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4D Chiral $N = 1$ Type I Vacua with and without $D5$-branes

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Abstract

In this paper we consider compactifications of type I strings on Abelian orbifolds. We discuss the tadpole cancellation conditions for the general case with $D9$-branes only. Such compactifications have (perturbative) heterotic duals which are also realized as orbifolds (with non-standard embedding of the gauge connection). The latter have extra twisted states that become massive once orbifold singularities are blown-up. This is due to the presence of *perturbative* heterotic superpotential with couplings between the extra twisted states, the orbifold blow-up modes, and (sometimes) untwisted matter fields. Anomalous $U(1)$ (generically present in such models) also plays an important role in type I-heterotic (tree-level) duality matching. We illustrate these issues on a particular example of $\mathbf{Z}_3 \otimes \mathbf{Z}_3$ orbifold type I model (and its heterotic dual). The model has $N = 1$ supersymmetry, $U(4)^3 \otimes SO(8)$ gauge group, and chiral matter. We also consider compactifications of type I strings on Abelian orbifolds with both $D9$- and $D5$-branes. We discuss tadpole cancellation conditions for a certain class of such models. We illustrate the model building by considering a particular example of type I theory compactified on $\mathbf{Z}_6$ orbifold. The model has $N = 1$ supersymmetry, $[U(6) \otimes U(6) \otimes U(4)]^2$ gauge group, and chiral matter. We also propose the candidate heterotic dual. However, the precise matching (if any) of the two models remains an open question.

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I. INTRODUCTION

Recently it has become clear that type I-heterotic duality [1] can be extended to $N = 1$ cases in four dimensions [2,3]. In particular, two cases with $D9$-branes (but no $D5$-branes) have been worked out in detail. Thus, Ref [2] studied type I-heterotic duality matching for the Z-orbifold model of Ref [4]. There it was realized that although naively there is a discrepancy between the type I and heterotic spectra at the orbifold points, it disappears once orbifold singularities are blown-up and (anomalous) gauge symmetries are broken. The origin of this discrepancy is the following. If the type I orbifold model contains only $D9$-branes, then the open string sector gives rise to charged matter fields identical to those in the untwisted sector of the corresponding heterotic dual. The heterotic untwisted sector also contains gauge singlets which are geometric moduli. These are found in the untwisted closed string sector of the type I model. Since heterotic strings are closed strings, the heterotic orbifold model also contains twisted sectors. These give rise to certain singlets charged under $U(1)$ factors in the gauge group, but neutral under the non-Abelian symmetries. Their duals are found in the twisted closed string sectors of the type I model. The heterotic twisted sectors also contain matter fields charged under the non-Abelian gauge groups. These have no type I counterparts. This is precisely the descrepancy between the type I and heterotic spectra mentioned above.

The descrepancy is resolved by the fact that in the heterotic string model, there always is a *perturbative* superpotential such that after appropriate Higgsing the extra twisted matter fields (charged under non-Abelian gauge groups) become heavy and decouple from the massless spectrum. This Higgsing always involves blowing-up orbifold singularities, and can sometimes also involve breaking of (anomalous) gauge symmetries. Thus, in the Z-orbifold case [4] studied in Ref [2] the relevent terms in the superpotential are *renormalizable* Yukawa couplings of the form ST^2 (this is only a symbolic way of writing these couplings as certain indices are suppressed), where S are the orbifold blow-up modes, whereas T are the extra twisted matter fields. In Ref [3] we worked out another chiral $N = 1$ type I model on $\mathbf{Z}_7$ orbifold. There the couplings responsible for making the extra twisted matter fields heavy were *non-renormalizable* couplings of the form ST^2Q^2, where Q stands for untwisted matter fields acquiring vevs in order to break anomalous $U(1)$ gauge symmetry present in that model.

In this paper we give a prescription (following from tadpole cancellation conditions given in Appendix A) for contructing type I models with only $D9$-branes on general Abelian orbifolds (the latter have odd order or else $D5$-branes would be present). Constructing their heterotic duals is not difficult. For a given type I model on T^{2d}/G symmetric Abelain orbifold (of odd order), one works out the Chan-Paton matrices γ_g ($g \in G$) according to the prescription given in Appendix A. Then the heterotic dual is constructed by compactifying the Spin(32)/$\mathbf{Z}_2$ heterotic string on T^{2d}/G with gauge connection (shift) in each twisted sector (labeled by) g given by γ_g. The duality matching always goes as described above. To illustrate the rules and duality matching, we consider type I compactification on $\mathbf{Z}_3 \otimes \mathbf{Z}_3$ orbifold (which is not a $\mathbf{Z}_N$ orbifold).

We also generalize the rules of Appendix A to a certain class of orbifolds with both $D9$- and $D5$-branes (the orbifold has even order in this case). To illusterate the rules we construct a type I compactification on $\mathbf{Z}_6$ orbifold. The model has $N = 1$ supersymmetry,

$U(6) \otimes U(6) \otimes U(4)$ gauge group coming from the 99 strings, $U(6) \otimes U(6) \otimes U(4)$ gauge group coming from the 55 strings, and chiral matter coming from all three open string sectors (99, 55 and 59). This model has anomalous $U(1)$ gauge symmetry.

We propose a candidate heterotic dual (and discuss some issues relevant for possible duality), and give the massless spectrum for this model. The precise matching (if any) between the type I and heterotic model, however, remains an open question and is currently under investigation.

The paper is organized as follows. In section II we construct type I compactification on $\mathbf{Z}_3 \otimes \mathbf{Z}_3$ orbifold. In section III we construct the heterotic dual. In section IV we give perturbative superpotentials for these models. In section V we discuss the moduli space, and explain the matching between type I and heterotic tree-level massless spectra. In section VI we construct type I compactification on $\mathbf{Z}_6$ orbifold. In section VII we propose the candidate heterotic dual, and discuss the potential problems with duality matching. In section VIII we give conclusions and remarks. Appendix A contains the rules for constructing type I compactifications with only $D9$-branes on general Abelian orbifolds (of odd order). Appendix B generalizes these rules to certain type I orbifolds (of even order) with both $D9$- and $D5$-branes.

While completing the manuscript we were informed [5] that another group has obtained results overlaping with some of ours. The former will soon appear.

II. TYPE I STRING ON $\mathbf{Z}_3 \otimes \mathbf{Z}_3$ ORBIFOLD

In this section we discuss the construction of the type I model on $\mathbf{Z}_3 \otimes \mathbf{Z}_3$ orbifold. Let us start from the type IIB string model compactified on the six-torus $T^2 \otimes T^2 \otimes T^2$, where each of the two-tori T^2 has a $\mathbf{Z}_3$ rotational symmetry. This model has $N = 8$ supersymmetry. Let us now consider the symmetric $\mathbf{Z}_3 \otimes \mathbf{Z}_3$ orbifold model generated by the twists

$$T_3 = (\theta, \theta, 0 || \theta, \theta, 0) \ , \qquad (1)$$

$$T_3' = (0, \theta, \theta || 0, \theta, \theta) \ . \qquad (2)$$

Here θ is a $2\pi/3$ rotation of a complex boson (we have complexified the six real bosons into three complex bosons). The double vertical line separates the right- and left-movers of the string. The resulting model has $N = 2$ space-time supersymmetry. This model has the following moduli. There are 8 NS-NS fields $\phi, B_{\mu\nu}, B_{i\bar{i}}, g_{i\bar{i}}$, and 8 R-R fields $\phi', B'_{\mu\nu}, B'_{i\bar{i}}, C'_{\mu\nu i\bar{i}}$.

Let us now consider the orientifold projection of this model. The closed string sector (which is simply the subspace of the Hilbert space of the original type IIB spectrum invariant under the orientifold projection Ω) contains the $N = 1$ supergravity multiplet, and 3 untwisted (the NS-NS fields that survive the Ω projection are $g_{i\bar{i}}$, whereas the R-R fields that are kept are $B'_{i\bar{i}}$; note that the NS-NS field ϕ and the R-R field $B'_{\mu\nu}$ also survive and, enter in the dilaton supermultiplet) and 81 twisted chiral supermultiplets (which are neutral under the gauge group of the model). For consistency (*i.e.*, tadpole cancellation; see Appendix A for details), we must include the open string sector. Note that in this model we only have $D9$-branes but no $D5$-branes since the orbifold group does not contain an order two element. Thus, we only have 99 open strings. The gauge group consistent with tadpole cancellation then is $U(4) \otimes U(4) \otimes U(4) \otimes SO(8)$. The 99 open strings also give rise to the following chiral matter fields: $(\overline{\mathbf{4}},\mathbf{1},\mathbf{1},\mathbf{8}_v)(+1,0,0)_L$, $(\mathbf{1},\overline{\mathbf{4}},\overline{\mathbf{4}},\mathbf{1})(0,-1,-1)_L$ and $(\mathbf{6},\mathbf{1},\mathbf{1},\mathbf{1})(+2,0,0)_L$. In addition, there are fields that can be obtained by permuting the three $U(4)$'s (this permutation must be accompanied by changing the irrep of the second and the third $U(4)$ to its complex conjugate). Here the first four entries in bold font indicate the irreps of the $SU(4)\otimes SU(4)\otimes SU(4)\otimes SO(8)$ subgroup, whereas the $U(1)^3$ charges are given in the parenthesis. The subscript L indicates the space-time helicity of the corresponding fermionic fields. The massless spectrum of this model is summarized in Table I.

Note that the $U(1)^3$ gauge symmetry is anomalous. We can form a linear combination of these $U(1)$'s such that only one of them is anomalous (this combination is given by $Q_1-Q_2-Q_3$, where $Q_{1,2,3}$ are the first, second and third $U(1)$ charges, respectively). The total $U(1)$ anomaly is $+36$. By the generalized Green-Schwarz mechanism [6] some of the fields charged under $U(1)$ will acquire vevs to cancel the Fayet-Illiopoulos D-term.

III. HETEROTIC STRING ON $\mathbf{Z}_3\otimes\mathbf{Z}_3$ ORBIFOLD

In this section we give the construction of the heterotic string model that is (candidate) dual to the type I model considered in the previous section. Let us start from the Narain model with $N=4$ space-time supersymmetry in four dimensions. Let the momenta of the internal (6 right-moving and 22 left-moving) world-sheet bosons span the (even self-dual) Narain lattice $\Gamma^{6,22}=(\Gamma^{2,2})^3\otimes\Gamma^{16}$. Here Γ^{16} is the Spin(32)/$\mathbf{Z}_2$ lattice, whereas the lattice $\Gamma^{2,2}$ (that corresponds to a two-torus T^2) is spanned by the momenta $(p_R||p_L)$ with

$$p_{L,R}=\frac{1}{2}m_i\tilde{e}^i\pm n^ie_i\ . \tag{3}$$

Here m_i and n^i are integers, $e_i\cdot e_j=g_{ij}$ is the constant background metric, and $e_i\cdot\tilde{e}^j=\delta_i{}^j$. Note that we could have included the constant anti-symmetric background tensor field B_{ij}, but for now we will set it equal to zero.

This Narain model has the gauge group $SO(32)\otimes U(1)^6$. The first factor $SO(32)$ comes from the Γ^{16} lattice (the 480 roots of length squared 2), and 16 oscillator excitations of the corresponding world-sheet bosons (the latter being in the Cartan subalgebra of $SO(32)$). The factor $U(1)^6$ comes from the oscillator excitations of the six left-moving world-sheet bosons corresponding to $\Gamma^{6,6}=(\Gamma^{2,2})^3$. Note that there are also six additional vector bosons coming from the oscillator excitations of the right-moving world-sheet bosons corresponding to $\Gamma^{6,6}$. These vector bosons are part of the $N=4$ supergravity multiplet.

Next consider the $\mathbf{Z}_3\otimes\mathbf{Z}_3$ orbifold model (with non-standard embedding of the gauge connection) obtained via twisting the above Narain model by the following $\mathbf{Z}_3$ twists:

$$T_3=(\theta,\theta,0||\theta,\theta,0|(\frac{1}{3})^4(-\frac{1}{3})^4 0^8)\ , \tag{4}$$

$$T_3'=(0,\theta,\theta||0,\theta,\theta|(\frac{1}{3})^2 0^2(-\frac{1}{3})^2 0^2(\frac{1}{3})^2(-\frac{1}{3})^2 0^4)\ . \tag{5}$$

$$\tag{6}$$

Here θ is a $2\pi/3$ rotation of a complex boson (we have complexified the original six real bosons into three complex ones). Thus, the first three entries correspond to the $\mathbf{Z}_3$ twists of the three right-moving complex bosons (coming from the six-torus). The double vertical line separates the right- and left-movers. The first three left-moving entries correspond to the $\mathbf{Z}_3$ twists of the three left-moving complex bosons (coming from the six-torus). The single vertical line separates the latter from the sixteen real bosons corresponding to the Γ^{16} lattice. The latter are written in the $SO(32)$ basis. Thus, for example, $(+1,-1,0^{14})$ is a root of $SO(32)$ with length squared two. There are 480 roots like this in the Γ^{16} lattice, and they are descendents of the identity irrep of $SO(32)$. The lattice Γ^{16} also contains one of the spinor irreps as well. Thus, we will choose this spinor irrep to contain the momentum states of the form $(\pm\frac{1}{2},...,\pm\frac{1}{2})$ with even number of plus signs.

Now we are ready to discuss the orbifold model generated by the twists T_3 and T_3' . This model has $N=1$ space-time supersymmetry, and gauge group $U(4)\otimes U(4)\otimes U(4)\otimes SO(8)$, the same as the type I model discussed in the previous section. The untwisted sector gives rise to the $N=1$ supergravity multiplet coupled to the $N=1$ Yang-Mills gauge multiplet in the adjoint of $U(4)\otimes U(4)\otimes U(4)\otimes SO(8)$. The matter fields in the untwisted sector are the same as those in the open string sector of the type I model. There are also chiral multiplets neutral under the gauge group: $3(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(0,0,0)_L$. Note that these contain 6 scalar fields that are the left-over geometric moduli whose vevs parametrize the moduli space $[SU(1,1,\mathbf{Z})\backslash SU(1,1)/U(1)]^3$. (This is the subspace of the original Narain moduli space $SO(6,6,\mathbf{Z})\backslash SO(6,6)/SO(6)\otimes SO(6)$ that is invariant under the twist.) Actually, the (perturbative) moduli space of this model is larger, and we will return to this point later on.

Next, consider the twisted sector. Since both T_3 and T_3' are of order 3, their respective contributions to the one-loop partition function can have a non-trivial relative $\mathbf{Z}_3$ phase between them which we denote as $\phi(T_3,T_3')$ (*i.e.*, $\phi(T_3,T_3')=0,1/3,2/3$). The states that survive the T_3 projection in the T_3' sector have T_3 phase $\phi(T_3,T_3')$, and the states that survive the T_3 projection in the inverse twisted sector $(T_3')^{-1}$ have T_3 phase $-\phi(T_3,T_3')$. String consistency requires that the states that survive the T_3' projection in the T_3 sector must have T_3' phase $-\phi(T_3,T_3')$. Similarly, the states that survive the T_3' projection in the inverse twisted sector $(T_3)^{-1}$ must have T_3' phase $\phi(T_3,T_3')$. The twisted sector field content depends on the relative phase $\phi(T_3,T_3')$ and so it gives rise to three different heterotic string models. It turns out that the model with $\phi(T_3,T_3')=2/3$ is dual to the type I model described in the previous section. Here, we list out the twisted sector field content of this model. We have the following chiral supermultiplets: $9(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(\pm4/3,\pm4/3,0)_L$, $9(\mathbf{1},\mathbf{6},\mathbf{1},\mathbf{1})(+4/3,-2/3,0)$ and $9(\mathbf{6},\mathbf{1},\mathbf{1},\mathbf{1})(+2/3,-4/3,0)$ together with fields obtained by permuting the three $U(4)$'s (this permutation must be accompanied by changing the irrep of the second and the third $U(4)$ to its complex conjugate). In addition, there are $27(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(-4/3,+4/3,+4/3)_L$ and $27(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{8_s})(+2/3,-2/3,-2/3)_L$ coming from T_3+T_3' sector (and its conjugate). Here we note that the factors "9" and "27" come from the number of fixed points in different sectors of the $\mathbf{Z}_3\times\mathbf{Z}_3$ orbifold we are considering. The 27 singlets and the 27 $\mathbf{8_s}$ of $SO(8)$ are all projected out in the other two models ($\phi(T_3,T_3')=0,1/3$), and hence their spectra do not match with the type I model.

We summarize the massless spectrum of the heterotic string model which is the candidate dual of the type I model in Table II. Note that the $U(1)^3$ gauge symmetry is anomalous. Again, only one linear combination of the three $U(1)$'s is anomalous. Thus, the contributions of the untwisted and twisted sectors into the trace anomaly are $+36$ and $27\times(+36)$, respectively, so that the total trace anomaly is $+972$. By the generalized Green-Schwarz mechanism [6] some of the fields charged under $U(1)$ will acquire vevs to cancel the Fayet-Illiopoulos D-term.

IV. SUPERPOTENTIAL

In this section we discuss the perturbative superpotentials for the type I and heterotic string models discussed in the previous sections. Studying the couplings and flat directions in these superpotentials will enable us to make the type I-heterotic duality map more precise.

Let us start from the type I model of section II. We refer the reader to Table I for the massless spectum as well as our notation. Note that perturbatively the 81 chiral singlets coming from the closed string sector are flat. This can be explicitly seen by computing the scattering amplitudes for these modes within the framework of the conformal field theory of orbifolds [8]. On the other hand, the matter fields coming from the 99 open string sector have three (and, of course, some higher) point couplings. The lowest order superpotential can be written as (the calculation of the type I superpotential is completely analogous to that of the heterotic one in the untwisted sector)

$$W_I = \lambda \epsilon_{abc} \mathrm{Tr}(P_a P_b Q_c) + \dots \ . \tag{7}$$

Due to the presence of the anomalous $U(1)$, some of the fields that are charged under this $U(1)$ must acquire vevs to cancel the Fayet-Illiopoulos D-term. This results in breakdown of gauge symmetry, yet the space-time supersymmetry is preserved.

Now let us turn to the heterotic string model. The superpotential of this model is more involved than that of the type I model as there are non-trivial couplings between the twisted sector fields. The superpotential for the heterotic string model thus reads (here we are only interested in the general structure of the *non-vanishing* terms):

$$\begin{aligned} W_H = \quad & \lambda' \epsilon_{abc} \mathrm{Tr}(P_a P_b Q_c) + \Lambda^{(\alpha\alpha'\alpha'')(\beta\beta'\beta'')(\gamma\gamma'\gamma'')} \mathrm{Tr}(S_{\alpha\beta\gamma} T_{\alpha'\beta'\gamma'} T_{\alpha''\beta''\gamma''}) \\ & + \Lambda_1^{(\alpha\alpha'\alpha'')(\beta\beta')(\gamma\gamma'')} \mathrm{Tr}(S_{\alpha\beta\gamma} T^{1+}_{\alpha'\beta'} T^{3+}_{\alpha''\gamma''}) + \Lambda_2^{(\alpha\alpha')(\beta\beta'\beta'')(\gamma\gamma'')} \mathrm{Tr}(S_{\alpha\beta\gamma} T^{1-}_{\alpha'\beta'} T^{2-}_{\beta''\gamma''}) \\ & + \Lambda_3^{(\alpha\alpha'')(\beta\beta')(\gamma\gamma'\gamma'')} \mathrm{Tr}(S_{\alpha\beta\gamma} T^{2+}_{\beta'\gamma'} T^{3-}_{\alpha''\gamma''}) + \dots \ . \end{aligned} \tag{8}$$

(The notation for the fields is given in Table II.) The couplings Λ, Λ_1, Λ_2 and Λ_3 are non-vanishing if the orbifold *space group* selection rules are satisifed. These rules read as follows. $\Lambda^{(\alpha\alpha'\alpha'')(\beta\beta'\beta'')(\gamma\gamma'\gamma'')} \neq 0$ if and only if $\alpha = \alpha' = \alpha''$ or $\alpha \neq \alpha' \neq \alpha'' \neq \alpha$, and similarly for the β- and γ-indices. $\Lambda_1^{(\alpha\alpha'\alpha'')(\beta\beta')(\gamma\gamma'')} \neq 0$ if and only if $\alpha = \alpha' = \alpha''$ or $\alpha \neq \alpha' \neq \alpha'' \neq \alpha$, $\beta = \beta'$ and $\gamma = \gamma''$. The selection rules for the Λ_2 and Λ_3 couplings are similar to those for the Λ_1 couplings. Here we note that, say, couplings $\Lambda^{(\alpha\alpha'\alpha'')(\beta\beta'\beta'')(\gamma\gamma'\gamma'')}$ with $\alpha \neq \alpha' \neq \alpha'' \neq \alpha$, and similarly for the β- and γ-indices, are exponentially suppressed in the limit of large volume compactification, whereas the couplings with $\alpha = \alpha' = \alpha''$, $\beta = \beta' = \beta''$ and $\gamma = \gamma' = \gamma''$ are not suppressed. This is because in the former case, the corresponding fields are coming from different fixed points so that upon taking them apart (in the limit of large volume of the orbifold) their coupling becomes weaker and weaker. Similar statements hold for the Λ_1, Λ_2 and Λ_3 couplings as well.

Notice that upon the singlets $S_{\alpha\beta\gamma}$ (which are 27 blow-up modes) acquiring vevs, the states $T_{\alpha\beta\gamma}$ and $T^{a\pm}_{\alpha\beta}$ become heavy and decouple from the massless spectrum. Thus, after blowing up the orbifold singularities on the heterotic side (combined with some of the untwisted charged matter fields acquiring vevs to cancel the D-term), we can match the massless spectrum to that of the type I model (where the charged matter must acquire vevs to cancel the effect of the anomalous $U(1)$). Note the crucial role of the perturbative superpotential in this matching. It is precisely such that all the extra fields on the heterotic side can be made massive by blowing up the orbifold.

V. MODULI SPACE

We now turn to the discussion of the moduli spaces for the type I and heterotic models considered in the previous sections. Let us start with the heterotic model. The (perturbative) moduli space of the corresponding Narain model before orbifolding is $SO(6,22,\mathbf{Z})\backslash SO(6,22)/SO(6)\otimes SO(22)$. After orbifolding we have two types of moduli: those coming from the untwisted sector, and those coming from the twisted sector. The untwisted sector moduli parametrize the coset $[SU(1,3,\mathbf{Z})\backslash SU(1,3)/SU(3)\otimes U(1)]^3$. The subspace $[SU(1,1,\mathbf{Z})\backslash SU(1,1)/U(1)]^3$ of this moduli space is parametrized by 6 neutral singlets ϕ_a that correspond to the left-over geometric moduli (coming from the constant metric g_{ij} and antisymmetric tensor B_{ij} fields). The other 12 moduli correspond to the flat directions in the superpotential for the fields P_a, Q_a and Φ_a. (These are the left-over moduli coming from the 6×16 Wilson lines A^I_i, $I=1,...,16$, in the original Narain model.)

Next, we turn to the twisted moduli of the heterotic string model. In the twisted sectors, we have the chiral superfields $S^{a\pm}_{\alpha\beta}$, $T^{a\pm}_{\alpha\beta}$, $S_{\alpha\beta\gamma}$ and $T_{\alpha\beta\gamma}$. At a generic point (upon giving appropriate vevs to the $S_{\alpha\beta\gamma}$ fields), the fields $T^{a\pm}_{\alpha\beta}$ and $T_{\alpha\beta\gamma}$ become massive (according to the couplings in the superpotential) and there is no superpotential for the singlets $S^{a\pm}_{\alpha\beta}$ and $S_{\alpha\beta\gamma}$ (the blow-up modes of the $\mathbf{Z}_3\otimes\mathbf{Z}_3$ orbifold). The singlets $S^{a\pm}_{\alpha\beta}$ are not charged under the anomalous $U(1)$. Hence, all of them survive the Higgsing process and they match with the corresponding singlets in the type I spectrum. However, the fields $S_{\alpha\beta\gamma}$ are charged under the anomalous $U(1)$. Therefore, a linear combination of them will be eaten in the super-Higgs mechanism. So, naively, not all the 27 chiral fields survive after Higgsing. This would pose a problem for matching of the type I and heterotic models. Note, however, that the type I model has anomalous $U(1)$, and to cancel the D-term one needs to give vevs to the corresponding charged fields. So, generically, the fields P_a, Q_a and Φ_a will acquire vevs (on the type I side) to break the anomalous $U(1)$. Thus, to match the type I and heterotic models we have to give vevs to the P_a, Q_a and Φ_a fields on the heterotic side as well. Then, we will have 27 neutral chiral superfields (which, after Higgsing, are a mixture of the original fields $S_{\alpha\beta\gamma}$ and the untwisted matter fields) on the heterotic side, which do correspond to the 27 neutral chiral superfields coming from the twisted closed string sector of the type I model. Thus, the matching is complete after giving appropriate vevs to *both* twisted *and* untwisted fields on the heterotic side, as well as giving appropriate vevs to open string sector matter fields, and 27 twisted closed string moduli. Upon breaking the anomalous $U(1)$, the dilaton may mix with other gauge singlets [7]. *A priori*, the mixing is different on the type I and the heterotic side. To make the matching precise, one generically has to appropriately tune the dilaton plus ϕ_a geometric moduli on both sides.

Thus, the moduli spaces (at generic points) of both type I and heterotic models are the same (at least at tree-level). They are described by the untwisted moduli of the heterotic string, or equivalently, the moduli coming from the untwisted closed string sector and the open string sector of the type I model (these parametrize the coset $[SU(1,3,\mathbf{Z})\backslash SU(1,3)/SU(3)\otimes U(1)]^3$), plus the 2×81 twisted moduli in the heterotic string model, or equivalently, the moduli coming from the twisted closed string sector of the type I model. The (perturbative) moduli space (of the heterotic model) is schematically depicted in Fig.1.

It is worth noticing the role of anomalous $U(1)$ in $N=1$ type I-heterotic duality. To cancel the Fayet-Illiopoulos D-term, fields that are charged under the anomalous $U(1)$ will generically acquire vevs. As a result, the extra twisted matter fields in the heterotic model are higgsed away and the matching of the massless spectra of the type I and heterotic models is precise. The appearance of massless twisted matter fields $T_{\alpha\beta\gamma}$ and $T^{a\pm}_{\alpha\beta}$ on the heterotic side is a perturbative effect. On the type I side this effect is non-perturbative, and reflects the fact that from type I point of view there is a (non-perturbative) singularity in the moduli space (or, more precisely, a singular subspace of the full moduli space). Notice that the fields $T_{\alpha\beta\gamma}$ and $T^{a\pm}_{\alpha\beta}$ in the heterotic model get heavy via couplings in the *perturbative* superpotential. This indicates the importance of perturbative superpotential in $N=1$ type I-heterotic duality.

VI. TYPE I STRING ON $\mathbf{Z}_6$ ORBIFOLD

In this section we discuss the compactification of type I strings on $\mathbf{Z}_6$ orbifold. Since $\mathbf{Z}_6$ has an order two element, we have to include both $D9$- and $D5$-branes. Let us start from the type IIB string model compactified on the six-torus $T^2\otimes T^2\otimes T^2$, where each of the two-tori has a $\mathbf{Z}_3$ and a $\mathbf{Z}_2$ rotational symmetry. This model has $N=8$ supersymmetry. Let us now consider the symmetric $\mathbf{Z}_6$ orbifold model generated by the twists

$$T_3 = (\theta,\theta,\theta||\theta,\theta,\theta)\ , \qquad (9)$$
$$T_2 = (\sigma,\sigma,0||\sigma,\sigma,0)\ . \qquad (10)$$

Here θ is a $2\pi/3$ rotation of a complex boson (we have complexified the six real bosons into three complex bosons). Similarly, σ is a π rotation of a complex boson. The double vertical line separates the right- and left-movers of the string. The resulting model has $N=2$ space-time supersymmetry. This model has the following moduli. This model has the following moduli. There are 12 NS-NS fields $\phi, B_{\mu\nu}, B_{i\bar{j}}, g_{i\bar{j}}$ $(i,j=1,2)$, $B_{3\bar{3}}, g_{3\bar{3}}$ and 12 R-R fields $\phi', B'_{\mu\nu}, B'_{i\bar{j}}, C'_{\mu\nu i\bar{j}}$ $(i,j=1,2)$, $B'_{3\bar{3}}, C'_{\mu\nu 3\bar{3}}$.

Let us now consider the orientifold projection of this model. The closed string sector (which is simply the subspace of the Hilbert space of the original type IIB spectrum invariant under the orientifold projection Ω) contains the $N=1$ supergravity multiplet, and 5 untwisted (the NS-NS fields that survive the Ω projection are $g_{i\bar{j}}$ $(i,j=1,2)$ and $g_{3\bar{3}}$, whereas the R-R fields that are kept are $B'_{i\bar{j}}$ $(i,j=1,2)$ and $B'_{3\bar{3}}$; note that the NS-NS field ϕ and the R-R field $B'_{\mu\nu}$ also survive and, enter in the dilaton supermultiplet) and 29 twisted chiral supermultiplets (which are neutral under the gauge group of the model). For consistency (*i.e.*, tadpole cancellation; see Appendix B for details), we must include the open string sector. In this model, there are both $D9$- and $D5$-branes. The gauge group consistent with tadpole cancellation then is $[U(6)\otimes U(6)\otimes U(4)]^2$. The gauge bosons of the first $U(6)\otimes U(6)\otimes U(4)$ factor come from the 99 sector whereas the gauge bosons of the second $U(6)\otimes U(6)\otimes U(4)$

come from the 55 sector. The 99 sector also gives rise to the following chiral matter fields charged under the first $U(6) \otimes U(6) \otimes U(4)$: $2(\mathbf{15}, \mathbf{1}, \mathbf{1})(+2, 0, 0)_L$, $2(\mathbf{1}, \overline{\mathbf{15}}, \mathbf{1})(0, -2, 0)_L$, $2(\overline{\mathbf{6}}, \mathbf{1}, \overline{\mathbf{4}})(-1, 0, -1)_L$, $2(\mathbf{1}, \mathbf{6}, \mathbf{4})(0, +1, +1)_L$, $(\mathbf{6}, \overline{\mathbf{6}}, \mathbf{1})(+1, -1, 0)_L$, $(\overline{\mathbf{6}}, \mathbf{1}, \mathbf{4})(-1, 0, +1)_L$ and $(\mathbf{1}, \mathbf{6}, \overline{\mathbf{4}})(0, +1, -1)_L$. Here the first three entries in bold font indicate the irreps of the $SU(6) \otimes SU(6) \otimes SU(4)$ subgroup, whereas the $U(1)^3$ charges are given in the parenthesis. The subscript L indicates the space-time helicity of the corresponding fermionic fields. The 55 sector gives rise to similar chiral matter fields but charged under the second $U(6) \otimes U(6) \otimes U(4)$. The fact that the 99 and 55 sector have the same spectrum follows from T-duality. The 59 sector provides the chiral matter fields charged under both 99 and 55 gauge groups. The massless spectrum of this model is summarized in Table III. Note that this model has anomalous $U(1)$.

VII. HETEROTIC STRING ON $\mathbf{Z}_6$ ORBIFOLD

In this section we propose the candidate heterotic dual of the type I model considered in the previous section. Let us start from the same Narain model with gauge group $SO(32) \otimes U(1)^6$ and $N = 4$ space-time supersymmetry in four dimensions as described in section III. Consider the $\mathbf{Z}_6$ orbifold model (with non-standard embedding of the gauge connection) obtained via twisting the above Narain model by the followind twists:

$$T_3 = (\theta, \theta, \theta || \theta, \theta, \theta | (\frac{1}{3})^6 (-\frac{1}{3})^6 0^4) \ , \tag{11}$$

$$T_2 = (\sigma, \sigma, 0 || \sigma, \sigma, 0 | (\frac{1}{4})^3 (-\frac{1}{4})^3 (-\frac{1}{4})^3 (\frac{1}{4})^3 (\frac{1}{4})^2 (-\frac{1}{4}) (\frac{3}{4})) \ . \tag{12}$$

Here θ is a $2\pi/3$ rotation of a complex boson (we have complexified the original six real bosons into three complex ones). Thus, the first three entries correspond to the $\mathbf{Z}_3$ twists of the three right-moving complex bosons (coming from the six-torus). Similarly, σ is a π rotation of a complex boson. The double vertical line separates the right- and left-movers. The first three left-moving entries correspond to the twists of the three left-moving complex bosons (coming from the six-torus). The single vertical line separates the latter from the sixteen real bosons corresponding to the Γ^{16} lattice. The latter are written in the $SO(32)$ basis just as in section III.

Now we are ready to discuss the orbifold model generated by the twists T_3 and T_2. This model has $N = 1$ space-time supersymmetry, and gauge group $U(6) \otimes U(6) \otimes U(4)$, the same as the 99 sector of the type I model discussed in the previous section. The untwisted sector gives rise to the $N = 1$ supergravity multiplet coupled to the $N = 1$ Yang-Mills gauge multiplet in the adjoint of $U(6) \otimes U(6) \otimes U(4)$. The charged matter fields in the untwisted sector are the same as those in the 99 open string sector of the type I model. There are also chiral multiplets neutral under the gauge group: $5(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{1})(0, 0, 0)_L$. Note that these contain 10 scalar fields that are the left-over geometric moduli whose vevs parametrize the moduli space $[SU(1, 1, \mathbf{Z}) \backslash SU(1, 1)/U(1)] \otimes [SU(2, 2, \mathbf{Z}) \backslash SU(2, 2)/SU(2) \otimes SU(2) \otimes U(1)]$. (This is the subspace of the original Narain moduli space $SO(6, 6, \mathbf{Z}) \backslash SO(6, 6)/SO(6) \otimes SO(6)$ that is invariant under the twist.)

Next, we list out the twisted sector field content of the model. In T_3 sector, we have the following chiral multiplets: $15(\mathbf{1}, \mathbf{1}, \mathbf{1})(-2, +2, 0)_L$, $15(\mathbf{1}, \mathbf{1}, \mathbf{6})(+1, -1, 0)_L$ and

$12(\mathbf{1},\mathbf{1},\mathbf{1})(+1,-1,\pm 2)_L$. The factor 15 comes from the 15 linear combinations of the fixed points (out of the 27 fixed points under $\mathbf{Z}_3$ twist) that are invariant under the $\mathbf{Z}_2$ twist. Similarly, the factor 12 comes from the the remaining linear combinations of the fixed points that have non-zero $\mathbf{Z}_2$ phase. The T_6 sector contributes the following chiral multiplets: $3(\mathbf{6},\mathbf{1},\mathbf{1})(-3/2,-1/2,+1)_L$ and $3(\mathbf{1},\overline{\mathbf{6}},\mathbf{1})(+1/2,+3/2,-1)_L$. Similarly, in T_2 sector, we have the following chiral multiplets: $5(\mathbf{1},\mathbf{1},\overline{\mathbf{4}})(+3/2,+3/2,0)_L$, $5(\mathbf{1},\mathbf{1},\mathbf{4})(-3/2,-3/2,0)_L$, $6(\overline{\mathbf{6}},\mathbf{1},\mathbf{1})(+1/2,+3/2,+1)_L$, $5(\mathbf{6},\mathbf{1},\mathbf{1})(-1/2,-3/2,-1)_L$, $5(\mathbf{1},\overline{\mathbf{6}},\mathbf{1})(+3/2,+1/2,+1)_L$ and $6(\mathbf{1},\mathbf{6},\mathbf{1})(-3/2,-1/2,-1)_L$. We summarize the massless spectrum of this heterotic string model (the candidate dual of the Type I model considered in the previous section) in Table IV.

Let us discuss some of the issues that would need to be resolved in order to understand whether the heterotic string model constructed in this section is dual (at least at tree-level) to the type I model of the previous section. First, we note that by construction the untwisted sector of the heterotic model is the same as 99 open string sector plus the untwisted closed string sector of the type I model. There are two types of differences, however. First, in order to make the matching possible, the 55 gauge group in the type I model must be broken at least to some residual Abelian factors. The latter, if present, could be further broken as they may turn out to be anomalous as in the six-dimensional example studied in Ref [9]. But getting rid of the non-Abelian gauge symmetry seems to be more non-trivial. In Ref [9] this was done by placing the $D5$-branes at *different* orbifold fixed points. By T-duality this can be understood as turning on *discrete* Wilson lines in the 99 sector. In the $\mathbf{Z}_2$ case of Ref [9] there exist exactly as many discrete Wilson lines compatible with $\mathbf{Z}_2$ orbifold as required to break the gauge symmetry to $U(1)^{16}$. Then the latter being anomalous are broken, and the corresponding 16 hypermultiplets coming from the $\mathbf{Z}_2$ twisted closed string sector of the type I model (these states do not have heterotic counterparts) are eaten in the super-Higgs mechanism. One would expect something similar to be the case in the $\mathbf{Z}_6$ model as well, but it seems to be problematic to break the non-Abelian gauge symmetry completely just with the discrete Wilson lines as there are tighter constraints on the latter in the $\mathbf{Z}_6$ case. Also, in the heterotic $\mathbf{Z}_6$ model there are extra twisted states. It remains to be seen whether they become massive via couplings in the superpotential. On the other hand, there are extra singlets in (some of) the type I twisted closed string sectors, and it is unclear whether there is a mechanism that gets rid of them. Also note that the $\mathbf{Z}_2$ twisted sector in the heterotic $\mathbf{Z}_6$ model has multiplicities 6, 5 and 5 (coming from $16 = 6 + 5 + 5$) for the numbers of fixed points. The sector corresponding to this on the type I side is the 59 sector, where the $D5$-brane Chan-Paton indices can be viewed as the corresponding multiplicities. The latter, however, come out to be 6, 6 and 4. These and other issues need to be resolved in order to make the matching precise, and are currently under investigation.

VIII. CONCLUSIONS

In this paper we discussed the rules for type I compactifications with only $D9$-branes on general Abelian orbifolds (of odd order), and gave the prescription for constructing their heterotic duals. We discussed the issues involved in the duality matching, and illustrated them on a particular example of $\mathbf{Z}_3 \otimes \mathbf{Z}_3$ orbifold. We also generalized the rules for type I model building to certain cases with both $D9$- and $D5$-branes present. The example of

$\mathbf{Z}_6$ orbifold we consider in this paper would be an interesting arena for testing the validity of type I-heterotic duality. We pointed out some problems to be resolved in order to make matching with the candidate heterotic dual we have proposed and constructed. Work in these directions is in progress.

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APPENDIX A: TADPOLES FOR $D9$-BRANES

In this appendix we discuss the tadpole cancellation constraints for orbifold compactifications of type I strings without $D5$-branes. In particular, we confine our attention to general Abelian $\mathbf{Z}_{n_1}\otimes\mathbf{Z}_{n_2}\otimes\cdots\otimes\mathbf{Z}_{n_k}$ orbifolds where $n_1,n_2,\ldots,n_k$ are all odd integers (if the orbifold group contains an order two element R, then the sector ΩR would contain $D5$-branes). The constraints for orbifold compactifications with $D5$-branes will be given in the next section.

There are two kinds of constraints we need to consider. The first one comes from the cancellation of the untwisted tadpoles for the $D9$-branes. This constraint is the same in all dimensions and leads to the statement that there are 16 $D9$-branes *not* counting the orientifold images. (This last statement is only correct if the NS-NS antisymmetric background B_{ij} is set equal to zero [13]; see below.) The other constraint comes from the cancellation of the twisted tadpoles for the $D9$-branes. The twisted tadpoles have been computed for certain cases in six dimensions [10,11], and in four dimensions [4,12]. The cases of $\mathbf{Z}_N$ orbifolds (odd prime N) in general D dimensions have been discussed in [3]. Here, we generalize these tadpole cancellation conditions to general Abelian orbifolds with no order two element.

Consider compactification on T^{2d}/G where $G=\mathbf{Z}_{n_1}\otimes\mathbf{Z}_{n_2}\otimes\cdots\otimes\mathbf{Z}_{n_k}$ for odd integers $n_1,n_2,\ldots,n_k$. Let $g_1,g_2,\ldots,g_k$ be the generators of $\mathbf{Z}_{n_1},\mathbf{Z}_{n_2},\ldots,\mathbf{Z}_{n_k}$ respectively. The corresponding twists are given by

$$T_{g_a}=(t^a_1,t^a_2,...,t^a_d||t^a_1,t^a_2,...,t^a_d)\ . \tag{A1}$$

Here t^a_i are fractional numbers taking values in $\{0,1/n_a,2/n_a,...,(n_a-1)/n_a\}$. A given t^a_i corresponds to a twist of the i-th complex boson by a $2\pi t^a_i$ rotation. (We have complexified the $2d$ real bosons into d complex bosons.) The double vertical line separates the right- and left-movers of the string. Because we are considering symmetric orbifold, the right- and left-moving twists are the same. The elements $g(\alpha)$ of the the orbifold group G can be written as $g(\alpha)=g_1^{\alpha_1}g_2^{\alpha_2}\cdots g_k^{\alpha_k}$, where $\alpha_a=0,1,...,n_a-1$. The corresponding twist $T_{g(\alpha)}$ is defined by $t_i(\alpha)=\sum_a\alpha_a t^a_i$ (mod 1), so that $0\leq t_i(\alpha)<1$. We note that the consistency of the orbifold requires that for each element $g(\alpha)$ the expression

$$\prod_{i=1}^{d} 4\sin^2(\pi t_i(\alpha)) \ , \tag{A2}$$

where the factors with $t_i(\alpha) = 0$ are not included in the product, be an integer. In fact the latter is nothing but the number of fixed points (tori) in the $T_{g(\alpha)}$ twisted sector.

The orbifold action on Chan-Paton factors is described by the unitary matrices $\gamma_{g(\alpha)}$ that act on the string end-points. In our case $\gamma_{g(\alpha)}$ is a 16×16 matrix (note that it is *not* a 32×32 matrix because we have chosen *not* to count the orientifold images of the $D9$-branes). Since the orbifold is Abelian, we can simultaneously diagonalize the matrices $\gamma_{g(\alpha)}$ for each $g(\alpha) \in G$. Then, the most general form of $\gamma_{g(\alpha)}$ is given by

$$\gamma_{g(\alpha)} = \mathrm{diag}([\omega(\alpha)]^{\ell_1(\alpha)}, [\omega(\alpha)]^{\ell_2(\alpha)]}, \dots, [\omega(\alpha)]^{\ell_{16}(\alpha)}) \ , \tag{A3}$$

where $\omega(\alpha) \equiv \exp(2\pi i/N(\alpha))$. Here $N(\alpha)$ is the order of $g(\alpha)$, *i.e.*, the smallest integer such that $N(\alpha)t_i(\alpha) \in \mathbf{Z}$ for $i = 1, \dots, d$.

The twisted tadpole cancellation conditions are thus

$$\mathrm{Tr}(\gamma_{g(\alpha)}) = 16p(\alpha) \ , \quad p(\alpha) \equiv \prod_{i=1}^{d} (-1)^{N(\alpha)t_i(\alpha)} \cos(\pi t_i(\alpha)) \tag{A4}$$

for each $g(\alpha) \in G$.

Let us illustrate the above constraint by the $\mathbf{Z}_3 \otimes \mathbf{Z}_3$ orbifold considered in this paper:

$$T_{g_1} = (1/3, 1/3, 0||1/3, 1/3, 0) \ , \tag{A5}$$
$$T_{g_2} = (0, 1/3, 1/3||0, 1/3, 1/3) \ . \tag{A6}$$

The tadpole cancellation conditions read:

$$\begin{aligned} \mathrm{Tr}(\gamma_{g_1}) &= 4 \\ \mathrm{Tr}(\gamma_{g_2}) &= 4 \\ \mathrm{Tr}(\gamma_{g_1 g_2^2}) &= 4 \\ \mathrm{Tr}(\gamma_{g_1 g_2}) &= -2 \ . \end{aligned} \tag{A7}$$

The conditions for all other $g \in G$ can be derived from the ones above. The solutions for γ_{g_1} and γ_{g_2} are then:

$$\begin{aligned} \gamma_{g_1} &= \mathbf{I}_8 \otimes \omega \mathbf{I}_4 \otimes \omega^2 \mathbf{I}_4 \\ \gamma_{g_2} &= \mathbf{I}_4 \otimes \omega \mathbf{I}_2 \otimes \omega^2 \mathbf{I}_2 \otimes \mathbf{I}_2 \otimes \omega \mathbf{I}_2 \otimes \mathbf{I}_2 \otimes \omega^2 \mathbf{I}_2 \end{aligned} \tag{A8}$$

where $\mathbf{I}_n$ denotes the $n \times n$ unit matrix. Thus, the gauge group is $U(4)^3 \otimes SO(8)$.

Here we also list a few non-supersymmetric models (that have never been discussed previously to the best of our knowledge):

• 6D $\mathbf{Z}_3 \otimes \mathbf{Z}_3$ orbifold. The twists read:

$$T_{g_1} = (1/3, 0||1/3, 0) \ , \tag{A9}$$
$$T_{g_2} = (0, 1/3||0, 1/3) \ . \tag{A10}$$

The gauge group is $U(8) \otimes U(8)$.
• 4D $\mathbf{Z}_3 \otimes \mathbf{Z}_3$ orbifold. The twists read:

$$T_{g_1} = (1/3, 0, 0||1/3, 0, 0) \ , \tag{A11}$$
$$T_{g_2} = (0, 1/3, 1/3||0, 1/3, 1/3) \ . \tag{A12}$$

The gauge group is $U(4) \otimes U(4) \otimes U(8)$.
• 4D $\mathbf{Z}_3 \otimes \mathbf{Z}_3 \otimes \mathbf{Z}_3$ orbifold. The twists read:

$$T_{g_1} = (1/3, 0, 0||1/3, 0, 0) \ , \tag{A13}$$
$$T_{g_2} = (0, 1/3, 0||0, 1/3, 0) \ , \tag{A14}$$
$$T_{g_2} = (0, 0, 1/3||0, 0, 1/3) \ . \tag{A15}$$

The gauge group is $U(4)^4$.
• 4D $\mathbf{Z}_3 \otimes \mathbf{Z}_5$ orbifold. The twists read:

$$T_{g_1} = (1/3, 0, 0||1/3, 0, 0) \ , \tag{A16}$$
$$T_{g_2} = (0, 1/5, 2/5||0, 1/5, 2/5) \ . \tag{A17}$$

The gauge group is $U(4)^4$.
• 4D $\mathbf{Z}_9$ orbifold. The twist reads:

$$T_g = (1/9, 2/9, 4/9||1/9, 2/9, 4/9) \ . \tag{A18}$$

The gauge group is $U(4)^4$.

Finally, we would like to consider the cases with non-zero NS-NS antisymmetric background B_{ij}. Although there are no massless scalars corresponding to these in type I theory (recall that there B_{ij} fields are projected out of the spectrum after orientifolding), B_{ij} can have certain quantized values. The quantization is due to the fact that to have a consistent orientifold the corresponding type IIB spectrum must be left-right symmetric. At generic values of B_{ij} this symmetry is destroyed. There are, however, certain discrete B_{ij} backgrounds compatible with the orientifold projection [13]. The effect of non-zero B_{ij} background is that the rank of the gauge group coming from the $SO(32)$ (*i.e.*, Chan-Paton) factor is reduced, depending on the rank r (which is always even) of the matrix B_{ij}. That is, the number of the $D9$-branes required by the tadpole cancellation condition is no longer 16 but $16/2^{r/2}$. All of the above formulas then get modified in the presence of rank r B_{ij} in an obvious way via replacing the factor 16 everywhere by $16/2^{r/2}$.

APPENDIX B: TADPOLES FOR $D9$- AND $D5$-BRANES

In this section we generalize the tadpole cancellation conditions discussed in Appendix A to the cases with both $D9$- and $D5$-branes present. In order to have $D5$-branes, the orbifold group G must contain an order two element R. In general, there may be more than one order two elements R_i in the orbifold group G. Then there will be corresponding $D5_i$-branes for each of these elements. Once the case with only one order two element is understood, the generalization to more general cases is relatively straightforward.

Let us, therefore, concentrate on the case where we have only one order two element $R \in G$. In fact, for now let us take the orbifold group $G = \mathbf{Z}_{2N} \sim \mathbf{Z}_2 \otimes \mathbf{Z}_N$,where N is odd. Note that $R \in \mathbf{Z}_2$. We can write the group elements as $G = \{1, g^1, ..., g^{N-1}, R, Rg^1, ..., Rg^{N-1}\} = \{(Rg)^k, k = 0, ..., 2N-1\}$ (note that $R^2 = 1$). The orientifold group element Ω gives rise to $D9$-branes, whereas the element ΩR gives rise to $D5$-branes. Let us denote the open string Chan-Paton matrices as γ_k for the $D9$-branes, and as $\tilde{\gamma}_k$ for the $D5$-branes. Here $k = 0, ..., 2N-1$, and the matrices with index k even correspond to the g^k twists, whereas those with the index k odd correspond to the Rg^k twists. Note that the untwisted Chan-Paton matrices are given by $\gamma_0 = \tilde{\gamma}_0 = \mathbf{I}_{16}$, where $\mathbf{I}_{16}$ is a 16×16 identity matrix. This latter fact follows from the untwisted tadpole cancellation conditions that require presence of 16 $D9$-branes and 16 $D5$-branes (we are not counting the orientifold images here).

Next, we need to understand the twisted tadpole cancellation conditions. Here we will write the general form of these conditions. The contributions to the twisted tadpoles come from the Klein bottle $\mathcal{K}$, cylinder $\mathcal{C}$ and Möbius strip $\mathcal{M}$ (the total twisted tadpole is the sum $\mathcal{T} = \mathcal{K} + \mathcal{C} + \mathcal{M}$):

$$\mathcal{K} = \sum_{k=1}^{2N-1} \mathcal{K}_k \ , \tag{B1}$$

$$\mathcal{C} = \sum_{k=1}^{2N-1} \left(a_k \mathrm{Tr}(\gamma_k)^2 - 2\mathrm{Tr}(\gamma_k)\mathrm{Tr}(\tilde{\gamma}_k) + \tilde{a}_k \mathrm{Tr}(\tilde{\gamma}_k)^2\right) \ , \tag{B2}$$

$$\mathcal{M} = \sum_{k=1}^{2N-1} \left(b_k \mathrm{Tr}(\gamma^{-1}_{\Omega_k}\gamma^T_{\Omega_k}) + \tilde{b}_k \mathrm{Tr}(\tilde{\gamma}^{-1}_{\Omega_k}\tilde{\gamma}^T_{\Omega_k})\right) \ . \tag{B3}$$

Here $\Omega_k \equiv \Omega(Rg)^k$. Note that in the Klein bottle $\mathcal{K}$ and Möbius strip $\mathcal{M}$ tadpoles we must *not* include the terms with $k = N$.

The Klein bottle contributions $\mathcal{K}_k$, as well as the coefficients $a_k, \tilde{a}_k, b_k, \tilde{b}_k$, are given by certain numerical factors independent of the Chan-Paton matrices. Note that according to the composition algebra for the matrices γ_{Ω_k} and $\tilde{\gamma}_{\Omega_k}$ we have:

$$\mathrm{Tr}(\gamma^{-1}_{\Omega_k}\gamma^T_{\Omega_k}) = \epsilon_k \mathrm{Tr}(\gamma_{2k}) \ , \tag{B4}$$

$$\mathrm{Tr}(\tilde{\gamma}^{-1}_{\Omega_k}\tilde{\gamma}^T_{\Omega_k}) = \tilde{\epsilon}_k \mathrm{Tr}(\tilde{\gamma}_{2k}) \ , \tag{B5}$$

where ϵ_k and $\tilde{\epsilon}_k$ take values ± 1. Next note that since $(Rg)^{k+2N} = (Rg)^k$, we can rewrite the total twisted tadpole $\mathcal{T}$ as a sum of two pieces $\mathcal{T} = \mathcal{T}_{\mathrm{odd}} + \mathcal{T}_{\mathrm{even}}$, where

$$\mathcal{T}_{\mathrm{odd}} = \sum_{k=1}^{N} (a_{2k-1}\mathrm{Tr}(\gamma_{2k-1})^2 - 2\mathrm{Tr}(\gamma_{2k-1})\mathrm{Tr}(\tilde{\gamma}_{2k-1}) + \tilde{a}_{2k-1}\mathrm{Tr}(\tilde{\gamma}_{2k-1})^2) \ , \tag{B6}$$

$$\begin{aligned} \mathcal{T}_{\mathrm{even}} = & \sum_{k=1}^{N-1} (a_{2k}\mathrm{Tr}(\gamma_{2k})^2 - 2\mathrm{Tr}(\gamma_{2k})\mathrm{Tr}(\tilde{\gamma}_{2k}) + \tilde{a}_{2k}\mathrm{Tr}(\tilde{\gamma}_{2k})^2) + \\ & \sum_{k=1}^{N-1} (b^\prime_k \mathrm{Tr}(\gamma_{2k}) + \tilde{b}^\prime_k \mathrm{Tr}(\tilde{\gamma}_{2k})) + \\ & \sum_{k=1}^{N-1} (\mathcal{K}_k + \mathcal{K}_{k+N}) \ . \end{aligned} \tag{B7}$$

Here $b'_k \equiv b_k\epsilon_k + b_{k+N}\epsilon_{k+N}$, and similarly for $\tilde{b}'_k$. From the above expressions it is clear that in order for the twisted tadpoles to cancel it must be the case that they cancel in $\mathcal{T}_{\rm odd}$ and $\mathcal{T}_{\rm even}$ independently. In fact, the tadpoles must factor into sums of perfect squares. Thus, for $\mathcal{T}_{\rm odd}$ we have

$$\mathcal{T}_{\rm odd} = \sum_{k=1}^{N} a_{2k-1}({\rm Tr}(\gamma_{2k-1}) - \tilde{a}_{2k-1}{\rm Tr}(\tilde{\gamma}_{2k-1}))^2 \ , \tag{B8}$$

where we have used the identity $a_k\tilde{a}_k = 1$ which follows from the T-duality of $D9$- and $D5$-branes. The T-duality also implies that we can set (up to an irrelevant overall phase) ${\rm Tr}(\gamma_{2k-1}) = {\rm Tr}(\tilde{\gamma}_{2k-1})$, and as an immediate consequence we get the following twisted tadpole cancellation conditions:

$${\rm Tr}(\gamma_{Rg^k}) = {\rm Tr}(\tilde{\gamma}_{Rg^k}) = 0 \ , \quad k = 0, ..., N-1. \tag{B9}$$

The rest of the twisted tadpole cancellation conditions are for the Chan-Paton matrices ${\rm Tr}(\gamma_{g^k}) = {\rm Tr}(\tilde{\gamma}_{g^k})$, $k = 1, ..., N-1$. These tadpole cancellation conditions are exactly the same as for the $\mathbf{Z}_N$ orbifold with N odd, and we have already discussed them in the previous section.

It is clear now how to construct the Chan-Paton matrices for $\mathbf{Z}_2 \otimes \mathbf{Z}_N$ orbifolds (N is odd). The Chan-Paton matrices for the $D9$- and $D5$-branes are the same. Thus, we will confine our attention to the $D9$-brane matrices. Those for the twists g^k are given in the previous section. The matrix $\gamma_R = {\rm diag}(i \ (8 \ {\rm times}), -i \ (8 \ {\rm times}))$. Also, $\gamma_{Rg^k} = \gamma_R\gamma_{g^k}$. The form of γ_{g^k} is then fixed (up to equivalent representations) by the constraints on ${\rm Tr}(\gamma_{g^k})$ (this fixes the diagonal elements up to certain permutations) and ${\rm Tr}(\gamma_{Rg^k})$ (this fixes the positions of the diagonal elements in γ_{g^k} in the basis where $\gamma_R = {\rm diag}(i \ (8 \ {\rm times}), -i \ (8 \ {\rm times}))$; there is still a residual permutational symmetry, but the latter is irrelevant).

Let us illustrate the above conditions by the $\mathbf{Z}_6$ orbifold considered in this paper. The twists read:

$$T_g = (1/3, 1/3, 1/3||1/3, 1/3, 1/3) \ , \tag{B10}$$
$$T_R = (1/2, 1/2, 0||1/2, 1/2, 0) \ . \tag{B11}$$

We have the following Chan-Paton matrices:

$$\gamma_R = {\rm diag}(i \ (8 \ {\rm times}), -i \ (8 \ {\rm times})) \ , \tag{B12}$$
$$\begin{aligned}\gamma_g = {\rm diag}(&\exp(2\pi i/3) \ (3 \ {\rm times}), \exp(-2\pi i/3) \ (3 \ {\rm times}), 1 \ (2 \ {\rm times}), \\ &\exp(2\pi i/3) \ (3 \ {\rm times}), \exp(-2\pi i/3) \ (3 \ {\rm times}), 1 \ (2 \ {\rm times})) \ ,\end{aligned} \tag{B13}$$
$$\begin{aligned}\gamma_{Rg} = {\rm diag}(&\exp(-\pi i/6) \ (3 \ {\rm times}), \exp(-5\pi i/6) \ (3 \ {\rm times}), i \ (2 \ {\rm times}), \\ &\exp(5\pi i/6) \ (3 \ {\rm times}), \exp(\pi i/6) \ (3 \ {\rm times}), -i \ (2 \ {\rm times})) \ .\end{aligned} \tag{B14}$$

Thus, the gauge group is $U(6) \otimes U(6) \otimes U(4)$ in the 99 sector. It is the same in the 55 sector if all the $D5$-branes are located at the same fixed point.

Note that although so far we have discussed only the case of $\mathbf{Z}_2 \otimes \mathbf{Z}_N$ orbifolds (N is odd), the tadplole cancellation conditions are staightforward to generalize to $\mathbf{Z}_2 \otimes \mathbf{Z}_{n_1} \otimes \mathbf{Z}_{n_2} \otimes \cdots \otimes \mathbf{Z}_{n_k}$ orbifolds, where $n_1, n_2, ..., n_k$ are odd integers. (Here we imply type I orbifolds with $D9$- and $D5$-branes only.) It is also straightforward to consider cases where instead of $\mathbf{Z}_2$ we have $\mathbf{Z}_{2^m}$, such as $\mathbf{Z}_4$, $\mathbf{Z}_8$ and $\mathbf{Z}_{12}$. We will not consider these cases here, however, as they appear to have new non-trivial issues involved which we will discuss elsewhere. (It would also be interesting to generalize the above conditions along the lines of Ref [14].)

TABLES

Sector	Field	$SU(4)\otimes SU(4)\otimes SU(4)\otimes SO(8)\otimes U(1)^3$	Comments
Closed Untwisted	ϕ_a	$3(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(0,0,0)_L$	$a=1,2,3$
Closed Twisted	$S^{1k}_{\alpha\beta}$	$18(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(0,0,0)_L$	$k=1,2,\ \alpha,\beta=1$ to 3
	$S^{2k}_{\beta\gamma}$	$18(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(0,0,0)_L$	$k=1,2,\ \beta,\gamma=1$ to 3
	$S^{3k}_{\alpha\gamma}$	$18(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(0,0,0)_L$	$k=1,2,\ \alpha,\gamma=1$ to 3
	$S_{\alpha\beta\gamma}$	$27(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(0,0,0)_L$	$\alpha,\beta,\gamma=1$ to 3
Open	P_1	$(\overline{\mathbf{4}},\mathbf{1},\mathbf{1},\mathbf{8}_v)(-1,0,0)_L$	
	P_2	$(\mathbf{1},\mathbf{4},\mathbf{1},\mathbf{8}_v)(0,+1,0)_L$	
	P_3	$(\mathbf{1},\mathbf{1},\mathbf{4},\mathbf{8}_v)(0,0,+1)_L$	
	Q_1	$(\mathbf{1},\overline{\mathbf{4}},\overline{\mathbf{4}},\mathbf{1})(0,-1,-1)_L$	
	Q_2	$(\mathbf{4},\mathbf{1},\overline{\mathbf{4}},\mathbf{1})(+1,0,-1)_L$	
	Q_3	$(\mathbf{4},\overline{\mathbf{4}},\mathbf{1},\mathbf{1})(+1,-1,0)_L$	
	Φ_1	$(\mathbf{6},\mathbf{1},\mathbf{1},\mathbf{1})(+2,0,0)_L$	
	Φ_2	$(\mathbf{1},\mathbf{6},\mathbf{1},\mathbf{1})(0,-2,0)_L$	
	Φ_3	$(\mathbf{1},\mathbf{1},\mathbf{6},\mathbf{1})(0,0,-2)_L$	

TABLE I. The massless spectrum of the type I model with $N=1$ space-time supersymmetry and gauge group $SU(4)\otimes SU(4)\otimes SU(4)\otimes SO(8)\otimes U(1)^3$ discussed in section II. The gravity, dilaton and gauge supermultiplets are not shown.

Sector	Field	$SU(4)^3 \otimes SO(8) \otimes U(1)^3$	$(H_1, H_2, H_3)_{-1}$	$(H_1, H_2, H_3)_{-1/2}$
Untwisted	ϕ_1	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(0,0,0)_L$	$(0,+1,0)$	$(+\frac{1}{2},+\frac{1}{2},+\frac{1}{2})$
	ϕ_2	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(0,0,0)_L$	$(-1,0,0)$	$(-\frac{1}{2},-\frac{1}{2},+\frac{1}{2})$
	ϕ_3	$(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(0,0,0)_L$	$(0,0,-1)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
	P_1	$(\overline{\mathbf{4}},\mathbf{1},\mathbf{1},\mathbf{8}_v)(-1,0,0)_L$	$(0,+1,0)$	$(+\frac{1}{2},+\frac{1}{2},+\frac{1}{2})$
	P_2	$(\mathbf{1},\mathbf{4},\mathbf{1},\mathbf{8}_v)(0,+1,0)_L$	$(-1,0,0)$	$(-\frac{1}{2},-\frac{1}{2},+\frac{1}{2})$
	P_3	$(\mathbf{1},\mathbf{1},\mathbf{4},\mathbf{8}_v)(0,0,+1)_L$	$(0,0,-1)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
	Q_1	$(\mathbf{1},\overline{\mathbf{4}},\overline{\mathbf{4}},\mathbf{1})(0,-1,-1)_L$	$(0,+1,0)$	$(+\frac{1}{2},+\frac{1}{2},+\frac{1}{2})$
	Q_2	$(\mathbf{4},\mathbf{1},\overline{\mathbf{4}},\mathbf{1})(+1,0,-1)_L$	$(-1,0,0)$	$(-\frac{1}{2},-\frac{1}{2},+\frac{1}{2})$
	Q_3	$(\mathbf{4},\overline{\mathbf{4}},\mathbf{1},\mathbf{1})(+1,-1,0)_L$	$(0,0,-1)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
	Φ_1	$(\mathbf{6},\mathbf{1},\mathbf{1},\mathbf{1})(+2,0,0)_L$	$(0,+1,0)$	$(+\frac{1}{2},+\frac{1}{2},+\frac{1}{2})$
	Φ_2	$(\mathbf{1},\mathbf{6},\mathbf{1},\mathbf{1})(0,-2,0)_L$	$(-1,0,0)$	$(-\frac{1}{2},-\frac{1}{2},+\frac{1}{2})$
	Φ_3	$(\mathbf{1},\mathbf{1},\mathbf{6},\mathbf{1})(0,0,-2)_L$	$(0,0,-1)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
Twisted T_3, $2T_3$	$S^{1+}_{\alpha\beta}$	$9(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(+4/3,+4/3,0)_L$	$(-\frac{1}{3},+\frac{2}{3},0)$	$(+\frac{1}{6},+\frac{1}{6},+\frac{1}{2})$
	$S^{1-}_{\alpha\beta}$	$9(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(-4/3,-4/3,0)_L$	$(-\frac{2}{3},+\frac{1}{3},0)$	$(-\frac{1}{6},-\frac{1}{6},+\frac{1}{2})$
	$T^{1+}_{\alpha\beta}$	$9(\mathbf{1},\mathbf{6},\mathbf{1},\mathbf{1})(+4/3,-2/3,0)_L$	$(-\frac{1}{3},+\frac{2}{3},0)$	$(+\frac{1}{6},+\frac{1}{6},+\frac{1}{2})$
	$T^{1-}_{\alpha\beta}$	$9(\mathbf{6},\mathbf{1},\mathbf{1},\mathbf{1})(+2/3,-4/3,0)_L$	$(-\frac{2}{3},+\frac{1}{3},0)$	$(-\frac{1}{6},-\frac{1}{6},+\frac{1}{2})$
Twisted T'_3, $2T'_3$	$S^{2+}_{\beta\gamma}$	$9(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(+4/3,0,+4/3)_L$	$(0,+\frac{2}{3},-\frac{1}{3})$	$(+\frac{1}{2},+\frac{1}{6},+\frac{1}{6})$
	$S^{2-}_{\beta\gamma}$	$9(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(-4/3,0,-4/3)_L$	$(0,+\frac{1}{3},-\frac{2}{3})$	$(+\frac{1}{2},-\frac{1}{6},-\frac{1}{6})$
	$T^{2+}_{\beta\gamma}$	$9(\mathbf{1},\mathbf{1},\mathbf{6},\mathbf{1})(+4/3,0,-2/3)_L$	$(0,+\frac{2}{3},-\frac{1}{3})$	$(+\frac{1}{2},+\frac{1}{6},+\frac{1}{6})$
	$T^{2-}_{\beta\gamma}$	$9(\mathbf{6},\mathbf{1},\mathbf{1},\mathbf{1})(2/3,0,-4/3)_L$	$(0,+\frac{1}{3},-\frac{2}{3})$	$(+\frac{1}{2},-\frac{1}{6},-\frac{1}{6})$
Twisted $T_3+2T'_3$, $2T_3+T'_3$	$S^{3+}_{\alpha\gamma}$	$9(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(0,+4/3,-4/3)_L$	$(-\frac{1}{3},0,-\frac{2}{3})$	$(+\frac{1}{6},-\frac{1}{2},-\frac{1}{6})$
	$S^{3-}_{\alpha\gamma}$	$9(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(0,-4/3,+4/3)_L$	$(-\frac{2}{3},0,-\frac{1}{3})$	$(-\frac{1}{6},-\frac{1}{2},+\frac{1}{6})$
	$T^{3+}_{\alpha\gamma}$	$9(\mathbf{1},\mathbf{6},\mathbf{1},\mathbf{1})(0,-2/3,-4/3)_L$	$(-\frac{1}{3},0,-\frac{2}{3})$	$(+\frac{1}{6},-\frac{1}{2},-\frac{1}{6})$
	$T^{3-}_{\alpha\gamma}$	$9(\mathbf{1},\mathbf{1},\mathbf{6},\mathbf{1})(0,-4/3,-2/3)_L$	$(-\frac{2}{3},0,-\frac{1}{3})$	$(-\frac{1}{6},-\frac{1}{2},+\frac{1}{6})$
Twisted $T_3+T'_3$, $2T_3+2T'_3$	$S_{\alpha\beta\gamma}$	$27(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})(-4/3,+4/3,+4/3)_L$	$(-\frac{1}{3},+\frac{1}{3},-\frac{1}{3})$	$(+\frac{1}{6},-\frac{1}{6},+\frac{1}{6})$
	$T_{\alpha\beta\gamma}$	$27(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{8_s})(2/3,-2/3,-2/3)_L$	$(-\frac{1}{3},+\frac{1}{3},-\frac{1}{3})$	$(+\frac{1}{6},-\frac{1}{6},+\frac{1}{6})$

TABLE II. The massless spectrum of the heterotic model with $N = 1$ space-time supersymmetry and gauge group $SU(4) \otimes SU(4) \otimes SU(4) \otimes SO(8) \otimes U(1)^3$ discussed in section III. The H-charges in both the -1 picture and the $-1/2$ picture are also given. The gravity, dilaton and gauge supermultiplets are not shown.

Sector	$[SU(6)\otimes SU(6)\otimes SU(4)\otimes U(1)^3]^2$	$(H_1,H_2,H_3)_{-1}$	$(H_1,H_2,H_3)_{-1/2}$
Closed Untwisted	$5(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})(0,0,0;0,0,0)_L$		
Closed $\mathbf{Z}_3$ Twisted	$15(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})(0,0,0;0,0,0)_L$		
Closed $\mathbf{Z}_6$ Twisted	$3(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})(0,0,0;0,0,0)_L$		
Closed $\mathbf{Z}_2$ Twisted	$11(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})(0,0,0;0,0,0)_L$		
	$2(\mathbf{15},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})(+2,0,0;0,0,0)_L$	$(+1,0,0)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
		$(0,+1,0)$	$(-\frac{1}{2},+\frac{1}{2},-\frac{1}{2})$
	$2(\mathbf{1},\overline{\mathbf{15}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})(0,-2,0;0,0,0)_L$	$(+1,0,0)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
		$(0,+1,0)$	$(-\frac{1}{2},+\frac{1}{2},-\frac{1}{2})$
	$2(\overline{\mathbf{6}},\mathbf{1},\overline{\mathbf{4}};\mathbf{1},\mathbf{1},\mathbf{1})(-1,0,-1;0,0,0)_L$	$(+1,0,0)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
		$(0,+1,0)$	$(-\frac{1}{2},+\frac{1}{2},-\frac{1}{2})$
Open 99	$2(\mathbf{1},\mathbf{6},\mathbf{4};\mathbf{1},\mathbf{1},\mathbf{1})(0,+1,+1;0,0,0)_L$	$(+1,0,0)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
		$(0,+1,0)$	$(-\frac{1}{2},+\frac{1}{2},-\frac{1}{2})$
	$(\mathbf{6},\overline{\mathbf{6}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{1})(+1,-1,0;0,0,0)_L$	$(0,0,+1)$	$(-\frac{1}{2},-\frac{1}{2},+\frac{1}{2})$
	$(\overline{\mathbf{6}},\mathbf{1},\mathbf{4};\mathbf{1},\mathbf{1},\mathbf{1})(-1,0,+1;0,0,0)_L$	$(0,0,+1)$	$(-\frac{1}{2},-\frac{1}{2},+\frac{1}{2})$
	$(\mathbf{1},\mathbf{6},\overline{\mathbf{4}};\mathbf{1},\mathbf{1},\mathbf{1})(0,+1,-1;0,0,0)_L$	$(0,0,+1)$	$(-\frac{1}{2},-\frac{1}{2},+\frac{1}{2})$
	$2(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{15},\mathbf{1},\mathbf{1})(0,0,0;+2,0,0)_L$	$(+1,0,0)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
		$(0,+1,0)$	$(-\frac{1}{2},+\frac{1}{2},-\frac{1}{2})$
	$2(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\overline{\mathbf{15}},\mathbf{1})(0,0,0;0,-2,0)_L$	$(+1,0,0)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
		$(0,+1,0)$	$(-\frac{1}{2},+\frac{1}{2},-\frac{1}{2})$
	$2(\mathbf{1},\mathbf{1},\mathbf{1};\overline{\mathbf{6}},\mathbf{1},\overline{\mathbf{4}})(0,0,0;-1,0,-1)_L$	$(+1,0,0)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
		$(0,+1,0)$	$(-\frac{1}{2},+\frac{1}{2},-\frac{1}{2})$
Open 55	$2(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{6},\mathbf{4})(0,0,0;0,+1,+1)_L$	$(+1,0,0)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
		$(0,+1,0)$	$(-\frac{1}{2},+\frac{1}{2},-\frac{1}{2})$
	$(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{6},\overline{\mathbf{6}},\mathbf{1})(0,0,0;+1,-1,0)_L$	$(0,0,+1)$	$(-\frac{1}{2},-\frac{1}{2},+\frac{1}{2})$
	$(\mathbf{1},\mathbf{1},\mathbf{1};\overline{\mathbf{6}},\mathbf{1},\mathbf{4})(0,0,0;-1,0,+1)_L$	$(0,0,+1)$	$(-\frac{1}{2},-\frac{1}{2},+\frac{1}{2})$
	$(\mathbf{1},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{6},\overline{\mathbf{4}})(0,0,0;0,+1,-1)_L$	$(0,0,+1)$	$(-\frac{1}{2},-\frac{1}{2},+\frac{1}{2})$
	$(\mathbf{6},\mathbf{1},\mathbf{1};\mathbf{6},\mathbf{1},\mathbf{1})(+1,0,0;+1,0,0)_L$	$(+\frac{1}{2},+\frac{1}{2},0)$	$(0,0,-\frac{1}{2})$
	$(\mathbf{1},\mathbf{6},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{4})(0,+1,0;0,0,+1)_L$	$(+\frac{1}{2},+\frac{1}{2},0)$	$(0,0,-\frac{1}{2})$
Open 59	$(\mathbf{1},\mathbf{1},\mathbf{4};\mathbf{1},\mathbf{6},\mathbf{1})(0,0,+1;0,+1,0)_L$	$(+\frac{1}{2},+\frac{1}{2},0)$	$(0,0,-\frac{1}{2})$
	$(\overline{\mathbf{6}},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\overline{\mathbf{4}})(-1,0,0;0,0,-1)_L$	$(+\frac{1}{2},+\frac{1}{2},0)$	$(0,0,-\frac{1}{2})$
	$(\mathbf{1},\overline{\mathbf{6}},\mathbf{1};\mathbf{1},\overline{\mathbf{6}},\mathbf{1})(0,-1,0;0,-1,0)_L$	$(+\frac{1}{2},+\frac{1}{2},0)$	$(0,0,-\frac{1}{2})$
	$(\mathbf{1},\mathbf{1},\overline{\mathbf{4}};\overline{\mathbf{6}},\mathbf{1},\mathbf{1})(0,0,-1;-1,0,0)_L$	$(+\frac{1}{2},+\frac{1}{2},0)$	$(0,0,-\frac{1}{2})$

TABLE III. The massless spectrum of the type I $\mathbf{Z}_6$ orbifold model with $N=1$ space-time supersymmetry and gauge group $[SU(6)\otimes SU(6)\otimes SU(4)\otimes U(1)^3]^2$ discussed in section VI. The H-charges in both the -1 picture and the $-1/2$ picture for states in the open string sector are also given. The gravity, dilaton and gauge supermultiplets are not shown.

Sector	$SU(6)\otimes SU(6)\otimes SU(4)\otimes U(1)^3$	$(H_1,H_2,H_3)_{-1}$	$(H_1,H_2,H_3)_{-1/2}$
Untwisted	$2(\mathbf{1},\mathbf{1},\mathbf{1})(0,0,0)_L$	$(+1,0,0)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
	$2(\mathbf{1},\mathbf{1},\mathbf{1})(0,0,0)_L$	$(0,+1,0)$	$(-\frac{1}{2},+\frac{1}{2},-\frac{1}{2})$
	$1(\mathbf{1},\mathbf{1},\mathbf{1})(0,0,0)_L$	$(0,0,+1)$	$(-\frac{1}{2},-\frac{1}{2},+\frac{1}{2})$
	$2(\mathbf{15},\mathbf{1},\mathbf{1})(+2,0,0)_L$	$(+1,0,0)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
		$(0,+1,0)$	$(-\frac{1}{2},+\frac{1}{2},-\frac{1}{2})$
	$2(\mathbf{1},\overline{\mathbf{15}},\mathbf{1})(0,-2,0)_L$	$(+1,0,0)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
		$(0,+1,0)$	$(-\frac{1}{2},+\frac{1}{2},-\frac{1}{2})$
	$2(\overline{\mathbf{6}},\mathbf{1},\overline{\mathbf{4}})(-1,0,-1)_L$	$(+1,0,0)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
		$(0,+1,0)$	$(-\frac{1}{2},+\frac{1}{2},-\frac{1}{2})$
	$2(\mathbf{1},\mathbf{6},\mathbf{4})(0,+1,+1)_L$	$(+1,0,0)$	$(+\frac{1}{2},-\frac{1}{2},-\frac{1}{2})$
		$(0,+1,0)$	$(-\frac{1}{2},+\frac{1}{2},-\frac{1}{2})$
	$(\mathbf{6},\overline{\mathbf{6}},\mathbf{1})(+1,-1,0)_L$	$(0,0,+1)$	$(-\frac{1}{2},-\frac{1}{2},+\frac{1}{2})$
	$(\overline{\mathbf{6}},\mathbf{1},\mathbf{4})(-1,0,+1)_L$	$(0,0,+1)$	$(-\frac{1}{2},-\frac{1}{2},+\frac{1}{2})$
	$(\mathbf{1},\mathbf{6},\overline{\mathbf{4}})(0,+1,-1)_L$	$(0,0,+1)$	$(-\frac{1}{2},-\frac{1}{2},+\frac{1}{2})$
T_3	$15(\mathbf{1},\mathbf{1},\mathbf{1})(-2,+2,0)_L$	$(+\frac{1}{3},+\frac{1}{3},+\frac{1}{3})$	$(-\frac{1}{6},-\frac{1}{6},-\frac{1}{6})$
	$12(\mathbf{1},\mathbf{1},\mathbf{6})(+1,-1,0)_L$	$(+\frac{1}{3},+\frac{1}{3},+\frac{1}{3})$	$(-\frac{1}{6},-\frac{1}{6},-\frac{1}{6})$
	$15(\mathbf{1},\mathbf{1},\mathbf{1})(+1,-1,\pm 2)_L$	$(+\frac{1}{3},+\frac{1}{3},+\frac{1}{3})$	$(-\frac{1}{6},-\frac{1}{6},-\frac{1}{6})$
T_6	$3(\mathbf{6},\mathbf{1},\mathbf{1})(-3/2,-1/2,+1)_L$	$(+\frac{1}{6},+\frac{1}{6},+\frac{2}{3})$	$(-\frac{1}{3},-\frac{1}{3},+\frac{1}{6})$
	$3(\mathbf{1},\overline{\mathbf{6}},\mathbf{1})(+1/2,+3/2,-1)_L$	$(+\frac{1}{6},+\frac{1}{6},+\frac{2}{3})$	$(-\frac{1}{3},-\frac{1}{3},+\frac{1}{6})$
T_2	$5(\mathbf{1},\mathbf{1},\overline{\mathbf{4}})(+3/2,+3/2,0)_L$	$(+\frac{1}{2},+\frac{1}{2},0)$	$(0,0,-\frac{1}{2})$
	$5(\mathbf{1},\mathbf{1},\mathbf{4})(-3/2,-3/2,0)_L$	$(+\frac{1}{2},+\frac{1}{2},0)$	$(0,0,-\frac{1}{2})$
	$6(\overline{\mathbf{6}},\mathbf{1},\mathbf{1})(+1/2,+3/2,+1)_L$	$(+\frac{1}{2},+\frac{1}{2},0)$	$(0,0,-\frac{1}{2})$
	$5(\mathbf{6},\mathbf{1},\mathbf{1})(-1/2,-3/2,-1)_L$	$(+\frac{1}{2},+\frac{1}{2},0)$	$(0,0,-\frac{1}{2})$
	$5(\mathbf{1},\overline{\mathbf{6}},\mathbf{1})(+3/2,+1/2,+1)_L$	$(+\frac{1}{2},+\frac{1}{2},0)$	$(0,0,-\frac{1}{2})$
	$6(\mathbf{1},\mathbf{6},\mathbf{1})(-3/2,-1/2,-1)_L$	$(+\frac{1}{2},+\frac{1}{2},0)$	$(0,0,-\frac{1}{2})$

TABLE IV. The massless spectrum of the heterotic $\mathbf{Z}_6$ orbifold model with $N=1$ space-time supersymmetry and gauge group $SU(6)\otimes SU(6)\otimes SU(4)\otimes U(1)^3$ discussed in section VII. The H-charges in both the -1 picture and the $-1/2$ picture are also given. The gravity, dilaton and gauge supermultiplets are not shown.

FIGURES

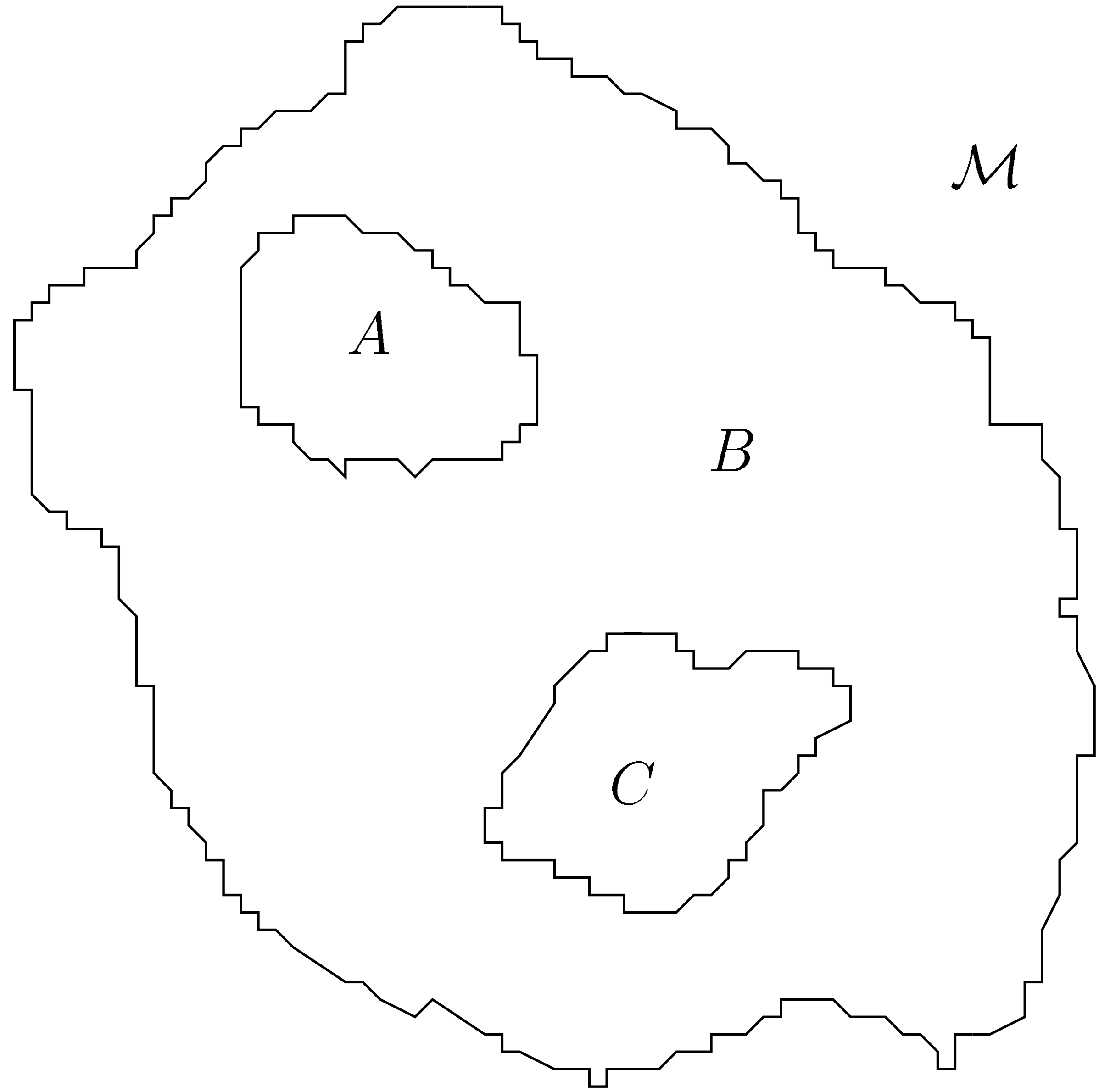

FIG. 1. A schematic picture of the (perturbative) moduli space $\mathcal{M}$ of the heterotic $\mathbf{Z}_3 \otimes \mathbf{Z}_3$ orbifold. This figure is taken from Ref [2] where the $\mathbf{Z}_3$ orbifold is discussed since the schematic picture of the moduli space for both $\mathbf{Z}_3$ and $\mathbf{Z}_3 \otimes \mathbf{Z}_3$ orbifolds are the same. Region A is the subspace corresponding to the type I model. Region C is the subspace where some or all of the $S_{\alpha\beta\gamma}$ vevs are zero and some or all of the $T_{\alpha\beta\gamma}$ and $T^{a\pm}_{\alpha\beta}$ fields are massless. Region B complements A and C in $\mathcal{M}$.

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