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Form-factors of exponential fields in the sine-Gordon model

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Abstract

An integral representation for form-factors of exponential fields $e^{ia\varphi}$ in the sine-Gordon model is proposed.

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In this letter we study the the sine-Gordon model defined by the Euclidean action

$$\mathcal{A}_{SG} = \int d^2x \left\{ \frac{1}{16\pi} \left(\partial_\nu \varphi\right)^2 - 2\mu \cos\left(\beta\varphi\right) \right\} . \tag{1}$$

Recently [1], [2], expectation values for exponential fields in this Quantum Field Theory (QFT) were proposed,

$$\begin{aligned} \langle\, e^{ia\varphi} \,\rangle \equiv \mathcal{G}_a = & \left[\frac{M\sqrt{\pi}\,\Gamma\left(\frac{1}{2-2\beta^2}\right)}{2\,\Gamma\left(\frac{\beta^2}{2-2\beta^2}\right)} \right]^{2a^2} \times \\ & \exp\left\{ \int_0^\infty \frac{dt}{t} \left[\frac{\sinh^2(2a\beta t)}{2\sinh(\beta^2 t)\,\sinh(t)\,\cosh\left((1-\beta^2)t\right)} - 2a^2\, e^{-2t} \right] \right\} , \end{aligned} \tag{2}$$

where M is the soliton mass. In writing (2) it was assumed that the exponential field is normalized in accordance with the short distance limiting form of the two-point function

$$\langle\, e^{ia\varphi}(x)\, e^{-ia\varphi}(y)\,\rangle \to |x-y|^{-2a^2} \qquad \text{as} \qquad |x-y| \to 0\ , \tag{3}$$

so that the field $e^{ia\varphi}(x)$ has the dimension $[\, mass\,]^{2a^2}$. The result (2) is expected to hold in the domain

$$\beta^2 < 1\ , \tag{4}$$

where the discrete symmetry $\varphi \to \varphi + 2\pi n \beta^{-1}$ $(n = \pm 1, \pm 2 ...)$ of (1) is spontaneously broken (and the QFT is massive) and $\langle ... \rangle$ in (2) means the expectation value over one of the ground states $|0\,\rangle$ in which the field $\varphi(x)$ is localized near 0. The expectation value (2) controls both short and long distance asymptotics of the two-point correlation function

$$\mathcal{G}_{aa'}\left(|x-y|\right) = \langle\, e^{ia\varphi}(x)\, e^{ia'\varphi}(y)\,\rangle \tag{5}$$

with $|a+a'| < \beta/2$. Indeed, if this inequality is satisfied the short distance limit of (5) is dominated by OPE

$$e^{ia\varphi}(x)\, e^{ia'\varphi}(y) \to \left|x-y\right|^{4aa'}\ e^{i(a+a')\varphi}(y) \qquad \text{as} \qquad |x-y| \to 0\ . \tag{6}$$

Therefore

$$\mathcal{G}_{aa'}(r) \quad \to \quad \left\{ \begin{array}{lll} \mathcal{G}_{a+a'}\ r^{4aa'} & \text{as} & r \to 0 \\ \mathcal{G}_a\ \mathcal{G}_{a'} & \text{as} & r \to \infty \end{array} \right. . \tag{7}$$

A systematic technique for analysis of the short distance expansion of (5) is the Conformal Perturbation Theory [3]. At the same time the most efficient way to study the large

r behavior is provided by the form-factor approach [4], [5], [6], [7], [8]. Up to now, the whole form-factor sets (for general values of β^2 from (4)) have been obtained for fields φ, $e^{\pm i\frac{\beta\varphi}{2}}$ and $e^{\pm i\beta\varphi}$ only [4], [6]. In this this paper we present form-factors of the exponential operators for arbitrary values of the parameters $0<\beta^2<1$ and a [1].

The technique, providing an integral representation for form-factors in the sine-Gordon model, was developed in [9][2]. For the exponential operators it is suggested that the matrix elements

$$\langle\, 0\,|\, e^{ia\varphi}\,|\, A_{\sigma_{2n}}(\theta_{2n})...A_{\sigma_1}(\theta_1)\,\rangle\ ,\quad \sigma_k=\pm\quad \Big(\sum_{k=1}^{2n}\sigma_k=0\Big)\ ,$$

where $|\, A_{\sigma_{2n}}(\theta_{2n})...A_{\sigma_1}(\theta_1)\,\rangle$ are multi-soliton states [3], are given by the following bosonization procedure

$$\langle\, 0\,|\, e^{ia\varphi}\,|\, A_{\sigma_{2n}}(\theta_{2n})...A_{\sigma_1}(\theta_1)\,\rangle=\mathcal{G}_a\ \langle\langle\, Z_{\sigma_{2n}}(\theta_{2n})...Z_{\sigma_1}(\theta_1)\,\rangle\rangle\ , \tag{8}$$

with

$$\begin{aligned} Z_+(\theta)&=\sqrt{i\frac{\mathcal{C}_2}{4\mathcal{C}_1}}\ e^{\frac{a\theta}{\beta}}\ e^{i\phi(\theta)}\ ,\\ Z_-(\theta)&=\sqrt{i\frac{\mathcal{C}_2}{4\mathcal{C}_1}}\ e^{-\frac{a\theta}{\beta}}\ \Big\{ e^{\frac{i\pi}{2\beta^2}}\int_{C_+}\frac{d\gamma}{2\pi}\ e^{(1-\frac{2a}{\beta}-\frac{1}{\beta^2})(\gamma-\theta)}\ e^{-i\bar\phi(\gamma)}\, e^{i\phi(\theta)}\ -\\ &\qquad\qquad e^{-\frac{i\pi}{2\beta^2}}\int_{C_-}\frac{d\gamma}{2\pi}\ e^{(1-\frac{2a}{\beta}-\frac{1}{\beta^2})(\gamma-\theta)}\ e^{i\phi(\theta)}\, e^{-i\bar\phi(\gamma)}\Big\}\ . \end{aligned} \tag{9}$$

The averaging $\langle\langle\,...\,\rangle\rangle$ should be performed by Wick's theorem using the prescriptions

$$\begin{aligned} &\langle\langle\, e^{i\phi(\theta_2)}e^{i\phi(\theta_1)}\,\rangle\rangle=G(\theta_1-\theta_2)\ ,\\ &\langle\langle\, e^{i\phi(\theta_2)}e^{-i\bar\phi(\theta_1)}\,\rangle\rangle=W(\theta_1-\theta_2)=\frac{1}{G\big(\theta_1-\theta_2-i\frac{\pi}{2}\big)G\big(\theta_1-\theta_2+i\frac{\pi}{2}\big)}\ ,\\ &\langle\langle\, e^{-i\bar\phi(\theta_2)}e^{-i\bar\phi(\theta_1)}\,\rangle\rangle=\bar G(\theta_1-\theta_2)=\frac{1}{W\big(\theta_1-\theta_2-i\frac{\pi}{2}\big)W\big(\theta_1-\theta_2+i\frac{\pi}{2}\big)}\ . \end{aligned} \tag{10}$$

The functions and the constants here read explicitly

$$\begin{aligned} G(\theta)&=i\,\mathcal{C}_1\,\sinh\big(\frac{\theta}{2}\big)\,\exp\Big\{\int_0^\infty\frac{dt}{t}\ \frac{\sinh^2 t\big(1-\frac{i\theta}{\pi}\big)\,\sinh(t(\xi-1))}{\sinh(2t)\,\cosh(t)\,\sinh(t\xi)}\Big\}\ ,\\ W(\theta)&=-\frac{2}{\cosh(\theta)}\,\exp\Big\{-2\int_0^\infty\frac{dt}{t}\ \frac{\sinh^2 t\big(1-\frac{i\theta}{\pi}\big)\,\sinh(t(\xi-1))}{\sinh(2t)\,\sinh(t\xi)}\Big\}\ ,\\ \bar G(\theta)&=-\frac{\mathcal{C}_2}{4}\ \xi\,\sinh\big(\frac{\theta+i\pi}{\xi}\big)\,\sinh(\theta)\ , \end{aligned} \tag{11}$$

---

[1] The restriction on a is not completely clear at the moment.

[2] See also works [10], [11] on an interesting algebraic interpretation of this procedure.

[3] Our convention on the normalization of the states is $\langle\, A^{\sigma}(\theta)\,|\, A_{\sigma'}(\theta')\,\rangle=2\pi\,\delta^{\sigma}_{\sigma'}\,\delta(\theta-\theta')$.

$$
\begin{aligned}
\mathcal{C}_1 &= \exp\Big\{ -\int_0^\infty \frac{dt}{t}\ \frac{\sinh^2\left(\frac{t}{2}\right)\, \sinh(t(\xi-1))}{\sinh(2t)\, \cosh(t)\, \sinh(t\xi)}\ \Big\} = G(-i\pi)\ , \\
\mathcal{C}_2 &= \exp\Big\{ 4\int_0^\infty \frac{dt}{t}\ \frac{\sinh^2\left(\frac{t}{2}\right)\, \sinh(t(\xi-1))}{\sinh(2t)\, \sinh(t\xi)}\ \Big\} = \frac{4}{\left[W\left(i\frac{\pi}{2}\right)\xi \sin\left(\frac{\pi}{\xi}\right)\right]^2}\ ,
\end{aligned}
\tag{12}
$$

where

$$
\xi = \frac{\beta^2}{1-\beta^2}\ .
$$

In (9) we suggest that the integration contour $C_+(C_-)$ goes from $\gamma=-\infty$ to $\gamma=+\infty$ and the pole $\gamma=\theta+i\frac{\pi}{2}$ $(\gamma=\theta-i\frac{\pi}{2})$ lies below (above) the contour.

The integrals in (9) are worked out explicitly for the free fermion point $\beta^2=\frac{1}{2}$. In this case

$$
\begin{aligned}
Z_+(\theta) &= 2^{-1}\, e^{\frac{i\pi}{4}}\ e^{\sqrt{2}a\theta}\ e^{i\phi(\theta)}\ , \\
Z_-(\theta) &= -e^{-\frac{i\pi}{4}}\ e^{-\sqrt{2}a\theta}\ \Big\{ e^{-i\sqrt{2}\pi a} e^{-i\phi(\theta+i\pi)} + e^{i\sqrt{2}\pi a}\ e^{-i\phi(\theta-i\pi)} \Big\}
\end{aligned}
\tag{13}
$$

and

$$
\langle\langle\, e^{i\phi(\theta_2)} e^{i\phi(\theta_1)}\, \rangle\rangle = i \sinh\left(\frac{\theta_1-\theta_2}{2}\right)\ . \tag{14}
$$

With the formulas (13), (14) at hands it is easy to get the following simple expression

$$
\begin{aligned}
&\langle\, 0\, |\, e^{ia\varphi}\, |\, A_+(\theta_{2n})...A_+(\theta_{n+1})A_-(\theta_n)...A_-(\theta_1)\, \rangle = \mathcal{G}_a\ (-1)^{\frac{n(n+1)}{2}}\ \left(i\sin\left(\sqrt{2}\pi a\right)\right)^n \times \\
&\qquad e^{\sqrt{2}a\sum_{k=1}^n(\theta_{n+k}-\theta_k)}\ \frac{\prod_{1\le k<j\le n}\sinh\left(\frac{\theta_k-\theta_j}{2}\right)\ \sinh\left(\frac{\theta_{n+k}-\theta_{n+j}}{2}\right)}{\prod_{1\le k,j\le n}\cosh\left(\frac{\theta_{n+k}-\theta_j}{2}\right)}\ ,
\end{aligned}
\tag{15}
$$

which is in agreement with the results of [12], [13].

It is instructive to consider two-point form-factors more closely. From (8) we have the integral representation

$$
\langle\, 0\, |\, e^{ia\varphi}\, |\, A_\pm(\theta_2)A_\mp(\theta_1)\, \rangle = \mathcal{G}_a\ F^a_{\mp\pm}(\theta_1-\theta_2)\ , \tag{16}
$$

with

$$
\begin{aligned}
F^a_{-+}(\theta) &= -\frac{G(\theta)}{G(-i\pi)}\ e^{\frac{\theta+i\pi}{2\xi}}\ \Big\{ e^{\frac{2i\pi a}{\beta}}\ I_a(\theta) + I_a(-2i\pi-\theta) \Big\}\ , \\
F^a_{+-}(\theta) &= -\frac{G(\theta)}{G(-i\pi)}\ e^{-\frac{\theta+i\pi}{2\xi}}\ \Big\{ e^{-\frac{2i\pi a}{\beta}}\ I_a(\theta) + I_a(-2i\pi-\theta) \Big\}\ .
\end{aligned}
\tag{17}
$$

Here the argument of $I_a(-2i\pi-\theta)$ means the corresponding analytical continuation of the function $I_a(\theta)$. The latter is specified for real θ and

$$-\beta^{-1}+\frac{\beta}{2}<\Re e\, a<\frac{\beta}{2}$$

by the integral

$$I_a(\theta)=\frac{1}{\Big[W\big(i\frac{\pi}{2}\big)\xi\sin\big(\frac{\pi}{\xi}\big)\Big]^2}\int_{-\infty}^{\infty}\frac{d\gamma}{2\pi}\,W\big(\gamma+\frac{\theta}{2}-i\pi\big)W\big(\gamma-\frac{\theta}{2}-i\pi\big)\,e^{(1-\frac{2a}{\beta}-\frac{1}{\beta^2})\gamma}\,. \tag{18}$$

Eqs. (17) were checked against the known form-factors [4], [6]. In particular, if $a\to 0$

$$F^{a}_{\mp\pm}(\theta)=\pm ia\;F(\theta)+O\big(a^2\big)\,, \tag{19}$$

and $F(\theta)$ coincides with the two-particle form-factor of the sine-Gordon field φ. It can be evaluated by the method discussed in the Appendix 4 of [9] with the result,

$$F(\theta)=-\frac{G(\theta)}{G(-i\pi)}\;\frac{\pi}{\beta\cosh\big(\frac{\theta+i\pi}{2\xi}\big)\,\cosh\big(\frac{\theta}{2}\big)}\;. \tag{20}$$

Similarly, one can find

$$F^{\frac{\beta}{2}}_{\mp\pm}(\theta)=\frac{G(\theta)}{G(-i\pi)}\;\frac{2i\;e^{\mp\frac{\theta+i\pi}{2\xi}}}{\xi\sinh\big(\frac{\theta+i\pi}{\xi}\big)} \tag{21}$$

and

$$F^{\beta}_{\mp\pm}(\theta)=\frac{G(\theta)}{G(-i\pi)}\;\cot\big(\frac{\pi\xi}{2}\big)\;\frac{4i\cosh\big(\frac{\theta}{2}\big)\;e^{\mp\frac{\theta+i\pi}{2\xi}}}{\xi\sinh\big(\frac{\theta+i\pi}{\xi}\big)}\;. \tag{22}$$

Eqs. (20), (21), (22) agree with known expressions from [4], [6]. Notice that the functions (17) admit simpler forms for any integer and half-integer values of $\frac{a}{\beta}$. Under this condition, one can show that

$$\begin{aligned}F^{l\beta}_{\mp\pm}(\theta)=\frac{G(\theta)}{G(-i\pi)}\frac{4i\;e^{\mp\frac{\theta+i\pi}{2\xi}}}{\xi\sinh\big(\frac{\theta+i\pi}{\xi}\big)}\;\sum_{m=1}^{l}\,(-1)^{l-m}\cosh\big((m-\frac{1}{2})\theta\big)\times\\ \prod_{k=m-l,k\neq 0}^{m+l-1}\cot\big(\frac{\pi\xi k}{2}\big)\;,\end{aligned} \tag{23}$$

$$\begin{aligned}F^{(l+\frac{1}{2})\beta}_{\mp\pm}(\theta)=&\frac{G(\theta)}{G(-i\pi)}\frac{2i\;e^{\mp\frac{\theta+i\pi}{2\xi}}}{\xi\sinh\big(\frac{\theta+i\pi}{\xi}\big)}\;\Big\{\prod_{k=1}^{l}\cot^2\big(\frac{\pi\xi k}{2}\big)+\\ &2\sum_{m=1}^{l}\,(-1)^{l-m}\cosh(m\theta)\prod_{k=m-l,k\neq 0}^{m+l}\cot\big(\frac{\pi\xi k}{2}\big)\Big\}\;,\end{aligned} \tag{24}$$

for $l = 1, 2, \ldots$ and

$$F^{-l\beta}_{\mp\pm}(\theta) = F^{l\beta}_{\pm\mp}(\theta) \ ,$$
$$F^{-(l+\frac{1}{2})\beta}_{\mp\pm}(\theta) = F^{(l+\frac{1}{2})\beta}_{\pm\mp}(\theta) \ .$$

Significant simplifications also appear at the so-called reflectionless points,

$$\beta^2 = \frac{1}{n+1} \ , \quad n = 1, 2, \ldots \ ,$$

for generic a. Then,

$$F^{a}_{\mp\pm}(\theta) = \pm \frac{G(\theta)}{G(-i\pi)} \frac{2i\, n\, e^{\mp\frac{n(\theta+i\pi)}{2}}}{\sinh\big(n(\theta+i\pi)\big)} \sum_{p=1}^{n} e^{\pm\theta(p-\frac{1}{2}-\frac{a}{\beta})} \prod_{k=p-n}^{p-1} \frac{\sin\big(\frac{\pi}{n}(\frac{a}{\beta}-\frac{k}{2})\big)\ \cos\big(\frac{\pi k}{2n}\big)}{\cos^2\big(\frac{\pi(p-k-1)}{2n}\big)} \ . \tag{25}$$

If $0 < \beta^2 < \frac{1}{2}$, the spectrum of the model (1) contains a number of soliton-antisoliton bound states ("breathers") B_k, $k = 1, 2, \ldots, < 1/\xi$. The lightest of these bound states B_1 coincides with the particle associated with the field φ in perturbative treatment of the QFT (1). An existence of the B_1 bound state means that the operators (13) satisfy the requirement [14], [10]

$$Z_+(\theta_2) Z_-(\theta_1) \to \frac{i\,\Gamma^1_{+-}\ \Lambda\big(\theta_1 + i\frac{\pi}{2}(1-\xi)\big)}{\theta_2 - \theta_1 - i\pi(1-\xi)} \qquad \text{as} \quad \theta_2 - \theta_1 - i\pi(1-\xi) \to 0 \ , \tag{26}$$

where $\Gamma^1_{+-} = \sqrt{2\cot\big(\frac{\pi\xi}{2}\big)}$ [4] and [5]

$$\Lambda(\theta) = \frac{\lambda}{2\sin(\pi\xi)} \Big\{ \ e^{-i\frac{a\pi\xi}{\beta}}\ e^{-i\omega(\theta+i\frac{\pi}{2})} - e^{i\frac{a\pi\xi}{\beta}}\ e^{i\omega(\theta-i\frac{\pi}{2})} \ \Big\} \ , \tag{27}$$

with

$$\omega(\theta) = \phi\big(\theta + i\frac{\pi\xi}{2}\big) - \phi\big(\theta - i\frac{\pi\xi}{2}\big) \ ,$$
$$\lambda = 2\cos\frac{\pi\xi}{2}\sqrt{2\sin\frac{\pi\xi}{2}}\ \exp\Big\{ -\int_0^{\pi\xi} \frac{dt}{2\pi}\ \frac{t}{\sin(t)} \ \Big\} \ .$$

Clearly B_1 form-factors of the exponential field admit the representation

$$\langle\, 0\,|\, e^{ia\varphi}\,|\, B_1(\theta_n) \ldots B_1(\theta_1)\,\rangle = \mathcal{G}_a\ \langle\langle\, \Lambda(\theta_n)\ldots\Lambda(\theta_1)\,\rangle\rangle \ . \tag{28}$$

---

[4] The soliton-antisoliton amplitude [15] contains a simple pole in the physical strip corresponding to the B_1 particle with the residue $S^{+-}_{+-}(\theta) \to \frac{i\ (\Gamma^1_{+-})^2}{\theta - i\pi(1-\xi)}$, $\ \theta - i\pi(1-\xi) \to 0$.

[5] The operator $\Lambda(\theta)$ is closely related with a current of the "deformed Virasoro algebra" [16], [17], [18].

The averaging $\langle\langle\, ...\,\rangle\rangle$ here is performed by Wick's theorem and

$$\begin{aligned} &\langle\langle\, e^{i\omega(\theta)}\,\rangle\rangle = 1\ , \\ &\langle\langle\, e^{i\omega(\theta_2)} e^{i\omega(\theta_1)}\,\rangle\rangle = R(\theta_1-\theta_2)\ , \end{aligned} \tag{29}$$

where the function $R(\theta)$ for $-2\pi+\pi\xi < \Im m\,\theta < -\pi\xi$ is given by the integral

$$R(\theta) = \mathcal{N}\ \exp\Big\{ 8\int_0^\infty \frac{dt}{t}\ \frac{\sinh(t)\,\sinh\big(t\xi\big)\,\sinh\big(t(1+\xi\big)}{\sinh^2(2t)}\ \sinh^2 t\big(1-\frac{i\theta}{\pi}\big)\Big\}\ , \tag{30}$$

$$\mathcal{N} = \exp\Big\{ 4\int_0^\infty \frac{dt}{t}\ \frac{\sinh(t)\,\sinh(t\xi)\,\sinh\big(t(1+\xi)\big)}{\sinh^2(2t)}\ \Big\}\ . \tag{31}$$

Notice the useful relations

$$R(\theta)R(\theta\pm i\pi) = \frac{\sinh(\theta)}{\sinh(\theta)\mp i\sin(\pi\xi)}\ . \tag{32}$$

Using (28)-(32), one can easily derive the first B_1 form-factors of the exponential fields

$$\begin{aligned} &\langle\, 0\,|\, e^{ia\varphi}\,|\, B_1(\theta)\,\rangle = -i\,\mathcal{G}_a\ \lambda\,[a]\ , \\ &\langle\, 0\,|\, e^{ia\varphi}\,|\, B_1(\theta_2)B_1(\theta_1)\,\rangle = -\mathcal{G}_a\ \lambda^2\,[a]^2\ R(\theta_1-\theta_2)\ , \\ &\langle\, 0\,|\, e^{ia\varphi}\,|\, B_1(\theta_3)B_1(\theta_2)B_1(\theta_1)\,\rangle = i\,\mathcal{G}_a\ \lambda^3\,[a] \prod_{1\le k<j\le 3} R(\theta_k-\theta_j)\times \\ &\qquad\qquad \Big\{ [a]^2 + \frac{x_1x_2x_3}{(x_1+x_2)(x_2+x_3)(x_1+x_3)} \Big\}\ , \\ &\langle\, 0\,|\, e^{ia\varphi}\,|\, B_1(\theta_4)B_1(\theta_3)B_1(\theta_2)B_1(\theta_1)\,\rangle = \mathcal{G}_a\ \lambda^4\,[a]^2 \prod_{1\le k<j\le 4} R(\theta_k-\theta_j)\times \\ &\Big\{ [a]^2 + \frac{(x_1+x_2+x_3+x_4)^2 x_1x_2x_3x_4 + (x_1x_2x_3+x_1x_2x_4+x_1x_3x_4+x_2x_3x_4)^2}{(x_1+x_2)(x_1+x_3)(x_1+x_4)(x_2+x_3)(x_2+x_4)(x_3+x_4)} \Big\}\ , \end{aligned} \tag{33}$$

here $x_k = e^{\theta_k}$ $(k=1,2,3,4)$ and $[a] = \frac{\sin(\frac{\pi\xi a}{\beta})}{\sin(\pi\xi)}$. The B_1 breather form-factors admit (for fixed a) a power series expansion in β with finite radius of convergence. Therefore it is natural to assume that the form-factors of exponential fields in the sinh-Gordon model can be obtained from the expression in (28) by the continuation $\beta\to ib$. A simple check shows that the analytical continuation of (33) leads to the form-factors of the field $e^{a\varphi}$ in the sinh-Gordon model proposed in [19].

Finally, we note that form-factors of the heavier breathers follow from (28) by the well known bootstrap procedure [15], [6], [8].

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