (B12) vanishes due to the classical constraints (B7). The exact superpotential then is given by the tree-level expression (B2)

$$W_{exact} = W_{tree} = \lambda_2(Y_{121} + Y_{212}) = \lambda_2(Q_1 L_2 R_1 + Q_2 L_1 R_2) \ . \tag{B13}$$

The theory has Higgs and Abelian Coulomb phases. (The Abelian Coulomb phase of this theory was recently discussed in Ref [38]). The latter occurs when both Q_1 and Q_2 acquire vevs to break $SU(2) \otimes SU(2) \supset SU(2)_{diag} \supset U(1)$. Note that if we give a vev to Q_1 in the direction that breaks $SU(2) \otimes SU(2)$ to $SU(2)_{diag}$, and also to the $SU(2)_{diag}$ singlet in Q_2, we will get the $N = 1$ theory with $SU(2)_{diag}$ gauge group and one chiral matter supermultiplet transforming in the adjoint of $SU(2)_{diag}$ (plus two singlets that are not charged under $SU(2)_{diag}$ and therefore decouple). This is the $N = 2$ theory discussed in Ref [37] ($N = 2$ $SU(2)$ Yang-Mills theory with no matter). The Abelian Coulomb branch mentioned above is then that of this $N = 2$ theory.

2. The $\Lambda_1 >> \Lambda_2$ Limit

The above results can be obtained in a different way, namely, by considering the limit where the two scales Λ_1 and Λ_2 are very different. For definiteness let us take $\Lambda_1 >> \Lambda_2$. In this limit we can deduce the superpotential as a sum of contributions generated by the two groups [39].

In fact, let us be more general and consider the $SU(N)_1 \otimes SU(N)_2$ theory with the following chiral matter content: $(\mathbf{N}_c, \overline{\mathbf{N}}_c)$, $(\overline{\mathbf{N}}_c, \mathbf{N}_c)$, $(\mathbf{N}_c, \mathbf{1})$, $(\overline{\mathbf{N}}_c, \mathbf{1})$, $(\mathbf{1}, \mathbf{N}_c)$, $(\mathbf{1}, \overline{\mathbf{N}}_c)$. Note that for $N_c = 2$ we have the $[2, 1, 1]$ theory considered in the previous subsection. We choose to study a more general case here as for $SU(2)$ the fundamental is pseudo-real, and certain features of the theory (such as presence of baryons) become more obscure.

Let the scales Λ_1 and Λ_2 of the two $SU(N_c)$'s be very different: $\Lambda_1 >> \Lambda_2$. Then the second $SU(N_c)$ can be regarded as dynamically inactive. We, therefore, have the $SU(N_c)_1$ theory with $N_f = N_c + 1$ flavors Q^i, $\tilde{Q}_{\bar{\imath}}$ ($i = 1, \ldots, N_f$, and similarly for $\bar{\imath}$). Here $Q^i = (\mathbf{N}_c, \overline{\mathbf{N}}_c)$ for $i = 1, \ldots, N_c$, and $Q^{N_c+1} = (\mathbf{N}_c, \mathbf{1})$. The fields $\tilde{Q}_{\bar{\imath}}$ are given by the complex conjugates of Q^i.

For the $N_f = N_c + 1$ theory the quantum moduli space is the same as the classical one [40]. The latter is described by the mesons $M^i_{\bar{\jmath}} = Q^i \tilde{Q}_{\bar{\jmath}}$, and baryons $B_i = \epsilon_{ij_1 \ldots j_{N_c}} Q^{j_1} \cdots Q^{j_{N_c}}$ and $\tilde{B}^{\bar{\imath}} = \epsilon^{\bar{\imath}\bar{\jmath}_1 \ldots \bar{\jmath}_{N_c}} \tilde{Q}_{\bar{\jmath}_1} \cdots \tilde{Q}_{\bar{\jmath}_{N_c}}$. The classical constraints read:

$$\begin{aligned} &(M^{-1})^{\bar{\jmath}}_i \det(M) - B_i \tilde{B}^{\bar{\jmath}} = 0 \ , \\ &M^i_{\bar{\jmath}} B_i = 0 \ , \quad M^i_{\bar{\jmath}} \tilde{B}^{\bar{\jmath}} = 0 \ . \end{aligned} \tag{B14}$$

Away from the origin $M = B = \tilde{B} = 0$, all the states in M, B and $\tilde{B}$ are physical and couple via the following superpotential [40]:

$$\mathcal{W}_{eff} = -\frac{1}{\Lambda_1^{2N_c-1}} (\det(M) - M^i_{\bar{j}} B_i \tilde{B}^{\bar{j}}) \ . \tag{B15}$$

(The classical constraints (B14) arise from Eq (B15) as equations of motion for these fields.) Away from the origin the number of independent massless degrees of freedom is the same as in the classical theory since the components of M, B and $\tilde{B}$ which are classically constrained acquire masses via the couplings in $\mathcal{W}_{eff}$ [40].

It is instructive to figure out which fields are generically massless, and what their charges are under the second $SU(N_c)$. We have (the r.h.s. is the representation of $SU(N_c)_2$):

$$M^{N_c+1}_{\overline{N_c+1}} = \mathbf{1} \ , \tag{B16}$$
$$M^{i}_{\overline{N_c+1}} = \overline{\mathbf{N}}_c \ (i = 1, \ldots, N_c) \ , \tag{B17}$$
$$M^{N_c+1}_{\bar{j}} = \mathbf{N}_c \ (\bar{j} = 1, \ldots, N_c) \ , \tag{B18}$$
$$M^{i}_{\bar{j}} = \mathbf{1} \oplus \mathbf{Adj} \ (i, \bar{j} = 1, \ldots, N_c) \ , \tag{B19}$$
$$B_{N_c+1} = \mathbf{1} \ , \tag{B20}$$
$$\tilde{B}^{\overline{N_c+1}} = \mathbf{1} \ , \tag{B21}$$
$$B_i = \mathbf{N}_c \ (i = 1, \ldots, N_c) \ , \tag{B22}$$
$$\tilde{B}^{\bar{j}} = \overline{\mathbf{N}}_c \ (\bar{j} = 1, \ldots, N_c) \ . \tag{B23}$$

Here $\mathbf{Adj}$ is the $N_c^2 - 1$ dimensional adjoint representation of $SU(N_c)$.

Let us see if we can recover the Abelian Coulomb branch discussed in the previous subsection. Consider the branch in the moduli space of mesons M and baryons B and $\tilde{B}$ where the singlet B_{N_c+1} acquires a vev. Then, according to the superpotential (B15), the two singlets $M^{N_c+1}_{\overline{N_c+1}}$ and $\tilde{B}^{\overline{N_c+1}}$ pair up and acquire a mass of the order of the B_{N_c+1} vev. Also, a linear combination of the fundamentals $M^{N_c+1}_{\bar{j}}$ and B_i (note that in this case the singlet in $M^i_{\bar{j}}$ can also acquire a vev) pairs up with the antifundamental $\tilde{B}^{\bar{j}}$, and these fields acquire a mass. Thus, we are left with the gauge group $SU(N_c)_2$, two singlets, one adjoint and two "flavors" (here we are taking into account the fields $(\mathbf{1}, \mathbf{N}_c)$, $(\mathbf{1}, \overline{\mathbf{N}}_c)$). Let us compare this directly to the classical result in the $SU(N_c)_1 \otimes SU(N_c)_2$ model. Let us give a vev to the field $(\mathbf{N}_c, \overline{\mathbf{N}}_c)$ in the direction that breaks the gauge group to the diagonal subgroup $SU(N_c)_{diag}$. Then the matter fields are two singlets, one adjoint and two "flavors". If we have an appropriate tree-level superpotential as in the $SU(2)_1 \otimes SU(2)_2$ case, the two "flavors" will become heavy, and we will have $SU(N_c)$ theory with chiral matter supermultiplet transforming in the adjoint of the gauge group. (This is $N = 2$ $SU(N_c)$ Yang-Mills with no matter.) One can explicitly check that provided such a superpotential is present at the tree-level, the two flavors in our above description (with $\Lambda_1 >> \Lambda_2$) precisely decouple to match the classical result.

Finally, we would like to return to the $SU(2)_1 \otimes SU(2)_2$ case and give the meson and baryon fields in the notations of the previous subsection. Note that for $SU(2)$ there is no invariant distinction between the mesons and baryons (as the fundamental of $SU(2)$ is pseudo-real). Let us introduce the following meson fields of $SU(2)_1$ in the limit $\Lambda_1 >> \Lambda_2$:

$$\mathcal{O}_{ab\dot{\alpha}\dot{\beta}} \equiv \frac{1}{\sqrt{2}} \epsilon^{\alpha\beta} Q_{a\alpha\dot{\alpha}} Q_{b\beta\dot{\beta}} \ , \quad V_{ai\dot{\alpha}} \equiv \frac{1}{\sqrt{2}} \epsilon^{\alpha\beta} Q_{a\alpha\dot{\alpha}} L_{i\beta} \ , \quad \mathcal{L} \ . \tag{B24}$$

Then we have the following correspondence between these meson fields and the mesons M and baryons B and $\tilde{B}$ (here we explicitly put $N_c = 2$):

$$\mathcal{L} = M^3_3 \tag{B25}$$
$$V_{12\dot{\alpha}} = M^i_3, \quad V_{21\dot{\alpha}} = M^3_{\bar{j}} \ , \quad (i, \bar{j} = 1, 2) \ , \tag{B26}$$
$$V_{11\dot{\alpha}} = \tilde{B}^{\bar{j}}, \quad V_{22\dot{\alpha}} = B_i \ , \quad (i, \bar{j} = 1, 2) \ , \tag{B27}$$
$$\mathcal{O}_{11\dot{\alpha}\dot{\beta}} = B_3 \ , \quad \mathcal{O}_{22\dot{\alpha}\dot{\beta}} = \tilde{B}^{\bar{3}} \ , \tag{B28}$$
$$\mathcal{O}_{12\dot{\alpha}\dot{\beta}} = M^i_{\bar{j}} \ , \quad (i, \bar{j} = 1, 2) \ . \tag{B29}$$

Note that in $\mathcal{O}_{11\dot{\alpha}\dot{\beta}}$ and $\mathcal{O}_{22\dot{\alpha}\dot{\beta}}$ the indices $\dot{\alpha}, \dot{\beta}$ must be antisymmetrized (this gives $SU(2)_2$ singlets), whereas in $\mathcal{O}_{12\dot{\alpha}\dot{\beta}}$ both antisymmetric ($SU(2)_2$ singlet) and symmetric ($SU(2)_2$ triplet) combinations are present.

3. Comments

We saw in the previous subsections that the classical moduli space is not modified in the hidden sector of the $S2$ model provided that the matter content is what we have been discussing so far. Therefore, supersymmetry with this matter content is *not* broken. One can ask a question whether at the string/anomalous $U(1)$ breaking scale by giving vevs to some of the fields Q_1, Q_2, L_1, L_2, R_1 and R_2 we can get a matter content (at the expense of breaking the gauge group) such that we have quantum modification in the moduli space of the resulting model once considered in the effective field theory. We already know that the fields Q_1 and Q_2 cannot do this. Since the spectrum is symmetric with respect to interchanging the two $SU(2)$'s, we are left to consider giving vevs to R_1 and R_2. Note that due to the D-flatness conditions if R_1 acquires a vev, then so must R_2. This breaks the gauge group from $SU(2)_1 \otimes SU(2)_2$ to $SU(2)_1$, *i.e.*, the $SU(2)_2$ subgroup is completely Higgsed. Note that due to the tree-level superpotential (B13), some of the matter fields become heavy, and (below the anomalous $U(1)$ breaking scale M_X) we are in fact left with one "flavor" of $SU(2)_1$ (coming from Q_1 and Q_2; L_1 and L_2 pair up with the other components of Q_1 and Q_2 and become heavy) plus a singlet. Note that we can arrive at this model (with completely Higgsed $SU(2)_2$ subgroup in the $S2$ model) starting from the $\Sigma 2$ model (see Table VI) and Higgsing the last two $U(1)$'s by giving vevs to the untwisted matter fields $(\mathbf{1}, \mathbf{1})(0, 0, +6, 0)$ and $(\mathbf{1}, \mathbf{1})(0, 0, -3, \pm 3)$. In fact, the massless spectrum and the tree-level superpotential of the resulting model are the same as those of the $S2$ model after the Higgsing described above.

Thus, below the anomalous $U(1)$ breaking scale M_X we have $SU(2)$ with one "flavor" (with $\tilde{b}_0 = 5$), whereas above the scale M_X we have $SU(2)$ with three "flavors" in the above scenario. We have already discussed this case in Appendix A where we saw that the supersymmetry breaking scale in this scenario comes out way too low.

APPENDIX C: HIDDEN SECTOR MATTER CONTENT OF THE $T5(1, 0)$ MODEL

Following the rules for computing superpotentials given in Ref [25], one can easily deduce the tree-level superpotential relevant for hidden sector dynamics (see first column of Table

III for notation):

$$W = \lambda \sum_{i,j=1}^{3} D_i (D_{i-} U_1 + D_{i+} U_2) \ . \tag{C1}$$

Upon the fields U_1 and U_2 acquiring vevs, D_i and 3 linear combinations of $D_{i\pm}$ become heavy and decouple. Below the corresponding scale we therefore have $SU(2) \otimes SU(2) \otimes SU(2)$ with (effectively) 6 "flavors" in each $SU(2)$ (some of these 6 "flavors" come as bi- and tri-fundamentals). The one-loop β-function coefficient for this matter content vanishes. Although the gauge group is asymptotically free in the two-loop order, the strong scale comes out way below the electroweak scale. The reader can readily verify that Higgsing the gauge group by giving vevs to bi- and/or tri-fundamentals cannot remedy this situation. We thus conclude that this model does not seem to give rise to the non-perturbative dynamics required by supersymmetry breaking at the phenomenologically desired scale.

APPENDIX D: SUSY GUT PHENOMENOLOGY OF $SU(6)$ MODELS

In this Appendix we provide some details on phenomenological properties of the $S1(1,1)$, $\Sigma1$ and $S1$ models. In particular, we study the Yukawa mass matrices. Our analysis of the corresponding superpotentials indicates that in the $S1(1,1)$ and $\Sigma1$ models there are two top-like generations of quarks, whereas in the $S1$ model generically there all three generations of quarks are top-like albeit with some fine-tuning one can achieve having only one top-like family.

Let us start with the branching rules of the $SU(6)$ irreps $\mathbf{35}$, $\mathbf{15}$, $\overline{\mathbf{6}}$ and $\mathbf{6}$ under the breaking $SU(6) \supset SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$:

$$\begin{aligned} \mathbf{35} = \ & (\mathbf{1},\mathbf{1})(0) + (\mathbf{1},\mathbf{1})(0) + (\mathbf{8},\mathbf{1})(0) + (\mathbf{1},\mathbf{3})(0) + (\mathbf{3},\mathbf{2})(-5/3) + (\overline{\mathbf{3}},\mathbf{2})(5/3) \\ & + (\overline{\mathbf{3}},\mathbf{1})(2/3) + (\mathbf{1},\mathbf{2})(1) + (\mathbf{3},\mathbf{1})(-2/3) + (\mathbf{1},\mathbf{2})(-1) \ , \end{aligned} \tag{D1}$$

$$\mathbf{15} = \ \overset{E}{(\mathbf{1},\mathbf{1})}(2) + \overset{H_u}{(\mathbf{3},\mathbf{1})}(-2/3) + \overset{U}{(\overline{\mathbf{3}},\mathbf{1})}(-4/3) + \overset{h_u}{(\mathbf{1},\mathbf{2})}(1) + \overset{Q}{(\mathbf{3},\mathbf{2})}(1/3) \ , \tag{D2}$$

$$\overline{\mathbf{6}} = \ \overset{L,\, h_d}{(\mathbf{1},\mathbf{2})}(-1) + \overset{D, H_d}{(\overline{\mathbf{3}},\mathbf{1})}(+2/3) + \overset{\tilde{\mathbf{s}}}{(\mathbf{1},\mathbf{1})}(0) \ , \tag{D3}$$

$$\mathbf{6} = \ \overset{h_u}{(\mathbf{1},\mathbf{2})}(1) + \overset{H_u}{(\mathbf{3},\mathbf{1})}(-2/3) + \overset{\mathbf{s}}{(\mathbf{1},\mathbf{1})}(0) \ . \tag{D4}$$

Here h_u, h_d denote Higgs doublets, H_u, H_d - Higgs triplets and the usual notations for chiral fermions are used. As discussed earlier, the possibility of h_u coming from $\mathbf{15}$ is disfavored for phenomenological reasons (doublet-triplet splitting in this case would require fine-tuning). Note that $\mathbf{6}$'s and $\overline{\mathbf{6}}$'s contain singlets $\mathbf{s}$ and $\tilde{\mathbf{s}}$. Two linear combinations (one coming from $\mathbf{6}$'s, and the other one coming from $\overline{\mathbf{6}}$'s) of these singlets vevs at a high scale. (The rest of them either become heavy via the couplings $\mathbf{6}\cdot\mathbf{6}\cdot\overline{\mathbf{6}}\cdot\overline{\mathbf{6}}$ or describe right-handed neutrinos and anti-neutrinos.) From the above equations one can see that the up-quark (meaning u, c, t)Yukawa matrix comes from the couplings $\mathbf{15}\cdot\mathbf{15}\cdot\mathbf{6}\cdot\mathbf{6}$ (times some power of the adjoint in certain cases), while the down-quark (meaning d, s, b) and lepton Yukawa matrices come from the couplings $\mathbf{15}\cdot\overline{\mathbf{6}}\cdot\overline{\mathbf{6}}$ in the superpotential. (The latter type of couplings are also responsible for decoupling of extra states in $\mathbf{15}$'s and $\overline{\mathbf{6}}$'s once a linear combination of $\tilde{\mathbf{s}}$ fields acquires a vev. This would, among other states, remove the extra h_u fields in $\mathbf{15}$'s.)

1. The $S1(1,1)$ Model

This model has five **15**'s Q_i and Q_0 ($i = ++, --, +-, -+$), two $\overline{\mathbf{15}}$'s $\tilde{Q}_\pm$, ten $\overline{\mathbf{6}}$'s $H_0^\pm$ and $H_i^\pm$, and four **6**'s $\tilde{H}_\pm^+$ and $\tilde{H}_\pm^-$ (see Table IV).

The **15**-$\overline{\mathbf{15}}$ pairing takes place via the $Q \cdot \tilde{Q} \cdot H \cdot \widetilde{H}$ couplings. The corresponding 4×2 matrix (Q_0 does not couple to $\tilde{Q}_\pm$) generically has rank two. (This is to be contrasted with the $T1(1,1)$ model, where the F-flatness constraints allow only one "generation-antigeneration" pair to decouple. In the $S1(1,1)$ model the F-flatness constraints relevant for the **15**-$\overline{\mathbf{15}}$ pairing can be read-off from the $H \cdot H \cdot \tilde{H} \cdot \tilde{H}$ term in the superpotential.) Thus two linear combinations of the fields Q_i become heavy. Writing down the up-quark Yukawa matrix for the remaining three **15**'s, namely, Q_0 and the two linear combinations of the fields Q_i that remain light, one obtains a rank-2 matrix, *i.e.*, two top-like heavy quarks and one light quark (the latter coming coming from Q_0). The corresponding term in the superpotential (see Appendix F) reads $(\tilde{H}_-^-\tilde{H}_+^+ + \tilde{H}_+^-\tilde{H}_-^+)(Q_{++}Q_{--} + Q_{+-}Q_{-+})$.

2. The $\Sigma1$ Model

This model has five **15**'s F_i and F_0 ($i = ++, --, +-, -+$), two $\overline{\mathbf{15}}$'s $\tilde{F}_\pm$, eight $\overline{\mathbf{6}}$'s $\tilde{S}_i$ and $\tilde{S}'_i$, and two **6**'s $S_\pm$ (see Table VI). This model is very similar to the $S1(1,1)$ model. Thus, the **15**-$\overline{\mathbf{15}}$ pairing takes place at a high scale (λ_{19} term in the superpotential - see Appendix F) leaving us with three **15**'s. These have a rank-2 up-quark Yukawa matrix via the coupling $\lambda_{23}S_+S_-(F_{++}F_{--} + F_{+-}F_{-+})$. λ_{21} and λ_{22} terms do not contribute to the Yukawa matrix because of the F-flatness condition $\partial W/\partial U_0 = U_{++}U_{--} + U_{+-}U_{-+} = 0$. Thus, just as in the $S1(1,1)$ model, we have two top-like quark families in the $\Sigma1$ model.

3. The $S1$ Model

This model has six **15**'s $F_\pm^A$, ($A = 1,2,3$), three $\overline{\mathbf{15}}$'s $\tilde{F}^A$, nine $\overline{\mathbf{6}}$'s $\tilde{S}_0^A$ and $\tilde{S}_\pm^A$, and three **6**'s S^A (see Table V).

Three **15**-$\overline{\mathbf{15}}$ pairs generically decouple at a high scale (via the couplings λ_{15} through λ_{24}). The rest of the **15**'s have a rank-3 up-quark Yukawa matrix (via the couplings λ_{25} through λ_{36}). Some fine-tuning is therefore required to have only one top-like generation of quarks.

APPENDIX E: SUPERPOTENTIALS FOR THE E_6 AND $SO(10)$ MODELS

In this Appendix we give superpotentials for the $E1$ and $T1(1,1)$ models. They were derived in Ref [25].

1. The E_6 Model

The superpotential for the $E1$ model (see Table I for notation) reads:

$$
\begin{aligned}
W = {} & \lambda_1 \chi_0 (\chi_{++}\chi_{--} + \chi_{+-}\chi_{-+})\Phi + \lambda_2 \tilde{\chi}_+ \tilde{\chi}_- (\chi_{++}\chi_{--} + \chi_{+-}\chi_{-+}) \\
& + \lambda_3 U_0 (U_{++}U_{--} + U_{+-}U_{-+}) + \lambda_4 [\chi_{++}^3 U_{--} + \chi_{+-}^3 U_{-+} + \chi_{-+}^3 U_{+-} + \chi_{--}^3 U_{++}]\Phi^2 \\
& + \lambda_5 (\chi_0)^3 U_0 D_+ D_- \Phi^2 + \lambda_6 [(\tilde{\chi}_+)^3 \tilde{U}_- + \tilde{\chi}_-^3 \tilde{U}_+] D_+ D_- \Phi + \dots ,
\end{aligned} \tag{E1}
$$

where traces over the irreps of the gauge group are implicit. Here $\lambda_k \equiv \lambda_k(\Phi^3, D_+D_-)$ are certain polynomials of their arguments

$$
\lambda_k(\Phi^3, D_+D_-) = \sum_{m,n} \lambda_{kmn} \Phi^{3m} (D_+D_-)^n \tag{E2}
$$

such that $\lambda_{km0}, \lambda_{k0n} \neq 0$.

2. The $T1(1,1)$ Model

The superpotential for the $T1(1,1)$ model can be deduced from that of the E_6 model by adjoint breaking $E_6 \supset SO(10) \otimes U(1)$. It reads (see Table II for notation):

$$
\begin{aligned}
W = {} & \lambda_1 \left[Q_0 (Q_{++}H_{--} + Q_{+-}H_{-+} + Q_{-+}H_{+-} + Q_{--}H_{++}) \right. \\
& + H_0 (Q_{++}Q_{--} + Q_{+-}Q_{-+}) + H_0 (H_{++}S_{--} + H_{+-}S_{-+} + H_{-+}S_{+-} + H_{--}S_{++}) \\
& \left. + S_0 (H_{++}H_{--} + H_{+-}H_{-+}) \right] (\Phi + \phi') \\
& + \lambda_2 \Big[\tilde{Q}_+ \tilde{Q}_- (Q_{++}Q_{--} + Q_{+-}Q_{-+}) \\
& + \tilde{Q}_+ \tilde{Q}_- (H_{++}S_{--} + H_{+-}S_{-+} + H_{-+}S_{+-} + H_{--}S_{++}) \\
& + (\tilde{Q}_+ \tilde{S}_- + \tilde{Q}_- \tilde{S}_+)(Q_{++}S_{--} + Q_{+-}S_{-+} + Q_{-+}S_{+-} + Q_{--}S_{++}) \\
& + (\tilde{H}_+ \tilde{S}_- + \tilde{H}_- \tilde{S}_+)(H_{++}S_{--} + H_{+-}S_{-+} + H_{-+}S_{+-} + H_{--}S_{++}) \\
& + (\tilde{H}_+ \tilde{S}_- + \tilde{H}_- \tilde{S}_+)(Q_{++}Q_{--} + Q_{+-}Q_{-+}) \\
& + (\tilde{Q}_+ \tilde{H}_- + \tilde{Q}_- \tilde{H}_+)(Q_{++}H_{--} + Q_{+-}H_{-+} + Q_{-+}H_{+-} + Q_{--}H_{++}) \\
& + \tilde{H}_+ \tilde{H}_- (H_{++}H_{--} + H_{+-}H_{-+}) + \tilde{S}_+ \tilde{S}_- (S_{++}S_{--} + S_{+-}S_{-+}) \Big] \\
& + \lambda_3 U_0 (U_{++}U_{--} + U_{+-}U_{-+}) \\
& + \lambda_4 \Big[(Q_{++}^2 H_{++} + H_{++}^2 S_{++}) U_{--} + (Q_{+-}^2 H_{+-} + H_{+-}^2 S_{+-}) U_{-+} \\
& + (Q_{-+}^2 H_{-+} + H_{-+}^2 S_{-+}) U_{+-} + (Q_{--}^2 H_{--} + H_{--}^2 S_{--}) U_{++} \Big] (\Phi + \phi')^2 + \dots ,
\end{aligned} \tag{E3}
$$

where the Clebsch-Gordan coefficients are suppressed. The field ϕ' is defined to be

$$
\phi' = \phi + \langle \phi \rangle \ . \tag{E4}
$$

Here $\lambda_k \equiv \lambda_k((\Phi + \phi')^3, D_+D_-)$ are defined in the same way as in the E_6 model.

APPENDIX F: SUPERPOTENTIALS FOR $SU(6)$ MODELS

In this Appendix we give superpotentials for the $S1(1,1)$, $S1$, $S2$, $\Sigma 1$ and $\Sigma 2$ models. The superpotential for the $S1(1,1)$ can be deduced from that of the E_6 model (see Appendix E) by adjoint breaking $E_6 \supset SU(6) \otimes U(1)$. The superpotentials for the $S1$ and $S2$ models were derived in Ref [25]. Those for the $\Sigma 1$ and $\Sigma 2$ models can be obtained following the rules of Ref [25].

1. The $S1(1,1)$ Model

The superpotential for the $S1(1,1)$ model (see Table IV for notation) reads:

$$
\begin{aligned}
W = &\lambda_1[Q_0(Q_{++}Q_{--} + Q_{+-}Q_{-+}) + Q_0 \sum_{i,j} z_{ij}(H^i_{++}H^j_{--} + H^i_{+-}H^j_{-+}) \\
&+ \sum_{i,j} z_{ij} H^i_0 (Q_{++}H^j_{--} + Q_{+-}H^j_{-+} + Q_{-+}H^j_{+-} + Q_{--}H^j_{++})](\Phi + \phi') \\
&+ \lambda_2[\tilde{Q}_+\tilde{Q}_-(Q_{++}Q_{--} + Q_{+-}Q_{-+}) \\
&+ \tilde{Q}_+\tilde{Q}_- \sum_{i,j} z_{ij}(H^i_{++}H^j_{--} + H^i_{+-}H^j_{-+}) + \sum_{i,j} z_{ij}\tilde{H}^i_+\tilde{H}^j_-(Q_{++}Q_{--} + Q_{+-}Q_{-+}) \\
&+ \sum_{i,j,k,l} y_{ijkl}\tilde{H}^i_+\tilde{H}^j_-(H^k_{++}H^l_{--} + H^k_{+-}H^l_{-+}) \\
&+ \sum_{i,j} z_{ij}(\tilde{Q}_+\tilde{H}^i_- + \tilde{Q}_-\tilde{H}^i_+)(Q_{++}H^j_{--} + Q_{+-}H^j_{-+} + Q_{-+}H^j_{+-} + Q_{--}H^j_{++})] \\
&+ \lambda_3 U_0(U_{++}U_{--} + U_{+-}U_{-+}) \\
&+ \lambda_4\Big[(Q^3_{++} + Q_{++}H^+_{++}H^-_{++})U_{--} + (Q^3_{+-} + Q_{+-}H^+_{+-}H^-_{+-})U_{-+} \\
&+ (Q^3_{-+} + Q_{-+}H^+_{-+}H^-_{-+})U_{+-} + (Q^3_{--} + Q_{--}H^+_{--}H^-_{--})U_{++}\Big](\Phi + \phi')^2 + \ldots\,,
\end{aligned}
\qquad \text{(F1)}
$$

where the Clebsch-Gordan coefficients are suppressed, and the coefficients z_{ij} and y_{ijkl} are defined as follows: $z_{ij} = 1 - \delta_{ij}$, $y_{ijkl} = 1$ if $(i,j,k,l) = (+,+,-,-)$ or its permutations, and $y_{ijkl} = 0$ otherwise. Here $i,j = +,-$. The couplings λ_k are defined in the same way as in the $T1(1,1)$ model.

2. The $S1$ Model

The superpotential for the $S1$ model reads (see Table V for notation):

$$
\begin{aligned}
W = &\lambda_1 U_0(U_{++}U_{--} + U_{+-}U_{-+}) \; + \; \lambda_2 T_0(\tilde{T}_+U_{-+} + \tilde{T}_-U_{++}) \; + \; \lambda_3\tilde{T}_0T_-U_0 \; + \; \lambda_4\tilde{T}_0T_0\tilde{U}_- \\
&+ \lambda_5 \sum_{A,B,C} y_{ABC}[U_{++}(F^A_+F^B_+F^C_- + \frac{1}{3}F^A_-F^B_-F^C_-) + U_{-+}(F^A_+F^B_-F^C_- + \frac{1}{3}F^A_+F^B_+F^C_+)] \\
&+ [\lambda_6\Phi^2 + \lambda_7\Phi\phi + \lambda_8\phi^2] \sum_A \{U_{++}[(F^A_-)^3 + F^A_-(F^A_+)^2] + U_{-+}[(F^A_+)^3 + F^A_+(F^A_-)^2]\} \\
&+ \lambda_9 \sum_A [F^A_+\tilde{S}^A_- + F^A_-\tilde{S}^A_+]\tilde{S}^A_0 \; + \; [\lambda_{10}\Phi + \lambda_{11}\phi] \sum_{A,B,C} y_{ABC}(F^A_+\tilde{S}^B_- + F^A_-\tilde{S}^B_+)\tilde{S}^C_0 \\
&+ \lambda_{12}\tilde{U}_+T_+\tilde{T}_0 \sum_{A,B,C} y_{ABC}\tilde{F}^A\tilde{F}^B\tilde{F}^C \; + \; [\lambda_{13}\Phi + \lambda_{14}\phi]\tilde{U}_+T_+\tilde{T}_0 \sum_A (\tilde{F}^A)^3 \\
&+ \lambda_{15} \sum_{A,B} z_{AB}\tilde{F}^A S^B(F^A_+\tilde{S}^B_- + F^A_-\tilde{S}^B_+ + F^B_+\tilde{S}^A_- + F^B_-\tilde{S}^A_+) \\
&+ [\lambda_{16}\Phi + \lambda_{17}\phi] \sum_{A,B,C} y_{ABC}\tilde{F}^A S^A(F^B_+\tilde{S}^C_- + F^B_-\tilde{S}^C_+) \\
&+ [\lambda_{18}\Phi^2 + \lambda_{19}\Phi\phi + \lambda_{20}\phi^2] \sum_{A,B,C} y_{ABC}[F^A_+\tilde{S}^A_- + F^A_-\tilde{S}^A_+]\tilde{F}^B S^C
\end{aligned}
$$

$$
\begin{aligned}
&+ [\lambda_{21}\Phi^3 + \lambda_{22}\Phi^2\phi + \lambda_{23}\Phi\phi^2 + \lambda_{24}\phi^3] \ \sum_A [F_+^A \tilde{S}_-^A + F_-^A \tilde{S}_+^A] \tilde{F}^A S^A \\
&+ \lambda_{25} \sum_{A,B,C} y_{ABC} [(F_+^A)^2 + (F_-^A)^2] S^B S^C (U_{++}U_{+-} + U_{-+}U_{--}) \\
&+ \lambda_{26} \sum_{A,B,C} y_{ABC} F_+^A F_-^A S^B S^C (U_{++}U_{--} + U_{+-}U_{-+}) \\
&+ [\lambda_{27}\Phi + \lambda_{28}\phi] \sum_{A,B} z_{AB} F_+^A F_+^B S^A S^B (U_{++}U_{+-} + U_{-+}U_{--}) \\
&+ [\lambda_{29}\Phi^2 + \lambda_{30}\Phi\phi + \lambda_{31}\phi^2] \ \sum_{A,B,C} y_{ABC} F_+^A F_+^B (S^C)^2 (U_{++}U_{+-} + U_{-+}U_{--}) \\
&+ [\lambda_{32}\Phi + \lambda_{33}\phi] \ \sum_{A,B} z_{AB} F_+^A F_-^B S^A S^B (U_{++}U_{--} + U_{+-}U_{-+}) \\
&+ [\lambda_{34}\Phi^2 + \lambda_{35}\Phi\phi + \lambda_{36}\phi^2] \ \sum_{A,B,C} y_{ABC} F_+^A F_-^B (S^C)^2 (U_{++}U_{--} + U_{+-}U_{-+}) + \dots \, ,
\end{aligned} \tag{F2}
$$

where $\lambda_k \equiv \lambda_k(\Phi, \phi)$ are certain polynomials of their arguments

$$
\lambda_k(\Phi, \phi) = \sum_{m,n} \lambda_{kmn} \Phi^{3m-n} \phi^n \tag{F3}
$$

such that $\lambda_{km0}, \lambda_{k0n} \neq 0$. The coefficients y_{ABC} and z_{AB} are defined as follows: $y_{ABC} = \epsilon_{ABC}$, and $z_{AB} = 1 - \delta_{AB}$.

3. The $S2$ Model

The superpotential for the $S2$ model reads (see Table V for notation):

$$
\begin{aligned}
W = \ &\lambda_1 U_0 (d_{++}d_{--} + d_{+-}d_{-+}) \ + \ \lambda_2 (D_+ \tilde{d}_- \Delta_- + D_- \tilde{d}_+ \Delta_+) \\
&+ \lambda_3 U_0 \Delta_+ \Delta_- \sum_{A,B,C} y_{ABC} F^A F^B F^C \ + \ [\lambda_4 \Phi^2 + \lambda_5 \Phi\phi + \lambda_6 \phi^2] U_0 \Delta_+ \Delta_- \sum_A (F^A)^3 \\
&+ \lambda_7 \sum_A F^A [\tilde{S}_{++}^A \tilde{S}_{--}^A + \tilde{S}_{+-}^A \tilde{S}_{-+}^A] \ + \ [\lambda_8 \Phi + \lambda_9 \phi] \sum_{A,B,C} y_{ABC} F^A (\tilde{S}_{++}^B \tilde{S}_{--}^C + \tilde{S}_{-+}^B \tilde{S}_{+-}^C) \\
&+ \lambda_{10} \sum_{A,B} z_{AB} S_+^A S_-^B (\tilde{S}_{++}^A \tilde{S}_{--}^B + \tilde{S}_{-+}^A \tilde{S}_{+-}^B + \tilde{S}_{+-}^A \tilde{S}_{-+}^B + \tilde{S}_{--}^A \tilde{S}_{++}^B) \\
&+ [\lambda_{11}\Phi^3 + \lambda_{12}\Phi^2\phi + \lambda_{13}\Phi\phi^2 + \lambda_{14}\phi^3] \ \sum_A S_+^A S_-^A (\tilde{S}_{++}^A \tilde{S}_{--}^A + \tilde{S}_{+-}^A \tilde{S}_{-+}^A) \\
&+ [\lambda_{15}\Phi + \lambda_{16}\phi] \sum_{A,B,C} y_{ABC} S_+^A S_-^A (\tilde{S}_{++}^B \tilde{S}_{--}^C + \tilde{S}_{-+}^B \tilde{S}_{+-}^C) \\
&+ [\lambda_{17}\Phi^2 + \lambda_{18}\Phi\phi + \lambda_{19}\phi^2] \ \sum_{A,B,C} y_{ABC} S_+^B S_-^C (\tilde{S}_{++}^A \tilde{S}_{--}^A + \tilde{S}_{+-}^A \tilde{S}_{-+}^A) + \dots \, .
\end{aligned} \tag{F4}
$$

The couplings λ_k and the coefficients y_{ABC} and z_{AB} are defined in the same way as in the $S1$ model.

4. The $\Sigma1$ Model

The superpotential for the $\Sigma1$ model reads (see Table VI for notation):

$$
\begin{aligned}
W = {} & \lambda_1 U_0(U_{++}U_{--} + U_{+-}U_{-+}) \quad + \quad (\lambda_2\Phi + \lambda_3\phi)\, F_0(F_{++}F_{--} + F_{+-}F_{-+}) \\
& + \left(\lambda_4\Phi^2 + \lambda_5\Phi\phi + \lambda_6\phi^2\right)\left(U_{++}F^3_{--} + U_{+-}F^3_{-+} + U_{-+}F^3_{+-} + U_{--}F^3_{++}\right) \\
& + \left(\lambda_7\Phi^2 + \lambda_8\Phi\phi + \lambda_9\phi^2\right) U_0^2\,(U_{++}U_{--} + U_{+-}U_{-+})\, F_0^3 \\
& + (\lambda_{10}\Phi + \lambda_{11}\phi)\, U_0\,(U_{++}U_{--} + U_{+-}U_{-+})\left(\tilde{U}_+\tilde{F}^3_- + \tilde{U}_-\tilde{F}^3_+\right) \\
& + \left(\lambda_{12}\Phi^2 + \lambda_{13}\Phi\phi + \lambda_{14}\phi^2\right) U_0 F_0\left(\tilde{S}_{++}\tilde{S}_{--} + \tilde{S}_{+-}\tilde{S}_{-+}\right) \\
& + (\lambda_{15}\Phi + \lambda_{16}\phi)\, U_0\,(U_{++}U_{--} + U_{+-}U_{-+})\, F_0\left(\tilde{S}'_{++}\tilde{S}'_{--} + \tilde{S}'_{+-}\tilde{S}'_{-+}\right) \\
& + (\lambda_{17}\Phi + \lambda_{18}\phi)\,[F_{++}(\tilde{S}_{-+}\tilde{S}'_{--} + \tilde{S}_{--}\tilde{S}'_{-+}) + F_{+-}(\tilde{S}_{-+}\tilde{S}'_{+-} + \tilde{S}_{--}\tilde{S}'_{++}) \\
& \qquad + F_{-+}(\tilde{S}_{++}\tilde{S}'_{--} + \tilde{S}_{+-}\tilde{S}'_{-+}) + F_{--}(\tilde{S}_{++}\tilde{S}'_{+-} + \tilde{S}_{+-}\tilde{S}'_{++})] \\
& + \lambda_{19}[(F_{++}S_- + F_{+-}S_+)(\tilde{S}_{--}\tilde{F}_+ + \tilde{S}_{-+}\tilde{F}_-) + (F_{-+}S_- + F_{--}S_+)(\tilde{S}_{+-}\tilde{F}_+ + \tilde{S}_{++}\tilde{F}_-)] \\
& + \lambda_{20}U_0\,(U_{++}U_{--} + U_{+-}U_{-+})\, F_0\{S_+\left(\tilde{S}'_{++}\tilde{F}_- + \tilde{S}'_{+-}\tilde{F}_+\right) + S_-\left(\tilde{S}'_{-+}\tilde{F}_- + \tilde{S}'_{--}\tilde{F}_+\right)\} \\
& + (\lambda_{21}\Phi + \lambda_{22}\phi)\,(U_{++}U_{--} + U_{+-}U_{-+})\left(F_{+-}F_{--}S_+^2 + F_{++}F_{-+}S_-^2\right) \\
& + \lambda_{23}\,(F_{++}F_{--} + F_{+-}F_{-+})\, S_+S_- \\
& + \left(\lambda_{24}\Phi^2 + \lambda_{25}\Phi\phi + \lambda_{26}\phi^2\right)\left(U_0 + \tilde{U}_+\tilde{U}_-\right)\left(\tilde{F}_+^2\tilde{S}_{+-}\tilde{S}_{--} + \tilde{F}_-^2\tilde{S}_{++}\tilde{S}_{-+}\right) \\
& + (\lambda_{27}\Phi + \lambda_{28}\phi)\, U_0\,(U_{++}U_{--} + U_{+-}U_{-+})\left(\tilde{F}_+^2\tilde{S}'_{+-}\tilde{S}'_{--} + \tilde{F}_-^2\tilde{S}'_{++}\tilde{S}'_{-+}\right) \\
& + \left(\lambda_{29}\Phi^2 + \lambda_{30}\Phi\phi + \lambda_{31}\phi^2\right)\left(U_0 + \tilde{U}_+\tilde{U}_-\right)\tilde{F}_+\tilde{F}_-\left(\tilde{S}_{++}\tilde{S}_{--} + \tilde{S}_{+-}\tilde{S}_{-+}\right) \\
& + (\lambda_{32}\Phi + \lambda_{33}\phi)\, U_0\,(U_{++}U_{--} + U_{+-}U_{-+})\,\tilde{F}_+\tilde{F}_-\left(\tilde{S}'_{++}\tilde{S}'_{--} + \tilde{S}'_{+-}\tilde{S}'_{-+}\right) \\
& + (\lambda_{34}\Phi + \lambda_{35}\phi)\,(U_{++}U_{--} + U_{+-}U_{-+})\left(S_+^2\tilde{S}'_{-+}\tilde{S}'_{--} + S_-^2\tilde{S}'_{++}\tilde{S}'_{+-}\right) \\
& + (\lambda_{36}\Phi + \lambda_{37}\phi)\, S_+S_-\left(\tilde{S}'_{++}\tilde{S}'_{--} + \tilde{S}'_{+-}\tilde{S}'_{-+}\right) + \dots\ .
\end{aligned} \tag{F5}
$$

The couplings λ_k are defined in the same way as in the $S1$ model.

5. The $\Sigma 2$ Model

The superpotential for the $\Sigma 2$ model reads (see Table VI for notation):

$$
\begin{aligned}
W = {} & (\lambda_1\Phi + \lambda_2\phi)\,\Delta_+\Delta_- F^1F^2F^3 + \left(\lambda_3\Phi^2 + \lambda_4\Phi\phi + \lambda_5\phi^2\right)\Delta_+\Delta_-\sum_A U_A(F^A)^3 \\
& + \left(\lambda_6\Phi^2 + \lambda_7\Phi\phi + \lambda_8\phi^2\right)\sum_A U_AF^A\left(\tilde{S}^A_{++}\tilde{S}^A_{--} + \tilde{S}^A_{+-}\tilde{S}^A_{-+}\right) \\
& + (\lambda_9\Phi + \lambda_{10}\phi)\sum_{A,B,C} y_{ABC}F^A\left(\tilde{S}^B_{++}\tilde{S}^C_{--} + \tilde{S}^B_{+-}\tilde{S}^C_{-+}\right) \\
& + \left(\lambda_{11}\Phi^2 + \lambda_{12}\Phi\phi + \lambda_{13}\phi^2\right)\sum_A S^A_+S^A_-\left(\tilde{S}^A_{++}\tilde{S}^A_{--} + \tilde{S}^A_{+-}\tilde{S}^A_{-+}\right) \\
& + (\lambda_{14}\Phi + \lambda_{15}\phi)\sum_{A,B} z_{AB}S^A_+S^B_-\left(\tilde{S}^A_{++}\tilde{S}^B_{--} + \tilde{S}^A_{+-}\tilde{S}^B_{-+} + \tilde{S}^A_{-+}\tilde{S}^B_{+-} + \tilde{S}^A_{--}\tilde{S}^B_{++}\right) \\
& + (\lambda_{16}\Phi + \lambda_{17}\phi)\sum_{A,B,C} y_{ABC}S^A_+S^A_-U_BU_C\left(\tilde{S}^B_{++}\tilde{S}^C_{--} + \tilde{S}^B_{+-}\tilde{S}^C_{-+}\right)
\end{aligned}
$$

$$+\left(\lambda_{18}\Phi^2+\lambda_{19}\Phi\phi+\lambda_{20}\phi^2\right)\sum_{A,B,C} y_{ABC}U_A S_+^B S_-^C\left(\tilde{S}^A_{++}\tilde{S}^A_{--}+\tilde{S}^A_{+-}\tilde{S}^A_{-+}\right)+\dots \ . \tag{F6}$$

The couplings λ_k and the coefficients y_{ABC} and z_{AB} are defined in the same way as in the $S1$ model.

APPENDIX G: CONSTRUCTION OF SOME MODELS

In this appendix we give the construction of the models $\Sigma1$, $\Sigma2$ and $\Sigma3$ discussed in section II. As we proceed, we will for completeness review the construction of the E_6 model, one of the $SO(10)$ models (namely, $T1(1,1)$), and two other $SU(6)$ models (namely, $S1$ and $S2$), which were previously constructed in [22,23].

1. The Narain Model

Consider the Narain model with the momenta of the internal bosons spanning an even self-dual Lorentzian lattice $\Gamma^{6,22}=\Gamma^{6,6}\otimes\Gamma^{16}$. Here Γ^{16} is the Spin(32)/$\mathbf{Z}_2$ lattice. The $\Gamma^{6,6}$ is the momentum lattice corresponding to the compactification on the E_6 six-torus defined by $X_I=X_I+E_I$. The dot product of the vectors E_I defines the constant background metric $G_{IJ}=E_I\cdot E_J$ which is the same as the E_6 Cartan matrix. There is also the antisymmetric background field B_{IJ} given by $2B_{IJ}=\frac{1}{2}G_{IJ}$ for $I<J$ and $2B_{IJ}=-\frac{1}{2}G_{IJ}$ for $I>J$. The vectors E_I (and also their duals $\tilde{E}^I$ defined so that $E_I\cdot\tilde{E}^J={\delta_I}^J$) can be expressed in terms of the $SU(3)$ root and weight vectors e_i and $\tilde{e}^i$ $(i=1,2)$:

$$\begin{aligned}
&E_1=(e_1,0,0), \quad E_2=(e_2,0,0),\\
&E_3=(0,e_1,0), \quad E_4=(0,e_2,0),\\
&E_5=(\tilde{e}^2,-\tilde{e}^2,\tilde{e}^2), \quad E_6=(\tilde{e}^1,-\tilde{e}^1,\tilde{e}^1).
\end{aligned}$$

Following the notations of Ref [22] we will refer to this Narain model as $N(1,1)$. This model has $N=4$ space-time supersymmetry and $E_6\otimes SO(32)$ gauge group.

2. Wilson Lines

Next, we discuss the Wilson lines that will give the Narain models with $SO(10)^3$ and $(E_6)^3$ gauge subgroups. Here we are writing the Wilson lines as shift vectors in the $\Gamma^{6,22}$ lattice. The shift vectors U_1 and U_2 to be introduced are order-2 shifts that break $SO(32)$ to $SO(10)^3\otimes SO(2)$. For certain Wilson lines U_1 and U_2, however, the new (shifted) sectors introduce additional gauge bosons to enhance $SO(10)^3$ to $(E_6)^3$.

Thus, starting from the Narain lattice $N(1,1)$ we have two inequivalent choices:

- The $N1(1,1)$ model generated by the Wilson lines

$$\begin{aligned}
U_1&=(0,0,0||e_1/2,0,0)(\mathbf{s}|\mathbf{0}|\mathbf{0}|\overline{S})\ ,\\
U_2&=(0,0,0||e_2/2,0,0)(\mathbf{0}|\mathbf{s}|\mathbf{0}|\overline{S})\ .
\end{aligned}$$

This model has $SU(3)^2 \otimes (E_6)^3$ gauge symmetry. The U_1 and U_2 are order-2 ($\mathbf{Z}_2$) shifts. The first three entries correspond to the right-moving complex world-sheet bosons. The next three entries correspond to the left-moving complex world-sheet bosons. Together they form the six-torus. The remaining 16 left-moving world-sheet bosons generate the Spin(32)/$\mathbf{Z}_2$ lattice. The $SO(32)$ shifts are given in the $SO(10)^3 \otimes SO(2)$ basis. In this basis, $\mathbf{0}(0)$ stands for the null vector, $\mathbf{v}(V)$ is the vector weight, whereas $\mathbf{s}(S)$ and $\overline{\mathbf{s}}(\overline{S})$ are the spinor and anti-spinor weights of $SO(10)(SO(2))$. (For $SO(2)$, $V = 1$, $S = 1/2$ and $\overline{S} = -1/2$.) The unshifted sector provides gauge bosons of $SU(3)^2 \otimes U(1)^2 \otimes SO(10)^3 \otimes SO(2)$. The permutation symmetry of the three $SO(10)$'s is explicit here. There are additional gauge bosons from the new sectors. Recall that under $E_6 \supset SO(10) \otimes U(1)$,

$$\mathbf{78} = \mathbf{1}(0) + \mathbf{45}(0) + \mathbf{16}(3) + \overline{\mathbf{16}}(-3) \ . \tag{G1}$$

It is easy to see that the U_1, U_2 and $U_1 + U_2$ sectors provide the necessary $\mathbf{16}(3)$ and $\overline{\mathbf{16}}(-3)$ gauge bosons to the three $SO(10)$'s respectively. Consistency and the permutation symmetry of the three $SO(10)$'s implies the permutation symmetry of the three E_6's. The resulting Narain model $N1(1,1)$ has $N = 4$ SUSY and gauge group $SU(3)^2 \otimes (E_6)^3$.

• The $N2(1,1)$ model generated by the Wilson lines

$$U_1 = (0, e_1/2, e_1/2||e_1/2, 0, 0)(\mathbf{s}|\mathbf{0}|\mathbf{0}|\overline{S}) \ ,$$
$$U_2 = (0, e_2/2, e_2/2||e_2/2, 0, 0)(\mathbf{0}|\mathbf{s}|\mathbf{0}|\overline{S}) \ .$$

This model has $SU(3)^2 \otimes U(1)^2 \otimes SO(10)^3 \otimes SO(2)$ gauge symmetry.

3. The E_6 and $SO(10)$ Models

Before we describe the $\mathbf{Z}_6$ asymmetric orbifolds that lead to the E_6 model and the $SO(10)$ model $T1(1,1)$, we will introduce some notation. By θ we will denote a $2\pi/3$ rotation of the corresponding two real chiral world-sheet bosons. Thus, θ is a $\mathbf{Z}_3$ twist. Similarly, by σ we will denote a π rotation of the corresponding two chiral world-sheet bosons. Thus, σ is a $\mathbf{Z}_2$ twist. By $\mathcal{P}$ we will denote the outer-automorphism of the three $SO(10)$'s that arise in the breaking $SO(32) \supset SO(10)^3 \otimes SO(2)$. Note that $\mathcal{P}$ is a $\mathbf{Z}_3$ twist. Finally, by (p_1, p_2) we will denote the outer-automorphism of the corresponding two complex chiral world-sheet bosons. Note that (p_1, p_2) is a $\mathbf{Z}_2$ twist. The spin structures of the world-sheet fermions in the right-moving sector are fixed by the world-sheet supersymmetry consistency. (Again, more details can be found in Ref [18].)

Finally, we are ready to give the corresponding $\mathbf{Z}_3 \otimes \mathbf{Z}_2$ twists.

• The $E1$ model. Start from the $N1(1,1)$ model and perform the following twists:

$$T_3 = (\theta, \theta, \theta||\theta, e_1/3, 0)(\mathcal{P}|2/3) \ ,$$
$$T_2 = (\sigma, p_1, p_2||0, e_1/2, e_1/2)(0^{15}|0) \ .$$

This model has $SU(2)_1 \otimes (E_6)_3 \otimes U(1)^3$ gauge symmetry. The massless spectrum of the $E1$ model is given in Table I. The states are grouped according to where they come from, namely, the untwisted sector U, the $\mathbf{Z}_3$ twisted (*i.e.*, T_3 and $2T_3$) sector T3, the $\mathbf{Z}_6$ twisted

(*i.e.*, T_3+T_2 and $2T_3+T_2$) sector T6, and $\mathbf{Z}_2$ twisted (*i.e.*, T_2) sector T2.
• The $E2$ model. Start from the $N1(1,1)$ model and perform the following twists:

$$T_3 = (0, \theta, \theta||\theta, e_1/3, 0)(\mathcal{P}|2/3) \ ,$$
$$T_2 = (\sigma, p_1, p_2||0, e_1/2, e_1/2)(0^{15}|0) \ .$$

This model has $SU(2)_1 \otimes (E_6)_3 \otimes U(1)^3$ gauge symmetry. The massless spectrum of the $E2$ model is given in Table I.
Here we note that the $E1$ and $E2$ models are the same, in particular, they have the same tree-level massless spectra and interactions. The $E1 = E2$ model is what we refer to as the E_6 model.
• The $T1(1,1)$ model. Start from the $N2(1,1)$ model and perform the same twists as in the $E1$ model. This model has $SU(2)_1 \otimes SO(10)_3 \otimes U(1)^4$ gauge symmetry. The massless spectrum of the $T1(1,1)$ model is given in Table II.
• The $T2(1,1)$ model. Start from the $N2(1,1)$ model and perform the same twists as in the $E2$ model. This model has $SU(2)_1 \otimes SO(10)_3 \otimes U(1)^4$ gauge symmetry. The massless spectrum of the $T2(1,1)$ model is given in Table II.
Here we note that the $T1(1,1)$ and $T2(1,1)$ models are the same, in particular, they have the same tree-level massless spectra and interactions.
• The $T5(1,0)$ and $T6(1,0)$ models.
These models are the same as they have the same tree-level massless spectra and interactions. They can be constructed in a way similar to the $T1(1,1) = T2(1,1)$ model starting from a different Narain model. Here we simply give the massless spectra of these models in Table III and refer the reader to Ref [22] for details.

4. The $SU(6)$ Models

We can start from the $E1$ or $E2$ model to generate some of the $SU(6)$ models by adding Wilson lines. Let us first consider the following Wilson line

$$A_3 = (0, 0, 0||0, \tilde{e}^2, \tilde{e}^2)((\frac{1}{3}\frac{1}{3}\frac{1}{3}\frac{1}{3}\frac{2}{3})^3|0) \ . \tag{G2}$$

• The $S1$ model. Start from the $E1$ model and add the A_3 Wilson line. Choose the relative phase between the T_3 and A_3 sectors to be $\phi(T_3, A_3) = 0$ or $1/3$ (both choices give the same model). This model has $SU(3)_1 \otimes SU(6)_3 \otimes U(1)^3$ gauge symmetry. The massless spectrum of the $S1$ model is given in Table V.
• The $S2$ model. Start from the $E1$ model and add the A_3 Wilson line. Choose the relative phase between the T_3 and A_3 twisted sectors to be $\phi(T_3, A_3) = 2/3$. This model has $SU(2)_1 \otimes SU(2)_1 \otimes SU(6)_3 \otimes U(1)^3$ gauge symmetry. The massless spectrum of the $S2$ model is given in Table V.
• The $S3$ model. Start from the $E2$ model and add the A_3 Wilson line. Choose the relative phase between the T_3 and A_3 twisted sectors to be $\phi(T_3, A_3) = 0$ or $1/3$ (both choices give the same model). This model has $SU(3)_1 \otimes SU(6)_3 \otimes U(1)^3$ gauge symmetry. The massless spectrum of the $S3$ model was given in [23]. Note that $S3 = S1$.
• The $S4$ model. Start from the $E2$ model and add the A_3 Wilson line. Choose the

relative phase between the T_3 and A_3 twisted sectors to be $\phi(T_3, A_3) = 2/3$. This model has $SU(2)_1 \otimes SU(2)_1 \otimes SU(6)_3 \otimes U(1)^3$ gauge symmetry. The massless spectrum of the $S4$ model was given in [23]. Note that $S4 = S2$.

By modifying some of the relative phases between sectors of these models, and also the Narain lattice that we started with, one can build other $SU(6)$ (and $SU(5)$) models. In Table IV we give the massless spectrum of the $S1(1,1)$ model. For illustrative purposes we also present one of the $SU(5)$ models (which in Ref [23] was named $F11(1,0)$).

Next we turn to construction of additional $SU(6)$ models by modifying the $\mathbf{Z}_2$ twist.

• The $\Sigma 1A$ model. Start from the $N1(1,1)$ model and perform the following twists:

$$T_3 = (\theta, \theta, \theta || \theta, e_1/3, 0)(\mathcal{P}|2/3) \ ,$$
$$T_2 = (\sigma, p_1, p_2 || 0, e_1/2, 0)((\frac{1}{4})^{15}| -\frac{3}{4}) \ .$$

This model has $SU(3)_1 \otimes SU(2)_3 \otimes SU(6)_3 \otimes U(1)^2$ gauge symmetry. The massless spectrum of the $\Sigma 1A$ model is given in Table VIII.

• The $\Sigma 1B$ model. Start from the $N1(1,1)$ model and perform the following twists:

$$T_3 = (0, \theta, \theta || \theta, e_1/3, 0)(\mathcal{P}|2/3) \ ,$$
$$T_2 = (\sigma, p_1, p_2 || 0, e_1/2, 0)((\frac{1}{4})^{15}| -\frac{3}{4}) \ .$$

This model has $SU(3)_1 \otimes SU(2)_3 \otimes SU(6)_3 \otimes U(1)^2$ gauge symmetry. The massless spectrum of the $\Sigma 1B$ model is given in Table VIII.

• The $\Sigma 2A$ model. Start from the $N1(1,1)$ model and perform the following twists:

$$T_3 = (\theta, \theta, \theta || \theta, e_1/3, 0)(\mathcal{P}|2/3) \ ,$$
$$T_2 = (\sigma, p_1, p_2 || 0, 0, e_1/2)((\frac{1}{4})^{15}| -\frac{3}{4}) \ .$$

This model has $SU(2)_1 \otimes SU(2)_3 \otimes SU(6)_3 \otimes U(1)^3$ gauge symmetry. The massless spectrum of the $\Sigma 2A$ model is given in Table IX.

• The $\Sigma 2B$ model. Start from the $N1(1,1)$ model and perform the following twists:

$$T_3 = (0, \theta, \theta || \theta, e_1/3, 0)(\mathcal{P}|2/3) \ ,$$
$$T_2 = (\sigma, p_1, p_2 || 0, 0, e_1/2)((\frac{1}{4})^{15}| -\frac{3}{4}) \ .$$

This model has $SU(2)_1 \otimes SU(2)_3 \otimes SU(6)_3 \otimes U(1)^3$ gauge symmetry. The massless spectrum of the $\Sigma 2B$ model is given in Table IX.

Here we note that if we give vev to the triplet (adjoint) of $SU(2)_3$ (which is part of the horizontal symmetry) in both $\Sigma 1A$ and $\Sigma 1B$, we obtain the same model, which we will refer to as $\Sigma 1$. Similarly, if we give vev to the triplet of $SU(2)_3$ in both $\Sigma 2A$ and $\Sigma 2B$, we obtain the same model, which we will refer to as $\Sigma 2$. Turning on vev for the triplet of $SU(2)_3$ in these models is equivalent to adding the following Wilson line:

$$A'_3 = (0, 0, 0 || 0, 0, e_1/3)((\frac{1}{3}\frac{1}{3}\frac{1}{3}\frac{1}{3}\frac{2}{3})^3|0) \ .$$

The massless spectra of the models $\Sigma 1$ and $\Sigma 2$ are given in Table VI.
There is one other Wilson line we can add. This is precisely the A_3 Wilson line given by Eq (G2). This way we get one more model, which we will refer to as $\Sigma 3$.
• The $\Sigma 3$ model. Start from the $\Sigma 1A$ model and add the A_3 Wilson line. Choose the phase $\phi(T_3, A_3) = 2/3$ (other choices lead to models already constructed). The resulting model has $SU(4)_1 \otimes SU(6)_3 \otimes U(1)^2$ gauge symmetry. Its massless spectrum is given in the first column of Table VII.
• There is another way of obtaining the $\Sigma 3$ model. Start from the $\Sigma 1B$ model and add the A_3 Wilson line. Choose the phase $\phi(T_3, A_3) = 2/3$ (other choices lead to models already constructed). The resulting model has $SU(4)_1 \otimes SU(6)_3 \otimes U(1)^2$ gauge symmetry. Its massless spectrum is given in the second column of Table VII. Note that the two models in Table VII are the same, and we refer to this model as $\Sigma 3$.
We note that if we add the Wilson line A_3 to the $\Sigma 2A$ model, we get the $S2$ model. Similarly, if we add the Wilson line A_3 to the $\Sigma 2B$ model, we get the $S4$ model.

TABLES

M	Field	$E1$ $SU(2)\otimes E_6\otimes U(1)^3$	$E2$ $SU(2)\otimes E_6\otimes U(1)^3$
U	Φ $U_{+\pm}$, $U_{-\pm}$ U_0	$(\mathbf{1},\mathbf{78})(0,0,0)_L$ $2(\mathbf{1},\mathbf{1})(0,-3,\pm 3)_L$ $(\mathbf{1},\mathbf{1})(0,+6,0)_L$	$(\mathbf{1},\mathbf{78})(0,0,0)_L$ $(\mathbf{1},\mathbf{1})(\pm 3,+3,0)_L$ $(\mathbf{1},\mathbf{1})(0,+6,0)_L$ $(\mathbf{2},\mathbf{1})(0,0,\pm 3)_L$
$T3$	χ_0 $\chi_{+\pm}$, $\chi_{-\pm}$	$(\mathbf{1},\mathbf{27})(0,-2,0)_L$ $2(\mathbf{1},\mathbf{27})(0,+1,\pm 1)_L$	$(\mathbf{1},\mathbf{27})(0,-2,0)_L$ $(\mathbf{1},\overline{\mathbf{27}})(\pm 1,-1,0)_L$
$T6$	$\tilde{\chi}_\pm$	$(\mathbf{1},\overline{\mathbf{27}})(\pm 1,-1,0)_L$	$2(\mathbf{1},\mathbf{27})(0,+1,\pm 1)_L$
$T2$	$D_\pm$ $\widetilde{U}_\pm$	$(\mathbf{2},\mathbf{1})(0,0,\pm 3)_L$ $(\mathbf{1},\mathbf{1})(\pm 3,+3,0)_L$	$2(\mathbf{1},\mathbf{1})(0,-3,\pm 3)_L$
$U(1)$		$(1/\sqrt{6},\ 1/3\sqrt{2},\ 1/\sqrt{6})$	$(1/\sqrt{6},\ 1/3\sqrt{2},\ 1/\sqrt{6})$

TABLE I. The massless spectra of the two E_6 models $E1$ and $E2$ both with gauge symmetry $SU(2)_1\otimes(E_6)_3\otimes U(1)^3$. The $U(1)$ normalization radii are given at the bottom of the Table. The gravity, dilaton and gauge supermultiplets are not shown.

M	Field	$T1(1,1)$ $SU(2)\otimes SO(10)\otimes U(1)^4$	$T2(1,1)$ $SU(2)\otimes SO(10)\otimes U(1)^4$
U	Φ	$(\mathbf{1},\mathbf{45})(0,0,0,0)_L$	$(\mathbf{1},\mathbf{45})(0,0,0,0)_L$
	ϕ	$(\mathbf{1},\mathbf{1})(0,0,0,0)_L$	$(\mathbf{1},\mathbf{1})(0,0,0,0)_L$
	$U_{+\pm},\, U_{-\pm}$	$2(\mathbf{1},\mathbf{1})(0,-3,\pm3,0)_L$	$(\mathbf{1},\mathbf{1})(\pm3,+3,0,0)_L$
	U_0	$(\mathbf{1},\mathbf{1})(0,+6,0,0)_L$	$(\mathbf{1},\mathbf{1})(0,+6,0,0)_L$
			$(\mathbf{2},\mathbf{1})(0,0,\pm3,0)_L$
$T3$	Q_0	$(\mathbf{1},\mathbf{16})(0,-2,0,-1)_L$	$(\mathbf{1},\mathbf{16})(0,-2,0,-1)_L$
	H_0	$(\mathbf{1},\mathbf{10})(0,-2,0,+2)_L$	$(\mathbf{1},\mathbf{10})(0,-2,0,+2)_L$
	S_0	$(\mathbf{1},\mathbf{1})(0,-2,0,-4)_L$	$(\mathbf{1},\mathbf{1})(0,-2,0,-4)_L$
	$Q_{+\pm},\, Q_{-\pm}$	$2(\mathbf{1},\mathbf{16})(0,+1,\pm1,-1)_L$	$(\mathbf{1},\overline{\mathbf{16}})(\pm1,-1,0,+1)_L$
	$H_{+\pm},\, H_{-\pm}$	$2(\mathbf{1},\mathbf{10})(0,+1,\pm1,+2)_L$	$(\mathbf{1},\mathbf{10})(\pm1,-1,0,-2)_L$
	$S_{+\pm},\, S_{-\pm}$	$2(\mathbf{1},\mathbf{1})(0,+1,\pm1,-4)_L$	$(\mathbf{1},\mathbf{1})(\pm1,-1,0,+4)_L$
$T6$	$\widetilde{Q}_\pm$	$(\mathbf{1},\overline{\mathbf{16}})(\pm1,-1,0,+1)_L$	$2(\mathbf{1},\ \mathbf{16})(0,+1,\pm1,-1)_L$
	$\widetilde{H}_\pm$	$(\mathbf{1},\mathbf{10})(\pm1,-1,0,-2)_L$	$2(\mathbf{1},\ \mathbf{10})(0,+1,\pm1,+2)_L$
	$\widetilde{S}_\pm$	$(\mathbf{1},\mathbf{1})(\pm1,-1,0,+4)_L$	$2(\mathbf{1},\ \mathbf{1})(0,+1,\pm1,-4)_L$
$T2$	$D_\pm$	$(\mathbf{2},\mathbf{1})(0,0,\pm3,0)_L$	$2(\mathbf{1},\mathbf{1})(0,-3,\pm3,0)_L$
	$\widetilde{U}_\pm$	$(\mathbf{1},\mathbf{1})(\pm3,+3,0,0)_L$	
$U(1)$		$(1/\sqrt{6},\ 1/3\sqrt{2},\ 1/\sqrt{6},\ 1/6)$	$(1/\sqrt{6},\ 1/3\sqrt{2},\ 1/\sqrt{6},\ 1/6)$

TABLE II. The massless spectra of the two $SO(10)$ models $T1(1,1)$ and $T2(1,1)$ both with gauge symmetry $SU(2)_1\otimes SO(10)_3\otimes U(1)^4$. The $U(1)$ normalization radii are given at the bottom of the Table. The gravity, dilaton and gauge supermultiplets are not shown.

M	$T5(1,0)$ Field	$T5(1,0)$ $SU(2)^3 \otimes SO(10) \otimes U(1)^4$	$T6(1,0)$ $SU(2)^3 \otimes SO(10) \otimes U(1)^4$
		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{45})(0,0,0,0)_L$	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{45})(0,0,0,0)_L$
		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(\pm3,+3,0,0)_L$	$(\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})(\pm3,0,0,0)_L$
U	U_1	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(0,+6,0,0)_L$	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(0,+6,0,0)_L$
	U_2	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(0,0,+6,0)_L$	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(0,0,+6,0)_L$
		$(\mathbf{2},\mathbf{2},\mathbf{2},\mathbf{1})(0,0,+3,0)_L$	$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1})(0,-3,-3,0)_L$
			$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1})(0,\pm3,\mp3,0)_L$
		$(\mathbf{1},\mathbf{1},\mathbf{1},\overline{\mathbf{16}})(0,+2,+2,+1)_L$	$(\mathbf{1},\mathbf{1},\mathbf{1},\overline{\mathbf{16}})(0,+2,+2,+1)_L$
$T3$		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{10})(0,+2,+2,-2)_L$	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{10})(0,+2,+2,-2)_L$
		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(0,+2,+2,+4)_L$	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(0,+2,+2,+4)_L$
		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{16})(\pm1,+1,-2,-1)_L$	
		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{10})(\pm1,+1,-2,+2)_L$	
		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(\pm1,+1,-2,-4)_L$	
			$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{16})(\pm1,+1,-2,-1)_L$
$T6$			$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{10})(\pm1,+1,-2,+2)_L$
			$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(\pm1,+1,-2,-4)_L$
		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{16})(\pm1,-2,+1,-1)_L$	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{16})(\pm1,-2,+1,-1)_L$
		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{10})(\pm1,-2,+1+2)_L$	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{10})(\pm1,-2,+1+2)_L$
		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(\pm1,-2,+1,-4)_L$	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(\pm1,-2,+1,-4)_L$
$T2$		$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1})(\pm3,0,0,0)_L$	$(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1})(\pm3,0,0,0)_L$
		$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1})(\pm3,0,0,0)_L$	$(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{1})(\pm3,0,0,0)_L$
		$(\mathbf{2},\mathbf{1},\mathbf{1},\mathbf{1})(\pm3,0,0,0)_L$	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(\pm3,+3,0,0)_L$
		$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(\pm3,0,+3,0)_L$	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(\pm3,0,+3,0)_L$
	D_2	$(\mathbf{2},\mathbf{1},\mathbf{2},\mathbf{1})(0,-3,-3,0)_L$	$(\mathbf{2},\mathbf{1},\mathbf{2},\mathbf{1})(0,-3,-3,0)_L$
	$D_{2\pm}$	$(\mathbf{2},\mathbf{1},\mathbf{2},\mathbf{1})(0,\pm3,\mp3,0)_L$	$(\mathbf{2},\mathbf{1},\mathbf{2},\mathbf{1})(0,\pm3,\mp3,0)_L$
	D_3	$(\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{1})(0,-3,-3,0)_L$	$(\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{1})(0,-3,-3,0)_L$
	$D_{3\pm}$	$(\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{1})(0,\pm3,\mp3,0)_L$	$(\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{1})(0,\pm3,\mp3,0)_L$
		$(\mathbf{2},\mathbf{2},\mathbf{2},\mathbf{1})(0,+3,0,0)_L$	$(\mathbf{2},\mathbf{2},\mathbf{2},\mathbf{1})(0,+3,0,0)_L$
	D_1	$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1})(0,-3,-3,0)_L$	$(\mathbf{2},\mathbf{2},\mathbf{2},\mathbf{1})(0,0,+3,0)_L$
	$D_{1\pm}$	$(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1})(0,\pm3,\mp3,0)_L$	
$U(1)$		$(1/\sqrt{6},\ 1/3\sqrt{2},\ 1/3\sqrt{2},\ 1/6)$	$(1/\sqrt{6},\ 1/3\sqrt{2},\ 1/3\sqrt{2},\ 1/6)$

TABLE III. The massless spectra of the $T5(1,0)$ and $T6(1,0)$ models both with gauge symmetry $SU(2)_1^3 \otimes SO(10)_3 \otimes U(1)^4$. The $U(1)$ normalization radii are given at the bottom of the Table. The graviton, dilaton and gauge supermultiplets are not shown.

M	Fields	$S1(1,1)$ $SU(2)\otimes SU(6)\otimes U(1)^4$	$F11(1,0)$ $SU(2)^2\otimes SU(5)\otimes U(1)^4$
U	Φ	$(\mathbf{1},\mathbf{35})(0,0,0,0)_L$	$(\mathbf{1},\mathbf{1},\mathbf{24})(0,0,0,0)_L$
	$U_{+\pm}$, $U_{-\pm}$	$2(\mathbf{1},\mathbf{1},\mathbf{1})(0,-3,\pm 3,0)_L$	$2(\mathbf{1},\mathbf{1},\mathbf{1})(0,-3,\pm 3,0)_L$
	U_0	$(\mathbf{1},\mathbf{1},\mathbf{1})(0,+6,0,0)_L$	$(\mathbf{1},\mathbf{1},\mathbf{1})(0,+6,0,0)_L$
	ϕ	$(\mathbf{1},\mathbf{1},\mathbf{1})(0,0,0,0)_L$	$(\mathbf{1},\mathbf{1},\mathbf{1})(0,0,0,0)_L$
			$(\mathbf{1},\mathbf{3},\mathbf{1})(0,0,0,0)_L$
$T3$	Q_0	$(\mathbf{1},\mathbf{15})(0,-2,0,0)_L$	$(\mathbf{1},\mathbf{2},\overline{\mathbf{5}})(0,-2,0,+1)_L$
	H_0^+	$(\mathbf{1},\overline{\mathbf{6}})(0,-2,0,+1)_L$	$(\mathbf{1},\mathbf{2},\mathbf{1})(0,-2,0,-5)_L$
	H_0^-	$(\mathbf{1},\overline{\mathbf{6}})(0,-2,0,-1)_L$	$(\mathbf{1},\mathbf{1},\mathbf{10})(0,-2,0,-2)_L$
			$(\mathbf{1},\mathbf{1},\mathbf{5})(0,-2,0,+4)_L$
	$Q_{+\pm}$, $Q_{-\pm}$	$2(\mathbf{1},\mathbf{15})(0,+1,\pm 1,0)_L$	$2(\mathbf{1},\mathbf{2},\overline{\mathbf{5}})(0,+1,\pm 1,+1)_L$
	$H_{+\pm}^+$, $H_{-\pm}^+$	$2(\mathbf{1},\overline{\mathbf{6}})(0,+1,\pm 1,+1)_L$	$2(\mathbf{1},\mathbf{2},\mathbf{1})(0,+1,\pm 1,-5)_L$
	$H_{+\pm}^+$, $H_{-\pm}^+$	$2(\mathbf{1},\overline{\mathbf{6}})(0,+1,\pm 1,-1)_L$	$2(\mathbf{1},\mathbf{1},\mathbf{10})(0,+1,\pm 1,-2)_L$
			$2(\mathbf{1},\mathbf{1},\mathbf{5})(0,+1,\pm 1,+4)_L$
$T6$	$\widetilde{Q}_\pm$	$(\mathbf{1},\overline{\mathbf{15}})(\pm 1,-1,0,0)_L$	$(\mathbf{1},\mathbf{2},\mathbf{5})(\pm 1,-1,0,-1)_L$
	$\widetilde{H}_\pm^+$	$(\mathbf{1},\mathbf{6})(\pm 1,-1,0,+1)_L$	$(\mathbf{1},\mathbf{2},\mathbf{1})(\pm 1,-1,0,+5)_L$
	$\widetilde{H}_\pm^-$	$(\mathbf{1},\mathbf{6})(\pm 1,-1,0,-1)_L$	$(\mathbf{1},\mathbf{1},\overline{\mathbf{10}})(\pm 1,-1,0,+2)_L$
			$(\mathbf{1},\mathbf{1},\overline{\mathbf{5}})(\pm 1,-1,0,-4)_L$
$T2$	$D_\pm$	$(\mathbf{2},\mathbf{1},\mathbf{1})(0,0,\pm 3,0)_L$	$(\mathbf{2},\mathbf{1},\mathbf{1})(0,0,\pm 3,0)_L$
	$\widetilde{U}_\pm$	$(\mathbf{1},\mathbf{1},\mathbf{1})(\pm 3,+3,0,0)_L$	$(\mathbf{1},\mathbf{1},\mathbf{1})(\pm 3,+3,0,0)_L$
$U(1)$		$(1/\sqrt{6},1/3\sqrt{2},1/\sqrt{6},1/\sqrt{6})$	$(1/\sqrt{6},1/3\sqrt{2},1/\sqrt{6},1/3\sqrt{10})$

TABLE IV. The massless spectra of the $S1(1,1)$ and $F11(1,0)$ models with gauge groups $SU(2)_1\otimes SU(6)_3\otimes U(1)^4$ and $SU(2)_1\otimes SU(2)_3\otimes SU(5)_3\otimes U(1)^4$, respectively. The $U(1)$ normalization radii are given at the bottom of the Table. The gravity, dilaton and gauge supermultiplets are not shown.

M	Field	$S1$ $SU(3)\otimes SU(6)\otimes U(1)^3$	Field	$S2$ $SU(2)^2\otimes SU(6)\otimes U(1)^3$
U	Φ ϕ U_0 $U_{+\pm}$, $U_{-\pm}$ $\widetilde{T}_\pm$ T_0	$(\mathbf{1},\mathbf{35})(0,0,0)_L$ $(\mathbf{1},\mathbf{1})(0,0,0)_L$ $(\mathbf{1},\mathbf{1})(+6,0,0)_L$ $2(\mathbf{1},\mathbf{1})(-3,\pm3,\pm3)_L$ $2(\overline{\mathbf{3}},\mathbf{1})(+3,-3,+1)_L$ $(\mathbf{3},\mathbf{1})(0,0,-4)_L$	Φ ϕ U_0 $d_{+\pm}$, $d_{-\pm}$ $D_\pm$	$(\mathbf{1},\mathbf{1},\mathbf{35})(0,0,0)_L$ $(\mathbf{1},\mathbf{1},\mathbf{1})(0,0,0)_L$ $(\mathbf{1},\mathbf{1},\mathbf{1})(0,0,-6)_L$ $2(\mathbf{1},\mathbf{2},\mathbf{1})(\pm1,\mp3,+3)_L$ $(\mathbf{2},\mathbf{1},\mathbf{1})(\pm2,0,+3)_L$
$T3$	$F^A_\pm$ $\widetilde{S}^A_\pm$ $\widetilde{S}^A_0$	$6(\mathbf{1},\mathbf{15})(+1,-1,-1)_L$ $6(\mathbf{1},\overline{\mathbf{6}})(-2,+1,-1)_L$ $3(\mathbf{1},\overline{\mathbf{6}})(+1,0,+2)_L$	F^A $\widetilde{S}^A_{+\pm}$, $\widetilde{S}^A_{-\pm}$	$3(\mathbf{1},\mathbf{1},\mathbf{15})(0,0,+2)_L$ $6(\mathbf{1},\mathbf{1},\overline{\mathbf{6}})(\pm1,\mp1,-1)_L$
$T6$	S^A $\widetilde{F}^A$	$3(\mathbf{1},\mathbf{6})(+2,+1,+1)_L$ $3(\mathbf{1},\overline{\mathbf{15}})(-1,-1,+1)_L$	$S^A_\pm$	$3(\mathbf{1},\mathbf{1},\mathbf{6})(\pm1,\pm1,+1)_L$
$T2$	$T_\pm$ $\widetilde{T}_0$ $\widetilde{U}_\pm$	$(\mathbf{3},\mathbf{1})(\pm3,-3,-1)_L$ $(\overline{\mathbf{3}},\mathbf{1})(-3,+3,+1)_L$ $(\mathbf{1},\mathbf{1})(+3,\pm3,\mp3)_L$	$\Delta_\pm$ $\tilde{d}_\pm$	$(\mathbf{2},\mathbf{2},\mathbf{1})(\pm1,\mp3,0)_L$ $(\mathbf{1},\mathbf{2},\mathbf{1})(\pm1,\pm3,-3)_L$
$U(1)$		$(\frac{1}{3\sqrt{2}},\ \frac{1}{2\sqrt{3}},\ \frac{1}{2\sqrt{3}})$		$(\frac{1}{2},\ \frac{1}{2\sqrt{3}},\ \frac{1}{3\sqrt{2}})$

TABLE V. The massless spectra of the two $SU(6)$ models $S1$ and $S2$ with gauge symmetries $SU(3)_1\otimes SU(6)_3\otimes U(1)^3$ and $SU(2)_1\otimes SU(2)_1\otimes SU(6)_3\otimes U(1)^3$, respectively. Note that double signs (as in $(\mathbf{1},\mathbf{2},\mathbf{1})(\pm1,\pm3,-3)_L$) are correlated. The $U(1)$ normalization radii are given at the bottom of the table. The graviton, dilaton and gauge supermultiplets are not shown.

M	Field	$\Sigma1$ (from $\Sigma1A$ with A_3') $SU(3)\otimes SU(6)\otimes U(1)^3$	Field	$\Sigma2$ (from $\Sigma2A$ with A_3') $SU(2)\otimes SU(6)\otimes U(1)^4$
U	Φ	$(\mathbf{1},\mathbf{35})(0,0,0)_L$	Φ	$(\mathbf{1},\mathbf{35})(0,0,0,0)_L$
	ϕ	$(\mathbf{1},\mathbf{1})(0,0,0)_L$	ϕ	$(\mathbf{1},\mathbf{1})(0,0,0,0)_L$
	U_0	$(\mathbf{1},\mathbf{1})(0,+6,0)_L$	U_1	$(\mathbf{1},\mathbf{1})(0,0,+6,0)_L$
	$U_{+\pm},\ U_{-\pm}$	$2(\mathbf{1},\mathbf{1})(0,-3,\pm3)_L$	U_2	$(\mathbf{1},\mathbf{1})(0,0,-3,-3)_L$
			U_3	$(\mathbf{1},\mathbf{1})(0,0,-3,+3)_L$
$T3$	F_0	$(\mathbf{1},\mathbf{15})(0,-2,0)_L$	F_1	$(\mathbf{1},\mathbf{15})(0,0,-2,0)_L$
	$F_{+\pm},\ F_{-\pm}$	$2(\mathbf{1},\mathbf{15})(0,+1,\pm1)_L$	F_2	$(\mathbf{1},\mathbf{15})(0,0,+1,+1)_L$
			F_3	$(\mathbf{1},\mathbf{15})(0,0,+1,-1)_L$
	$\widetilde{S}'_{+\pm}$	$(\mathbf{1},\overline{\mathbf{6}})(\pm1,+1,+1)_L$	$\widetilde{S}^2_{+\pm},\ \widetilde{S}^2_{-\pm}$	$2(\mathbf{1},\overline{\mathbf{6}})(\pm1,0,+1,+1)_L$
	$\widetilde{S}'_{-\pm}$	$(\mathbf{1},\overline{\mathbf{6}})(\pm1,+1,-1)_L$	$\widetilde{S}^3_{+\pm},\ \widetilde{S}^3_{-\pm}$	$2(\mathbf{1},\overline{\mathbf{6}})(\pm1,0,+1,-1)_L$
	$\widetilde{S}_{+\pm},\ \widetilde{S}_{-\pm}$	$2(\mathbf{1},\overline{\mathbf{6}})(\pm1,-2,0)_L$	$\widetilde{S}^1_{+\pm},\ \widetilde{S}^1_{-\pm}$	$2(\mathbf{1},\overline{\mathbf{6}})(\pm1,0,-2,0)_L$
$T6$	$\widetilde{F}_{\pm}$	$(\mathbf{1},\overline{\mathbf{15}})(\pm1,-1,0)_L$	$S^1_{\pm}$	$(\mathbf{1},\mathbf{6})(0,\pm1,+2,0)_L$
	$S_{\pm}$	$(\mathbf{1},\mathbf{6})(0,+2,\pm1)_L$	$S^2_{\pm}$	$(\mathbf{1},\mathbf{6})(0,\pm1,-1,-1)_L$
			$S^3_{\pm}$	$(\mathbf{1},\mathbf{6})(0,\pm1,-1,+1)_L$
$T2$	$\widetilde{U}_{\pm}$	$(\mathbf{1},\mathbf{1})(\pm3,+3,0)_L$	$\Delta_{\pm}$	$(\mathbf{2},\mathbf{1})(\pm3,0,0,0)_L$
$U(1)$		$(1/\sqrt{6},\ 1/3\sqrt{2},\ 1/\sqrt{6})$		$(1/\sqrt{6},\ 1/\sqrt{6},\ 1/3\sqrt{2},\ 1/\sqrt{6})$

TABLE VI. The massless spectra of the two $SU(6)$ models $\Sigma1$ and $\Sigma2$ (obtained by adding A_3' Wilson line to the model $\Sigma1A$ and to the model $\Sigma1B$, respectively). The model $\Sigma1$ has gauge symmetry $SU(3)_1\otimes SU(6)_3\otimes U(1)^3$. The model $\Sigma2$ has gauge symmetry $SU(2)_1\otimes SU(6)_3\otimes U(1)^4$. The $U(1)$ normalization radii are given at the bottom of the Table. The gravity, dilaton and gauge supermultiplets are not shown.

M	$\Sigma3$ (from $\Sigma1A$ with A_3) $SU(4)\otimes SU(6)\otimes U(1)^2$	$\Sigma3$ (from $\Sigma1B$ with A_3) $SU(4)\otimes SU(6)\otimes U(1)^2$
U	$(\mathbf{1},\mathbf{35})(0,0)_L$ $(\mathbf{1},\mathbf{1})(0,0)_L$ $(\mathbf{1},\mathbf{1})(+6,0)_L$ $2(\mathbf{4},\mathbf{1})(-3,-3)_L$ $2(\overline{\mathbf{4}},\mathbf{1})(-3,+3)_L$	$(\mathbf{1},\mathbf{35})(0,0)_L$ $(\mathbf{1},\mathbf{1})(0,0)_L$ $(\mathbf{1},\mathbf{1})(+6,0)_L$ $(\mathbf{4},\mathbf{1})(+3,+3)_L$ $(\overline{\mathbf{4}},\mathbf{1})(+3,-3)_L$
$T3$	$3(\mathbf{1},\mathbf{15})(-2,0)_L$ $3(\mathbf{1},\overline{\mathbf{6}})(+1,\pm1)_L$	$3(\mathbf{1},\mathbf{15})(-2,0)_L$ $3(\mathbf{1},\overline{\mathbf{6}})(+1,\pm1)_L$
$T6$	————	————
$T2$	$(\mathbf{4},\mathbf{1})(+3,+3)_L$ $(\overline{\mathbf{4}},\mathbf{1})(+3,-3)_L$	$2(\mathbf{4},\mathbf{1})(-3,-3)_L$ $2(\overline{\mathbf{4}},\mathbf{1})(-3,+3)_L$
$U(1)$	$(1/3\sqrt{2},\ 1/\sqrt{12})$	$(1/3\sqrt{2},\ 1/\sqrt{12})$

TABLE VII. The massless spectrum of the $SU(6)$ model $\Sigma3$ constructed by adding A_3 Wilson line to the model $\Sigma1A$ (first column) and to the model $\Sigma1B$ (second column). The model $\Sigma3$ has gauge symmetry $SU(4)_1\otimes SU(6)_3\otimes U(1)^2$. The $U(1)$ normalization radii are given at the bottom of the Table. The gravity, dilaton and gauge supermultiplets are not shown.

M	$\Sigma 1A$ $SU(3)\otimes SU(2)\otimes SU(6)\otimes U(1)^2$	$\Sigma 1B$ $SU(3)\otimes SU(2)\otimes SU(6)\otimes U(1)^2$
U	$(\mathbf{1},\mathbf{1},\mathbf{35})(0,0)_L$ $(\mathbf{1},\mathbf{3},\mathbf{1})(0,0)_L$ $(\mathbf{1},\mathbf{1},\mathbf{1})(+6,0)_L$ $2(\mathbf{1},\mathbf{1},\mathbf{1})(-3,\pm 3)_L$ $2(\mathbf{1},\mathbf{2},\mathbf{20})(0,0)_L$	$(\mathbf{1},\mathbf{1},\mathbf{35})(0,0)_L$ $(\mathbf{1},\mathbf{3},\mathbf{1})(0,0)_L$ $(\mathbf{1},\mathbf{1},\mathbf{1})(+6,0)_L$ $(\mathbf{1},\mathbf{1},\mathbf{1})(+3,\pm 3)_L$ $2(\mathbf{1},\mathbf{2},\mathbf{20})(0,0)_L$
$T3$	$(\mathbf{1},\mathbf{1},\mathbf{15})(-2,0)_L$ $2(\mathbf{1},\mathbf{1},\mathbf{15})(+1,\pm 1)_L$ $(\mathbf{1},\mathbf{2},\overline{\mathbf{6}})(+1,\pm 1)_L$ $2(\mathbf{1},\mathbf{2},\overline{\mathbf{6}})(-2,0)_L$	$(\mathbf{1},\mathbf{1},\mathbf{15})(-2,0)_L$ $(\mathbf{1},\mathbf{1},\overline{\mathbf{15}})(-1,\pm 1)_L$ $(\mathbf{1},\mathbf{2},\overline{\mathbf{6}})(+1,\pm 1)_L$ $(\mathbf{1},\mathbf{2},\mathbf{6})(+2,0)_L$
$T6$	$(\mathbf{1},\mathbf{2},\overline{\mathbf{15}})(-1,0)_L$ $(\mathbf{1},\mathbf{1},\mathbf{6})(+2,\pm 1)_L$	$2(\mathbf{1},\mathbf{2},\mathbf{15})(+1,0)_L$ $2(\mathbf{1},\mathbf{1},\overline{\mathbf{6}})(-2,\pm 1)_L$
$T2$	$2(\mathbf{1},\mathbf{2},\mathbf{1})(-3,0)_L$ $(\mathbf{1},\mathbf{4},\mathbf{1})(+3,0)_L$ $(\mathbf{1},\mathbf{2},\mathbf{1})(+3,0)_L$	$2(\mathbf{1},\mathbf{2},\mathbf{1})(+3,0)_L$ $2(\mathbf{1},\mathbf{4},\mathbf{1})(-3,0)_L$
$U(1)$	$(1/3\sqrt{2},\ 1/\sqrt{6})$	$(1/3\sqrt{2},\ 1/\sqrt{6})$

TABLE VIII. The massless spectra of the two $SU(6)$ models $\Sigma 1A$ and $\Sigma 1B$ both with gauge symmetry $SU(3)_1\otimes SU(2)_3\otimes SU(6)_3\otimes U(1)^2$. The $U(1)$ normalization radii are given at the bottom of the Table. The gravity, dilaton and gauge supermultiplets are not shown.

M	$\Sigma 2A$ $SU(2)\otimes SU(2)\otimes SU(6)\otimes U(1)^3$	$\Sigma 2B$ $SU(2)\otimes SU(2)\otimes SU(6)\otimes U(1)^3$
U	$(\mathbf{1},\mathbf{1},\mathbf{35})(0,0,0)_L$ $(\mathbf{1},\mathbf{3},\mathbf{1})(0,0,0)_L$ $(\mathbf{1},\mathbf{1},\mathbf{1})(0,+6,0)_L$ $(\mathbf{1},\mathbf{1},\mathbf{1})(0,-3,\pm 3)_L$ $2(\mathbf{1},\mathbf{2},\mathbf{20})(0,0,0)_L$	$(\mathbf{1},\mathbf{1},\mathbf{35})(0,0,0)_L$ $(\mathbf{1},\mathbf{3},\mathbf{1})(0,0,0)_L$ $(\mathbf{1},\mathbf{1},\mathbf{1})(0,+6,0)_L$ $(\mathbf{1},\mathbf{1},\mathbf{1})(0,-3,\pm 3)_L$ $2(\mathbf{1},\mathbf{2},\mathbf{20})(0,0,0)_L$ $(\mathbf{2},\mathbf{1},\mathbf{1})(\pm 3,0,0)_L$
$T3$	$(\mathbf{1},\mathbf{1},\mathbf{15})(0,-2,0)_L$ $(\mathbf{1},\mathbf{1},\mathbf{15})(0,+1,\pm 1)_L$ $2(\mathbf{1},\mathbf{2},\overline{\mathbf{6}})(0,-2,0)_L$ $2(\mathbf{1},\mathbf{2},\overline{\mathbf{6}})(0,+1,\pm 1)_L$	$(\mathbf{1},\mathbf{1},\mathbf{15})(0,-2,0)_L$ $(\mathbf{1},\mathbf{1},\mathbf{15})(0,+1,\pm 1)_L$ $(\mathbf{1},\mathbf{2},\mathbf{6})(0,+2,0)_L$ $(\mathbf{1},\mathbf{2},\mathbf{6})(0,-1,\pm 1)_L$
$T6$	$(\mathbf{1},\mathbf{1},\mathbf{6})(\pm 1,+2,0)_L$ $(\mathbf{1},\mathbf{1},\mathbf{6})(\pm 1,-1,-1)_L$ $(\mathbf{1},\mathbf{1},\mathbf{6})(\pm 1,-1,+1)_L$	$2(\mathbf{1},\mathbf{1},\overline{\mathbf{6}})(\pm 1,-2,0)_L$ $2(\mathbf{1},\mathbf{1},\overline{\mathbf{6}})(\pm 1,+1,+1)_L$ $2(\mathbf{1},\mathbf{1},\overline{\mathbf{6}})(\pm 1,+1,-1)_L$
$T2$	$(\mathbf{2},\mathbf{2},\mathbf{1})(0,0,0)_L$ $(\mathbf{2},\mathbf{4},\mathbf{1})(0,0,0)_L$	$2(\mathbf{2},\mathbf{2},\mathbf{1})(0,0,0)_L$
$U(1)$	$(1/\sqrt{6},\ 1/3\sqrt{2},\ 1/\sqrt{6})$	$(1/\sqrt{6},\ 1/3\sqrt{2},\ 1/\sqrt{6})$

TABLE IX. The massless spectra of the two $SU(6)$ models $\Sigma 2A$ and $\Sigma 2B$ both with gauge symmetry $SU(2)_1\otimes SU(2)_3\otimes SU(6)_3\otimes U(1)^3$. The $U(1)$ normalization radii are given at the bottom of the Table. The gravity, dilaton and gauge supermultiplets are not shown.

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