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Possibilities of Experimental Proof of the Logical-Linguistic Origins of Quantum Mechanics

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Abstract

Two types of experiment are proposed to establish whether quantum indeterminism and interference of probabilities are of a logical-linguistic, not a physical, nature.

Introduction.

Analysis of the conceptual structure of quantum mechanics [1] reveals the following components:

1. A set of mutually exclusive classical logical statements about possible observables. These statements are represented by truth operators (density matrices of pure states) whose eigenvectors form a basis for a corresponding Hilbert vector space.

2. A symmetry group of possible linear transformations of truth operators—redefinitions of logical truth values—leading to the formation of that Hilbert space. And,

3. Concrete physical-geometrical symmetries (whose invariants are physical observables) that provide a physical meaning for those logical statements and their truth values, and for redefinitions of truth values.

All specifically quantum features of quantum mechanics—quantum uncertainty, indeterminism, and interference of probabilities—arise from the combination of logical-linguistic components (1) and (2). Physics, with its Planck constant, results from the inclusion of (3). In other words, qualitatively, the "quantum" in quantum mechanics is entirely of a logical-linguistic, and not a physical, nature. (This is reflected in the fact that commutators of quantum density matrices are not proportional to any physical constants.)

Taken literally, this qualitative result leads to the conclusion that logic and language *per se,* even when they do not describe quantum systems, can possess some quantum-like properties. The purpose of this paper is to formulate schematically logical-linguistic experiments that can prove (or disprove) such a radical conclusion. Proof will have a number of implications (discussed briefly at the end of this article) for physics, theory of mind, and the social sciences.

But before presenting actual experiments, we analyze a simple gedanken experiment in which a purely logical and linguistic situation unconnected with any physics leads to quantum-like outcomes that can be interpreted as uncertainty, indeterminism, and interference of probabilities. In parallel with this logical situation, we give an analogous quantum-mechanical one involving two projections of $\frac{1}{2}$ spin, S_z and S_x.

1. The gedanken experiment.

The experiment is a game in which we have a context-dependent *indivisible "quantum,"* p, (e.g., a prize, grant, or portion of a budget) that must be assigned to one and only one of four alternative projects, $P_j, j = 1,2,3,4$. These four projects fall into two pairs of mutually exclusive ("rival") projects, $\left(P_z, \overline{P}_z\right)$ and $\left(P_x, \overline{P}_x\right)$, where $P_z = P_1 = \overline{P}_3$, $P_x = P_2 = \overline{P}_4$, $\overline{P}_z = P_3$, and $\overline{P}_x = P_4$. Here Z and X are two different areas of activity proposed in the projects: P_z and $\overline{P}_z$ are two rival scientific projects, say, while P_x and $\overline{P}_x$ are two rival humanities projects (see Fig. 1). The magnitude of p is not important; the only important assumption is that such a "quantum" exists.

Under the conditions of the game, we should have no preferences for any of these projects until one of them is *nominated* for p. This nomination of P_k for p, pP_k, should come from an outsider. It is then assumed that this nomination cannot for some reason be implemented, and that we must substitute for P_k another project from any pair, $\left(P_j, \overline{P}_j\right)$, identified by the outsider. The only substitution-rule is that a rival project cannot be substituted for any nominated project (i.e., when some $P_\ell\left(\text{or } \overline{P}_\ell\right)$ is nominated, and a substitute choice has to be made from pair $\left(P_\ell, \overline{P}_\ell\right)$, it is forbidden to choose $\overline{P}_\ell\left(P_\ell\right))$.

The nomination of some P_k, together with the above rule of mutual exclusiveness of P_k and $\overline{P}_k$, creates an asymmetry relative to the axis perpendicular to the $\left(P_k, \overline{P}_k\right)$-axis of Fig. 1. Nevertheless, it does not create any logical arguments for choosing any $P_\ell, \overline{P}_\ell$ from a "perpendicular" pair, nor, according to the initial conditions, should we have such arguments before the nomination. For example, when scientific project P_z is nominated for p, this does not help us decide which humanities project, P_x or $\overline{P}_x$, should be substituted for P_z. Only one of them can be nominated for p in place of P_z, since p is indivisible. Therefore, when we are forced to choose between P_x and $\overline{P}_x$, our answer can only be random, with equal probabilities 0.5 for P_x and $\overline{P}_x$.

The probabilities can be measured if we are asked to answer different questions, $Q_{jk}, j, k = 1$, 2, 3, 4, put to us at random, every question appearing many times. Question Q_{jk} is the following:

Q_{jk}: "Suppose P_j has been nominated for p, but this nomination cannot be implemented. It is forbidden to substitute $\overline{P}_j$ for P_j. You can only choose a substitute from the pair $\left(P_k, \overline{P}_k\right)$. If you don't choose P_k, that will count as choosing its rival, $\overline{P}_k$. Do you choose P_k (answer 'yes' or 'no')?" (1)

Or, in short:

$$Q_{jk}: \quad \text{"} pP_j;\ \overline{\overline{P}}_j;\ \left(P_k \overline{P}_k\right);\ pP_k\ ?\ \left(not - P_k = \overline{P}_k\right)\text{."} \tag{1a}$$

In (1a), $\overline{\overline{P}}_j$ is a reminder (important only when $k = j$) that it is forbidden to substitute $\overline{P}_j$ for P_j. The identity in parenthesis is a reminder that rejection of P_k will count as nomination of $\overline{P}_k$ instead of P_j; this reminder is needed to protect the exact logical meaning of a "no" choice of P_k. Under the ideal conditions described above, the probabilities will obviously be:

$$\omega(P_z, P_z) = \omega(\overline{P}_z, \overline{P}_z) = \omega(P_x, P_x) = \omega(\overline{P}_x, \overline{P}_x) = 1 \tag{2}$$

$$\omega(P_z, \overline{P}_z) = \omega(\overline{P}_z, P_z) = \omega(P_x, \overline{P}_x) = \omega(\overline{P}_x, P_x) = 0 \tag{3}$$

$$\begin{aligned} &\omega(P_z, P_x) = \omega(\overline{P}_z, P_x) = \omega(P_z, \overline{P}_x) = \omega(\overline{P}_z, \overline{P}_x) = \\ &\omega(P_x, P_z) = \omega(\overline{P}_x, P_z) = \omega(P_x, \overline{P}_z) = \omega(\overline{P}_x, \overline{P}_z) = \tfrac{1}{2} \end{aligned} \tag{4}$$

First, we have here the phenomenon of indeterminism, which appears not as a result of lack of knowledge, but (from a logical-linguistic point of view) as a result of the mutual nontranslatability of the truth values of logical statements λ_j, λ_k, corresponding to any two projects, P_j, P_k, involving two different areas of activity,

$$\lambda_j: \quad \text{"Project } P_j \text{ is appropriate to nominate for } p\text{."} \tag{5}$$

(The analogous statement about $\overline{P}_j$ will be denoted as $\overline{\lambda}_j$.)

Indeed, if it is decided, for example, that scientific project P_z is appropriate to nominate for p, so λ_z is certainly true, then it is uncertain whether statement λ_x (about humanities project P_x) is true or false, since under the initial conditions we have no preferences. Correspondingly, it is uncertain whether $\overline{\lambda}_x$ is false or true, since the rules of the game are such that when λ_x is true, $\overline{\lambda}_x$ is false, and vice versa. In other words, there is no deterministic one-to-one correspondence between the truth values of λ_z and λ_x. This uncertainty of truth values is crucial for the phenomenon of indeterminism.

Second, we can see that there is no way to use the classical theory of probability to connect results (2) and (3) with result (4). For example, let P_z be nominated for p, but the nomination unable to be implemented. $\overline{P}_z$ is forbidden for substitution, so the probability of getting choice $\overline{P}_z$, $\omega(P_z,\overline{P}_z)=0$. However, the probabilities of getting choices P_x and $\overline{P}_x$ are both $\frac{1}{2}$, $\omega(P_z,P_x)=\omega(P_z,\overline{P}_x)=\frac{1}{2}$; and if either P_x or $\overline{P}_x$ is then nominated but cannot be implemented, then the probability of $\overline{P}_z$ being chosen is also $\frac{1}{2}$. Therefore, when chains of nominations and choices (yes-no decisions), $P_z \to P_x \to \overline{P}_z$ and $P_z \to \overline{P}_x \to \overline{P}_z$, appear (represented in Fig. 2 by solid lines), the final probability of $\overline{P}_z$ is not $\omega(P_z,\overline{P}_z)=0$ (dotted line of Fig. 2), but $\frac{1}{2}$. Also, the final probability of P_z calculated by summarizing two chains of nominations and choices, $P_z \to P_x \to P_z$ and $P_z \to \overline{P}_x \to P_z$ is $\frac{1}{2}$, whereas in (2) the probability $\omega(P_z,P_z)=1$.

The situation is strikingly similar to that of $\frac{1}{2}$ spin quantum physics when only two possible axes, z and x, are involved (with rotation symmetry around the y-axis), so only S_z and S_x spin projections are prepared and measured. The correspondence is the following (using the symbol ~ for "corresponds to"):

$$P_z \sim S_z=\tfrac{1}{2};\ \ \overline{P}_z \sim S_z=-\tfrac{1}{2};\ \ P_x \sim S_x=\tfrac{1}{2};\ \ \overline{P}_x \sim S_x=-\tfrac{1}{2} \tag{6}$$

Therefore

$$\left.\begin{array}{l} \lambda_z \sim \hat{\lambda}_z \equiv \tfrac{1}{2}\left(\hat{1}+\sigma_z\right),\ \ \overline{\lambda}_z \sim \hat{\overline{\lambda}}_z = \hat{1}-\hat{\lambda}_z \equiv \tfrac{1}{2}\left(\hat{1}-\sigma_z\right) \\ \lambda_x \sim \hat{\lambda}_x \equiv \tfrac{1}{2}\left(\hat{1}+\sigma_x\right),\ \ \overline{\lambda}_x \sim \hat{\overline{\lambda}}_x = \hat{1}-\hat{\lambda}_x \equiv \tfrac{1}{2}\left(\hat{1}-\sigma_x\right) \end{array}\right\} \tag{7}$$

where $\sigma_{x,z}$ are Pauli matrices, $\hat{1}$ is the identity matrix, and $\hat{\lambda}_x$ and $\hat{\lambda}_z$ are density matrices of the corresponding pure states, matrices that simultaneously are the truth operators of the logical statements "$S_x=\frac{1}{2}$" and "$S_z=\frac{1}{2}$." The truth values of these statements are 1 ("true") and 0 ("false"). The statement "$S_{x,z}=\frac{1}{2}$" is true $\left(\hat{\lambda}_{x,z}=1\right)$ when $\sigma_{x,z}=1$, and false $\left(\hat{\lambda}_{x,z}=0\right)$ when $\sigma_{x,z}=-1$. We can assign the same truth values, 1 and 0, to logical statements λ_j.

Question Q_{jk} corresponds in quantum physics to the case when the prepared state is $S_{z_j} = \frac{1}{2}$ ($z_1 \equiv z$, $z_2 \equiv x$, $z_3 \equiv -z$, $z_4 \equiv -x$), and the researcher "asks" whether projection $S_{z_k} = +\frac{1}{2}$ will appear if the spin projection is measured along the z_k-axis. p corresponds to a (quantized) polarization in one of four possible directions. The first nomination mentioned in question (1) is analogous to the preparation of the polarized state. The second nomination (the substitution) is analogous to the observation of a result of the measurement.

If the initial spin is directed along the $\vec{z}$-axis while the analyzer axis is $\vec{z}'$ (the z, x-axes are rotated around the y-axis), the Pauli matrices are transformed, $\sigma_z' = \cos\theta\sigma_z + \sin\theta\sigma_x$. From this, the probabilities of getting $s_z' = \pm\frac{1}{2}$ when $s_z = \frac{1}{2}$ are equal to

$$
\begin{aligned}
w\left(s_z = \tfrac{1}{2}; s_z' = \tfrac{1}{2}\right) &= tr\left(\hat{\lambda}_z \hat{\lambda}_z'\right) = \cos^2\frac{\theta}{2} \\
w\left(s_z = \tfrac{1}{2}; s_z' = -\tfrac{1}{2}\right) &= tr\left(\hat{\lambda}_z \hat{\bar{\lambda}}_z'\right) = \sin^2\frac{\theta}{2}\ ,
\end{aligned}
\tag{8}
$$

see (6), (7). As a result of rotational symmetry (around the y-axis), these probabilities depend only on angles between the z, z' axes. Let us have, in a quantum case, two sequential Stern-Gerlach apparatuses with axes z' and z'' and initial spin polarization along the z-axis, $(\vec{z}\vec{z}') = \cos\theta_1$, $(\vec{z}'\vec{z}'') = \cos\theta_2$. If we measure the polarization after the first SG, combine after that two decoherent branches of the beam into a single beam, and direct this recombined beam into the second SG, then the probability of getting $s_z'' = \frac{1}{2}$ is equal to the classical probability of a complex event containing two consecutive ones,

$$
\begin{aligned}
w\left(s_z = \tfrac{1}{2}; s_z'' = \tfrac{1}{2}\right)_{clas} &= w\left(s_z = \tfrac{1}{2}; s_z' = \tfrac{1}{2}\right) w\left(s_z' = \tfrac{1}{2}; s_z'' = \tfrac{1}{2}\right) + \\
&+ w\left(s_z = \tfrac{1}{2}; s_z' = -\tfrac{1}{2}\right) w\left(s_z' = -\tfrac{1}{2}; s_z'' = \tfrac{1}{2}\right) = \\
&= \cos^2\tfrac{\theta_1}{2}\cos^2\tfrac{\theta_2}{2} + \sin^2\tfrac{\theta_1}{2}\sin^2\tfrac{\theta_2}{2}
\end{aligned}
\tag{9}
$$

But if we combine those two branches without measuring s'_z, then the coherence is not destroyed and

$$w\left(s_z = \tfrac{1}{2}; s''_z = \tfrac{1}{2}\right) = \cos^2 \frac{\theta_1 + \theta_2}{2} = w_{clas} + \text{ interference term}, \tag{10}$$
$$\text{interference term } = -\tfrac{1}{2} \sin\theta_1 \sin\theta_2 .$$

Quantum angles $\theta_1 = \theta_2 = \frac{\pi}{2}$ correspond in our gedanken experiment to transitions like, for example, $P_z \to P_x \left(\text{or } \overline{P}_x\right) \to \overline{P}_z$, the final angle being $\theta_1 + \theta_2 = \pi$. The probability is quantum-like if there is a direct question $Q_{13} \equiv Q_{z\bar{z}}$ connecting $\overline{P}_z$ directly with P_z (vertical dotted line in Fig. 2) without intermediate questions about P_x or $\overline{P}_x$. In this case, according to (10), $\omega\left(P_z, \overline{P}_z\right) = = \cos^2 \frac{\pi}{2} = 0$. In the analogous quantum case we also have a "direct question" since the first SG magnet can be thrown off when there is no intermediate measurement. But if we destroy the interference of two coherent chains of thoughts or "logical trajectories" $P_z \to P_x \to \overline{P}_z$ and $P_z \to \overline{P}_x \to \overline{P}_z$ by asking intermediate questions $Q_{zx}, Q_{z\bar{x}}, Q_{x\bar{z}}, Q_{\bar{x}\bar{z}}$, then the final probability of the choice of $\overline{P}_z$ should be classical and, according to (9), $\omega = \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$. As for quantum angles $\theta_1 = -\theta_2 = \frac{\pi}{2}$, they correspond to transitions like, for example, $P_z \to P_x\left(\overline{P}_x\right) \to P_z$; we see that again, without intermediate questions, $\omega\left(P_z, P_z\right) = \cos^2 0 = 1$, a quantum formula for coherent "logical trajectories," and with intermediate questions, $\omega = \frac{1}{2}$, a classical formula.

The analogy with quantum Hilbert space (containing here only four vectors) in our demonstration example is shown in Fig. 3. We can put λ_j's there because in quantum mechanics we can put density matrices, $\hat{\lambda}_j$'s, instead of their eigenvectors (though it would be better to call such a space "matrix space" and not vector space). Note that in quantum mechanics, matrices $\hat{\lambda}_j$ and $\hat{\bar{\lambda}}_j$ are mutually orthogonal: $\hat{\lambda}_j \hat{\bar{\lambda}}_j = 0$; the scalar product in quantum "matrix space" can be defined as $tr\left(\hat{\lambda}_j \hat{\lambda}_k\right) \equiv w_{jk}$. The corresponding scalar product of two logical statements in our gedanken experiment, Fig. 3, equals $Tr\left(\lambda_j \middle| \lambda_k\right)$, the conditional truth of λ_k on the condition that λ_j is true; $0 \le Tr\left(\lambda_j \middle| \lambda_k\right) \le 1$. In quantum mechanics, such a value also exists, and

$Tr\left(a\,|\,b\right) = tr\left(\hat{a}\hat{b}\right) \equiv w_{ab}$ where a, b are statements about physical observables [1]. In our experiment, $Tr\left(\lambda_j|\lambda_k\right)$ can be measured independently of, and be different from ω_{jk} (see remark at end of Sec. 2).

Symmetry transformations in quantum Hilbert space cause interference effects. These transformations are unobservable in principle since observations lead to destruction of the phases and hence of the interference effects. Now, if we maintain our assumption that decisions about substitutions in our nonphysical example are analogous to observations in quantum mechanics, then the process of interferences of chains of possible but not implemented nominations (analogous to a quantum process of interferences of nonobservable trajectories) must be invisible in principle *between* our decisions, and be destroyed at the moment a decision is made.

To sum up the discussion thus far: In our demonstration, we have pairs of words with certain logical connections, like P_xP_x; $P_x\overline{P}_x$, and pairs of words with uncertain logical connections, like P_xP_z; $P_x\overline{P}_z$. In logically certain cases, decisions are predictable and can be formally represented as results of *interference of coherent chains of thoughts—logical trajectories—unobservable in principle* either by the researcher or the respondent. Logical uncertainties lead to unpredictable decisions. In the gedanken experiment, we have no coherent chains between words with uncertain logical connections. (If such chains are discovered in the proposed actual experiments, this will be a proof that the logic of thought is not classical, although deterministic classical logic is a part of this wider logic.)

Of course, this demonstration merely reflects the fact that language does not obey the rules of the classical theory of probability. But this is our point. The gedanken experiment strongly suggests that logical-linguistic phenomena not only violate probabilistic rules, but in certain situations do so in ways that make these phenomena similar to quantum physics phenomena. From the fact that there is no quantum mechanics beyond the behavior described above, and that logical-linguistic properties are built into the very foundation of quantum mechanics, we can conclude that basic features of language and logic provide the common ground for both. This conclusion should be subjected to rigorous experiments. In the following section we sketch two such types of experiment designed to observe interference of probabilities, if it exists, in non-physical areas.

2. Logical-linguistic experiment, Type I.

This experiment, based on the gedanken experiment, is similar to measurements of s'_z projections of a $\frac{1}{2}$ spin particle when its premeasured state corresponds to $s_z = \frac{1}{2}$, and a new z'-axis results from a rotation around the y-axis. This type of experiment seems the easiest way to expose quantum-like effects outside quantum physics and to understand, *inter alia*, which kinds of indivisible things can play a role in "quantization" similar to quantization in physics.

Now we need to have more than four *alternatives* (the projects in our gedanken experiment), $P_j, j = 1,2,..., 2L, L > 2$. Correspondingly, there are more than two *pairs of mutually exclusive alternatives*: $(P_1 P_{1+L}), (P_2 P_{2+L}), ..., (P_{L-1}, P_{2L-1})$,

$$P_{j+L} = \overline{P}_j;\ \overline{P}_{j+L} = \overline{\overline{P}}_j = P_j;\quad j,\ j+L \le 2L. \tag{11}$$

An *indivisible quantum*, p, (a grant in our gedanken experiment) should be assigned to one and only one of the P_j' s, either by *nomination* of some P_j for p, or by substitution of some P_k for P_j when the nomination of P_j for p cannot be implemented. Such substitution is itself the nomination of P_k for p (instead of P_j). It is forbidden to substitute $\overline{P}_j$ for P_j.

The experiment consists of the selection of all P_j' s by a researcher; the selection of a *respondent* whose task is to answer questions composed by the researcher and presented by a generator of random events; and, after all questions have been answered, the *ordering* of P_j's by the researcher according to statistical analysis of the answers. (The ordering will be described later.)

Here we have only one respondent instead of an "ensemble of identical respondents," since it is unlikely that such an ensemble can be found. In connection with this, a doubt may arise whether it is proper to do any statistical experiment similar to quantum mechanical experiments, by working with a single system (one respondent, in our case) instead of an ensemble: the generally accepted statistical interpretation of quantum wave functions is that they exclusively describe ensembles, not individual systems. However, this interpretation of the wave function is not correct, and even contains an internal contradiction. An "ensemble described by wave function ψ" is a set of identical particles whose identical individual wave functions are mutually incoherent; otherwise, this set would be coherent and not be described by ψ, but by either a symmetric or an antisymmetric wave function, Ψ, quite different from ψ. We see from this that we cannot even define the term "ensemble" without referring to individual wave functions, since we need to say that there are no correlations among phases of these functions. The only consistent statistical interpretation of ψ is that it is defined up to an unknown phase and describes every individual particle of an ensemble as well as the ensemble itself. In particular, ψ can describe the set of repeatedly prepared identical states, ψ, defined up to unknown phases, of the same particle—when such repetitions are technically possible.

Therefore, it will not violate the analogy with quantum mechanics if, in our logical-linguistic experiment, we repeat the same questions to the same respondent and delete his/her memory of answers by randomizing questions and limiting the time needed for answers. Every new question is analogous to the preparation of a new state of the same particle, and every answer is a yes-no decision similar to a measurement in quantum mechanics.

We assume in this experiment that the following conditions will be met:

(a) *Symmetry*. Before the questioning begins, the respondent has no preferences among the P_j's; that is, from the respondent's point of view, every P_j can be equally nominated for p. (This must be tested in advance.) But the existence of symmetry in the much deeper sense analogous to rotation symmetry in $\frac{1}{2}$ spin physics, where probabilities (8) depend only on the angle, θ, *between* axes, can be precisely checked only after the experiment is completed. Roughly speaking, this deeper symmetry in the context of our experiment requires the respondent to compare any two alternatives, P_j and P_k, without being influenced by environmental or other associations they may have. A partial manifestation of this symmetry will be that the exchange $P_j \leftrightarrow P_k$ gives $\omega_{jk} = \omega_{kj}$.

(b) *Substitutions*. If some P_j is nominated for p, then *any* P_k (with the exception of $\overline{P}_j = P_{j \pm L}$) may be substituted for P_j. If such substitution looks senseless to the respondent, then the corresponding pair of alternatives, $\left(P_k \overline{P}_k\right)$, was selected mistakenly by the researcher.

(c) *Nontranslatability and uncertainty*. The concept of translatability means the following. Let A and B be a pair of elementary logical statements. If there exists a computable function mapping one-to-one the truth values of A onto the truth values of B, then A and B are mutually translatable. For example, statements λ_j and $\overline{\lambda}_j$ from (5) are mutually translatable: when λ_j is true (false), $\overline{\lambda}_j$ is false (true) because the nominations of P_j and $\overline{P}_j$ for p are mutually exclusive; $\left(\lambda_j \overline{\lambda}_j\right)$ is a pair of statements that negate each other.

Condition (c) means that although condition (b) holds, λ_k and λ_j from two different pairs, $\left(\lambda_k \overline{\lambda}_k\right)$, $\left(\lambda_j \overline{\lambda}_j\right)$, $k \neq j$, should in principle not be mutually translatable. This means that when asked whether P_k or $\overline{P}_k$ can be nominated instead of P_j, $k \neq j$, the respondent is not certain, and this uncertainty cannot be reduced by any logical arguments or by any information about P_k, P_j in addition to that already presented. And since this question about substituting one alternative for another is the main question to the respondent in the experiment, we cannot precisely check fulfillment of condition (c) before the end of the experiment, although some preliminary tests may be possible. (We may say that the idea of this experiment is to prove the existence of nontranslatability in a nonphysical area, since nontranslatability in quantum mechanics provides a common, though not complete, basis for quantum uncertainty, indeterminism, and interference of probabilities.)

The main lines of the experiment are as follows:

Different $4L^2$ questions, $Q_{jk}, j,k = 1,...,2L,$ the same as (1), (1a), are presented to the respondent at random, and many times in order to get good statistics. The upper limit on the time needed to answer every question should be sufficiently short to eliminate correlations between different answers caused by the respondent's memory of previous answers (although this memory is suppressed when Q_{jk}'s come at random) [2]. The permitted answers are 1 ("yes") and 0 ("no"); "yes" ("no") for P_k counts simultaneously as "no" ("yes") for $\overline{P}_k$. The aim is to measure probabilities $\omega_{jk}, j,k = 1, 2,.., 2L;$ ω_{jk} is the probability of the respondent's answering "yes, P_k" to Q_{jk}.

Let all ω_{jk}'s be measured with a sufficient accuracy. (Note that while statistical error decreases over time with the number of random repetitions of the same questions, systematic error caused by any changes of the respondent's interpretation of P_j's will grow over time.) One of the symmetry conditions is the equality $\omega_{jk} = \omega_{kj}$. If these are not equal, then the results of the experiment are not valid for further analysis. If $\omega_{jk} = \omega_{kj}$ for all j, $k = 1, ..., 2L$,, then in our analysis of the results we will introduce the formal angles, θ_{jk}, by the formula

$$\omega_{jk} = \cos^2 \frac{\theta_{jk}}{2} \quad . \tag{12}$$

The ordering task referred to earlier is to place all P_j's along a circumference (see Fig. 4) in such an order that *all* ω_{jk}'s obey formula (12), where every θ_{jk} is defined by the positions of P_j and P_k. When trying an order, we should place P_k after P_j, if we guess that $\theta_{jk} > 0$, and P_k before P_j, if we guess that $\theta_{jk} < 0$, in accordance with the chosen direction of the θ-coordinate. (In Fig. 4 we show, as an example, only three pairs of alternatives, and only some angles between them.) If such ordering (with a reasonable level of confidence) is impossible, then the result of the experiment should be considered negative But if our hypothesis about quantum-like behavior in this situation is correct, then from (12), for every θ_{km} such that $\theta_{km} = \theta_{kj} + \theta_{jm}$,

$$\omega_{km} = \cos^2 \frac{\theta_{kj} + \theta_{jm}}{2} = \omega_{kj}\omega_{jm} + \omega_{k\bar{j}}\omega_{\bar{j}m} - 2\sqrt{\omega_{kj}\omega_{jm}\omega_{k\bar{j}}\omega_{\bar{j}m}}\ \text{sign}\left(\sin\theta_{kj}\sin\theta_{jm}\right), \qquad (13)$$

where $\omega_{k\bar{j}} = 1 - \omega_{kj}$; the last term describes the interference of two unobservable coherent chains of thoughts, $P_k \to P_j \to P_m$ and $P_k \to \overline{P}_j \to P_m$ or, in other words, $\lambda_k \to \lambda_j \to \lambda_m$ and $\lambda_k \to \overline{\lambda}_j \to \lambda_m$.

This formula for ω_{km}, $k,m = 1,2,\ldots,2L$, should be valid for *any chosen* j. It is automatically implemented for any $\omega_{mm}, m = 1,2,\ldots,2L$, since in such cases $\omega_{mm} = 1$ and we should assume $\theta_{mj} = -\theta_{jm}$. It also is automatically implemented for every $\omega_{k,k+L}$ and $\omega_{k+L,k}, k$, $k + L \le 2L$, since in such cases we should assume $\theta_{kj} + \theta_{jm} = \pi$, $\omega_{k,k+L} = \omega_{k+L,k} = 0$. But it would not be trivial at all if, for all other ω_{km}'s, $\omega_{km} = \cos^2\left(\theta_{kj} + \theta_{jm}\right)/2$, since this very formula describes the quantum-like interference of probabilities. If (13) is valid for all ω_{km}'s, then the result of the experiment should be considered positive.

Now let us translate all the above into a concrete example.

We have a game, "Where to go?" There are four roads, *NS* (North-South), *EW* (East-West), *NE-SW*, and *NW-SE*, with eight possible directions in which to walk: $N \equiv P_1 \equiv \overline{P}_5$, $NE \equiv P_2 \equiv$

$\equiv \overline{P}_6$, $E \equiv P_3 \equiv \overline{P}_7$, $SE \equiv P_4 \equiv \overline{P}_8$, $S \equiv P_5 \equiv \overline{P}_1$, $SW \equiv P_6 \equiv \overline{P}_2$, $W \equiv P_7 \equiv \overline{P}_3$, and $NW \equiv P_8 \equiv \overline{P}_4$, and an indivisible group of walkers, p, pointed in a direction chosen by the respondent. 64 questions (1a) are given at random to the respondent in a notation s/he will be trained to read fluently. A typical question,

$$Q_{4,7}:\ \ "pSE;\ \overline{NW};\ EW;\ pW?\ \ (\text{not}\text{-}W = E)," \tag{14}$$

would be spelled out in training to the respondent as: "The ideal direction to walk is *SE*, but it is blocked. The opposite direction, which is *NW*, is unsuitable. All other directions are suitable, but only the *EW* road is open. Will you choose direction *W*? (If not, then you are implicitly choosing direction E.)"

The remark in parenthesis in (14) is a useful precaution since, without it, "not-*W*" may mean to the respondent all kinds of alternatives different from *W*—instead of the exact logical negation of "yes, W ," which is $\overline{W}$, $\overline{W} = E$. The remark "All other directions are suitable" in the detailed explanation of (14) is a useful reminder that if some P_j is nominated but cannot be implemented, then *all* other alternatives are acceptable for substitution (except $\overline{P}_j$), and not only those closest to P_j. In the concrete case of question (14), the only road open is the *EW* road, and the direction closest to *SE* is *E,* not *W*. But since both directions are suitable, the respondent can choose *W*.

As already mentioned, every "not-*W*" should be counted as "yes, $\overline{W}$," i.e., "yes, E"; and in question $Q_{4,3} \equiv Q_{SE,E}$, every "not-*E*" should be counted as "yes, *W*." This approach preserves the conditions $\omega_{jk} + \omega_{j\bar{k}} = 1$, $\omega_{jk} + \omega_{\bar{j}k} = 1$, and automatically fixes small possible differences between probabilities ω_{jk} and $(1 - \omega_{j\bar{k}})$ that can appear without such a procedure. But if these differences are not small compared with the probabilities themselves, then the results of the experiment should be considered invalid.

As for the ordering of directions in this game, the usual reading of direction-words (common to researcher and respondent) will lead to such angles (defined by probabilities) that *NE* will certainly be placed between *N* and *E*, *NW* between *N* and *W*, and so on. We will therefore need to define precisely only numerical values of angles $\theta_{jk}, j, k = 1, \ldots, 8$; the general relative positions of our eight directions along the circumference of Fig. 4 will be clear in advance.

The needed statistics can also be estimated in advance by an assumption about expected minimal probabilities. For example, we can expect the measured angle $\theta_{E,S} \sim \frac{\pi}{2}$ and the measured angle $\theta_{SE,E} \sim \frac{\pi}{4}$; therefore, $\omega_{SE,E} = \cos^2 \frac{\theta_{SE,E}}{2} \sim 0.85$, and $\omega_{SE,W} = \sin^2 \frac{\theta_{SE,E}}{2} = \cos^2 \frac{\theta_{SE,W}}{2} \sim 0.15$. Given this estimate, we will probably need to repeat (randomly) every question in this game about 1,000 times.

Thus, in this game, if *every* three measured angles, $\theta_{k\ell}, \theta_{\ell m}, \theta_{km}, k, \ell, m = 1, 2, \ldots, 8$, $\cos^2 \frac{\theta_{ij}}{2} = \omega_{ij}$, obey the rule of interference for experiments of this type:

$$\theta_{km} = \theta_{k\ell} + \theta_{\ell m}, \tag{15}$$

then the existence of the interference of probabilities in a non-physical area will have been proved.

It would be useful to compare this experiment with a variant having slightly different questions and answers. Namely, instead of "Do you choose P_k?" in (1), we ask: "Estimate your confidence (by a number between 0 and 1) that P_k is the appropriate substitute for P_j," where it is understood that 0 means no confidence and 1 means 100% confidence. All other conditions and procedures are the same as in the original version, including the random repetitions of the same questions. However, in this version we do not measure the probabilities, ω_{jk}'s. We measure t_{jk}'s, $t_{jk} =$

$= Tr\left(\lambda_j|\lambda_k\right)$, the conditional truth that λ_k (see (5)) is true under the condition that λ_j is true. Therefore, there is no *direct* analogy between this experiment and a quantum mechanical one: in quantum mechanics, $t_{jk} = \omega_{jk}$, by definition, but we can measure only ω_{jk}.

The following is a more complicated kind of logical-linguistic experiment.

3. Logical-linguistic experiment, Type II: maximally close to EPR-Bohm experiments.

It is instructive to compare EPR-Bohm experiments in $\frac{1}{2}$ spin physics, and the closest possible experiment with nonphysical alternatives.

Consider the principal $\frac{1}{2}$ spin EPR-type experiment [3], in which two protons are prepared in the singlet $(L = S = 0)$ state. Correlations between spin projections of two particles moving inside two different channels are measured under the condition that physical information about a result of a single measurement of the spin projection in any of the channels cannot reach another channel before the spin projection in that channel is measured. The problem in understanding the correlations observed in this experiment lies not, of course, in their existence but in their non-classical character. We know that every single measurement in a given channel produces a new and unpredictable state of the particle in this channel—a state logically, hence deterministically, disconnected with the original two-particle state, which is destroyed by such measurement. Moreover, there are two unpredictable factors, classical and quantum, in a single channel measurement in this experiment: a choice of the z-axis, which is (classically) random; and the appearance of either a $+\frac{1}{2}$ or $-\frac{1}{2}$ spin projection along this axis, which is unpredictable in the quantum sense. As explained in [1], this second, quantum unpredictability is a consequence of the nontranslatability between truth values of the logical statement describing the original unobserved two-particle state and the logical statement describing the final state that results from the measurement. But whatever the explanation, the fact is that the original state is destroyed and the new state is created and defined in terms of spin projections along an axis chosen *at random*. Although this choice is unknown in the second channel, the particle in that channel behaves as if the direction of the z-axis chosen in the first channel is immediately known in the second channel, so the state of the first particle created by the measurement in the first channel immediately defines the state of the second particle in the second channel.

This phenomenon has led some physicists to speculate about the existence of faster-than-light interactions. The less exotic explanation given in [1] is that even before measurements, all possible logical correlations that can be checked by measurements are already contained implicitly in the logical truth operator (density matrix) of the statement describing the common state of the two particles. This statement was born together with the two-particle system that was prepared by the researcher and was developing in space/time, according to quantum mechanics equations, before the measurement. We may say that in this quantum EPR-Bohm experiment there exists a logical statement such that it is nonlocal in space/time and covers both spatially separated particles before the measurement. This statement defines the particles' behavior—in particular, all long-distance correlations between them.

Now, if we try to formulate a nonphysical experiment exactly analogous to the two-particle EPR-Bohm experiment we should include two spatially separated respondents. But the most crucial element of the EPR-Bohm experiment is a logical statement covering two spatially separated particles and defining all their behavior. And an analogous logical statement covering separated respondents does not exist, at least in the world that can be described scientifically. Therefore, an *exact* analogy of experiments is impossible. The only option is to work with a single respondent.

As a concrete example of the experiment, we will use a variant of the "Where to go?" game described in Sec. 2. Conditions (a), (b) and (c) are the same. The new problem is to prepare a situation analogous as much as possible to the one in physical experiment [3]. The quantum state in that experiment is such that two $\frac{1}{2}$ spins are always directed in opposite directions independently of the choice of the z-axis (the invariant scalar state). In our game, the direction of the "z-axis" is one of eight roads; there are two imaginary indivisible groups of walkers now (Group 1 and Group 2); and the respondent must now make not one but two decisions simultaneously, namely, choosing $P_j(1)$, a j-direction for Group 1, and $P_k(2)$, a k-direction for Group 2. To address the above problem, we will introduce into the training and testing of the respondent before the game a condition similar to that in physical experiment [3]: the two groups of walkers want to walk in opposite directions (whatever they may be), and the respondent must always direct them in opposite directions. Accordingly, the random-events generator names a road, $P_j - \overline{P}_j$, $j = 1, 2, \dots, 8$, open in both directions, and the respondent must quickly decide the direction in which each of the two groups will walk. S/he is prepared for the game if and only if, for every j, the probability of his/her answering "yes, $P_j(1)$ and yes, $\overline{\mathrm{P}}_j(2)$" equals the probability of his/her answering "not-$P_j(1)$ and not-$\overline{P}_j(2)$"—without such incorrect answers as "$P_j(1)$ and yes, $P_j(2)$."

In the actual game, the random-events generator names *two* roads, $P_j - \overline{P}_j$ and $P_k - \overline{P}_k$, $j, k = 1, \dots, 8$. Again, both groups always want to walk in opposite directions, and the respondent must decide in which direction each group of walkers will go, for example, $\overline{P}_j(1)P_k(2)$. But now, whatever direction the respondent chooses for Group 1, the opposite direction is immediately blocked for Group 2 if $k \neq j$, and not blocked if $k = j$. Thus, if $k = j$ and the respondent chooses $P_j(1)$ (or $\overline{P}_j(1)$), then s/he can and must choose $\overline{P}_j(2)$ (or $P_j(2)$). If $k \neq j$, then the respondent must use another, completely open road for Group 2, that is, $P_k - \overline{P}_k$, and must decide in which direction, P_k or $\overline{P}_k$, Group 2 will walk on that road. Let $k \neq j$ and the respondent choose $\overline{P}_j(1)$ for Group 1, for example. This choice, by the first condition of the game, entails the choice of $P_j(2)$ for Group 2; however, the ideal direction for Group 2, P_j, is immediately blocked, so the respondent must choose instead whether to direct Group 2 to P_k or to $\overline{P}_k$. This part of the game is similar to the game in Sec. 2, the only difference being that now the initial choice of direction for Group 2 (unsuccessful in this case) is made by the respondent and not by the generator of random events. The substitution of either $P_k(2)$ or $\overline{P}_k(2)$ for $P_j(2)$ is made by the respondent in both games.

There are 64 different pairs (j,k) in this game, and every pair of roads should be presented at random to the respondent about 1,000 times. The processing of the results after the game is finished consists of two independent operations designed to identify quantum-like phenomena:

1. *The ordering*. This is the same operation as in the Sec. 2 game, with the following technical difference: If the respondent's choice for Group 1 is $P_j(1)$, then his/her unsuccessful choice for Group 2 is $\overline{P}_j(2) = P_{j\pm L}(2)$. ($j+L$ for $0<j\leq L$; $j-L$ for $L < j \leq 2L$.) Therefore, the respondent's final choice (that is, substitution) for Group 2 corresponds to the transitions $P_{j\pm L} \to P_k$ with the probability $\omega_{j\pm L,k}$ or $P_{j\pm L} \to \overline{P}_k = P_{k\pm L}$ with the probability $\omega_{j\pm L,k\pm L}$. These very probabilities define the angles in formulae (12) and (15), and the corresponding ordering. If the ordering is impossible, it means that (15) is not valid for all these angles.

2. The second operation is to *check possible violations of the Bell inequalities.* For example, one type of Bell inequality in our case might be

$$\tfrac{1}{2}\sin^2\frac{\theta_{k\ell}}{2} \leq \tfrac{1}{2}\sin^2\frac{\theta_{\ell m}}{2} + \tfrac{1}{2}\sin^2\frac{\theta_{km}}{2}, \tag{16}$$

where θ_{jk} is defined by the formula

$$\omega_{jk} = \cos^2\frac{\theta_{jk}}{2}, \tag{17}$$

ω_{jk} is the probability of the choice $P_j(1)P_k(2)$, and every $\frac{1}{2}$ is the probability of choosing one of two directions for Group 1. (Note that the angles θ_{jk} defined by these probabilities are different from those defined in the ordering.) If the results are quantum-like, analogous to those of experiment [3], then the Bell inequalities of (16) are violated. For example, when $\theta_{k\ell} = 90°$, $\theta_{\ell m} = -45°$, and $\theta_{km} = 45°$, we have in (16) $0.25 \le 0.15$. Violation of those inequalities means that the classical theory of probability, based on the assumption that something unknown to us in the measuring procedure has led to stochastization of the respondent's choices, cannot be applied to our game.

The problem of nonlocality essential in EPR-Bohm experiments in quantum physics cannot be investigated in this logical-linguistic experiment for the reasons explained at the beginning of this section.

Conclusion.

If the proposed experiments give positive results, then the purely logical-linguistic nature of indeterminism and interference of probabilities will have been established, and all interpretations of quantum mechanics but logical-linguistic ones should be rejected. (Until now, only the hypothesis of local hidden parameters [4, 5] can be certainly rejected.) In addition, positive results will show that Niels Bohr's intuition about the existence of the phenomenon of complementarity beyond physics was correct, while the common assumption that the strange logic of quantum mechanics is an exceptional product of the microworld is not. It is also reasonable to suggest that after strong confirmation of the logical-linguistic basis of quantum mechanics, quantum field theory should be reformulated on that basis.

As for implications outside physics— in the theory of mind, as well as in the social sciences— we will have to recognize that the nontranslatability of different "viewpoints," and, following from it, intellectual conflicts and fundamental unpredictability, are unavoidable parts of mind and social life, although in "ordinary" life these quantum-like phenomena are probably strongly mixed with classical deterministic and chaotic phenomena. We will also have to recognize that any theory of mind must include the phenomenon of quantum-like interference of coherent logical trajectories.

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Figure Captions

Fig. 1 Two pairs of alternatives in the gedanken experiment.

Fig. 2 Possible chains of yes-no decisions in the gedanken experiment. Every arrow corresponds to probability 0.5. Dotted lines correspond to zero probability.

Fig. 3 An analogy to the Hilbert space of $\frac{1}{2}$ spin $\left(S_x, S_z\right)$ physics (in the gedanken nonphysical experiment).

Fig. 4 A hypothetical version of the ordering of alternatives along the circumference, $\cos^2 \frac{\theta_{k\ell}}{2} = \omega_{k\ell}$.