CLNS 96/1403

Spin Symmetry Without Heavy Quarks: Hyperon Form Factors In The Large N_c Limit

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(February 29, 1996)

Abstract

In the large N_c limit, all hyperon decays involving the same quark diagram $Q \to Q'$ are described by a single weak form factor $\eta_{QQ'}(w)$. No assumption on the mass of Q or Q' is necessary, making our results applicable to both $b \to c$ and $c \to s$ transitions. This same form factor describes both $\Lambda_Q \to \Lambda_{Q'}$ and $\Sigma_Q^{(*)} \to \Sigma_{Q'}^{(*)}$ transitions. The (non-)commutativity between the heavy quark and the large N_c limits is briefly discussed under the definite example of $\Lambda_b \to \Lambda_c$ decay.

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In the past few years, our understanding in the large N_c limit of QCD in the baryon sector has been greatly improved. It is now realized that a contracted SU(4) spin-flavor symmetry arises in the large N_c limit, and many important consequences of this result has been discussed in Ref. [1][1]. It turns out that baryon properties like the masses and the meson couplings are severely constrained by this spin-flavor symmetry. In this letter, the application of this symmetry to weak processes of hyperons are discussed. For hyperon we mean a baryon with a single s, c and b quark. It is found that the reduction of the numbers of form factors, a result usually obtained in the heavy quark limit, is reproduced, although no assumptions has been made on the masses of the quarks involved in the weak decay. As a result, our results are equally applicable to "heavy-to-heavy" (like $b \to c$) and "heavy-to-light" (like $c \to s$) decays. In the large N_c limit, a single universal weak form factor $\eta_{QQ'}(w)$ describes both $\Lambda_Q \to \Lambda_{Q'}$ and $\Sigma_Q^{(*)} \to \Sigma_{Q'}^{(*)}$, where Q and Q' can be b, c or s. Moreover, one can use flavor SU(3) to extend this result to decays to u and d quarks as well. This form factor $\eta_{QQ'}(w)$ in general depends on the quark species Q and Q' but is independent on the spin structure of the baryon and the current involved. However, $\eta_{QQ'}(w)$ is in general not normalized at any kinematic point unless we further assume some flavor symmetry between the parent and daughter quark.

Since the baryons are heavy in the large N_c limit, their velocities are well-defined. A weak transition from a spin-$\frac{1}{2}$ hyperon of velocity v to one also with spin $\frac{1}{2}$ but velocity v'

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[1]Besides this work by the San Diego group based on the "current algebra approach", there are other groups studying the large N_c limit of baryon dynamics in other formalisms, like the Harvard group and the Berkeley group. Our reference to the San Diego group results reflects the author's familiarity with that particular formalism and certainly does not imply that the works by other groups are inferior or irrelevant. The physics is independent of which formalism one works with, as the crux of the matter is the SU(4) spin-flavor symmetry, which is present in all different approaches.

is in general parametrized by six form factors.

$$
\begin{aligned}
&\langle B'_{Q'}(v',s')|\bar{Q}'\gamma^\mu Q|B_Q(v,s)\rangle \\
&= \bar{u}(v',s')(F_1(w)\gamma^\mu + F_2(w)v^\mu + F_3(w)v'^\mu)u(v,s),
\end{aligned}
\tag{1a}
$$

$$
\begin{aligned}
&\langle B'_{Q'}(v',s')|\bar{Q}'\gamma^\mu\gamma_5 Q|B_Q(v,s)\rangle \\
&= \bar{u}(v',s')(G_1(w)\gamma^\mu + G_2(w)v^\mu + G_3(w)v'^\mu)\gamma_5 u(v,s),
\end{aligned}
\tag{1b}
$$

where B_Q can be a Λ_Q or a Σ_Q hyperon. In general the set of form factors F's and G's governing the Λ and Σ sectors are unrelated. In additional to these twelve form factors (six for Λ and six for Σ) there are another eight form factors describing the $\Sigma_Q \to \Sigma^*_{Q'}$ transition. So in general one needs twenty form factor to describe all hyperon weak transitions involving the quark level process $Q \to Q'$. (The Σ^*_Q hyperons can decay electromagnetically to Σ_Q, so the $\Sigma^*_Q \to \Sigma^{(*)}_{Q'}$ transitions are physically insignificant.)

Now it is well known that, in the heavy quark limit, the number of independent form factors is dramatically reduced [2–5]. For example, for the decay $\Lambda_Q \to \Lambda_{Q'}$, there are only two independent form factors $\eta(w)$ and $\beta(w)$ when $m_Q \to \infty$,

$$
\begin{aligned}
F_1(w) &= \eta(w) - \beta(w), \quad F_2 = 2\beta(w), \quad F_3(w) = 0, \\
G_1(w) &= \eta(w) + \beta(w), \quad G_2 = 2\beta(w), \quad G_3(w) = 0.
\end{aligned}
\tag{2}
$$

Moreover, when we further assume $m_{Q'} \to \infty$, $\beta(w)$ vanishes identically and the weak decay is described by just a single universal form factor $\eta(w)$. The drastic simplification bases on the observation that, in the heavy quark limit, the heavy quark spin s_Q is conserved, i.e.,

$$
[s_Q, H_Q] = 0,
\tag{3}
$$

where H_Q is the hamiltonian of a baryon containing the heavy quark Q. Similar simplification is also possible for the $\Sigma_Q \to \Sigma_{Q'}$ transition, which is described by ten form factors when $m_Q \to \infty$ but only two when $m_{Q'}$ is also set to infinity. It must be emphasized that this reduction of form factors is just the consequence of the spin symmetry (3). As a result, this reduction of form factors will also occur at other limits of QCD where condition (3) is satisfied. We will see promptly that this is indeed the case for baryons in the large N_c limit.

The representations of the baryonic states in the large N_c limit has been studied in detail in Ref. [1]. It is found that the baryons states can be denoted by $|I, I_3; J, J_3; K\rangle$, where $\vec{K} = \vec{I} + \vec{J}$ is an additional operator satisfying the SU(2) commutation relations. We have $K = 0$ for baryons with just u and d quarks, while a hyperon with a single s, c or b quark has $K = \frac{1}{2}$. In the latter case K generates the rotation of the s, c or b quark in question, i.e., $\vec{K} = \vec{s}_Q$. Under this notation, the baryon hamiltonian in the large N_c limit has the following expansion in orders of $1/N_c$:

$$H = N_c M_0 + N_c^0 s_Q M_1 + N_c^{-1}(aI^2 + bJ^2 + cs_Q^2). \tag{4}$$

Since for hyperons with a single s, c or b quark we have $s_Q \equiv 1/2$, the first two terms are just constant numbers which commutes with $\vec{I}$, $\vec{J}$ and $\vec{K}$. The $\Sigma_Q - \Lambda_Q$ and $\Sigma_Q^* - \Sigma_Q$ splittings are determined by the parameters a and b respectively. It is evident that, in the large N_c limit, Λ_Q and $\Sigma_Q^{(*)}$ are degenerate, and the splittings enter only at order N_c^{-1}, two orders below the leading term. Hence, if we only keep the first two terms in the expansion, the hamiltonian is just a constant number (total degeneracy between all states) and condition (3) is satisfied. As a result, we have "heavy quark spin symmetry" although we have not placed any assumptions on the mass of the "heavy quark", which may as well be just a strange quark. This symmetry is simply the consequence of the *light quark* spin-flavor symmetry in the large N_c limit. Roughly speaking, this large N_c light quark spin symmetry means that the physics is invariant upon the flipping of the spin of any light quark. As a result, one can flip the spin of the heavy quark by first flipping the spins of all the light quarks, and then rotation the whole system by an angle π. The physics is invariant under both processes, and hence we have "heavy quark spin symmetry" without necessarily a heavy quark.

So we have come to see that all consequences of the heavy quark spin symmetry are also valid in the large N_c limit. For example, the heavy quark spin symmetry decrees that Σ_Q and Σ_Q^* are degenerate. Our study shows that they are also degenerate in the large N_c limit, in regardless of the mass of the heavy quark. In particular, the $\Sigma^* - \Sigma$ splitting is of order N_c^{-1}. This is in accordance with the experimental result

$$\Sigma^* - \Sigma \sim 200 \text{ MeV} \ll 400 \text{ MeV} \sim K^* - K, \tag{5}$$

as the $K^* - K$ splitting is not $1/N_c$ suppressed at all. Moreover, if one takes the unconfirmed measurement of Σ_c^* mass at 2530 MeV [6,7], one have a corresponding inequality in the charmed sector as well.

$$\Sigma_c^* - \Sigma_c \sim 75 \text{ MeV} \ll 145 \text{ MeV} \sim D^* - D. \tag{6}$$

Returning to the weak transition, with the spin symmetry we have the reduction of weak form factors as usual. In the large N_c limit, the $\Lambda_Q \to \Lambda_{Q'}$ transition is described by a single form factor $\eta(w)$.

$$\langle \Lambda_{Q'}(v')|\bar{Q}'\Gamma Q|\Lambda_Q(v)\rangle = \eta_{QQ'}(w)\bar{u}_{\Lambda_{Q'}}\Gamma u_{\Lambda_Q}, \tag{7}$$

where the $\Sigma_{Q'}^{(*)} \to \Sigma_Q^{(*)}$ transitions are controlled by two form factors.

$$\begin{aligned} &\langle \Sigma_{Q'}^{(*)}(v')|\bar{Q}'\Gamma Q|\Sigma_Q^{(*)}(v)\rangle \\ &= \left(\zeta_{1QQ'}(w)g_{\mu\nu} + \zeta_{2QQ'}(w)v_\nu v'_\mu\right) \bar{u}^\nu_{\Sigma_{Q'}^{(*)}}(v')\,\Gamma\, u^\mu_{\Sigma_Q^{(*)}}(v), \end{aligned} \tag{8}$$

where $u^\nu_{\Sigma_Q^*}(v')$ is the Rarita–Schwinger spinor vector for a spin-$\frac{3}{2}$ particle and $u^\mu_{\Sigma_Q}(v,s)$ is defined by

$$u^\mu_{\Sigma_Q}(v) = \frac{(\gamma^\mu + v^\mu)\gamma_5}{\sqrt{3}} u_{\Sigma_Q}(v) \tag{9}$$

and similarly for $u^\nu_{\Sigma_{Q'}^{(*)}}(v')$. Since we have not utilized the heavy quark limit, these reduction of form factors are equally applicable for "heavy-to-heavy" transitions like $\Lambda_b \to \Lambda_c$ as well as "heavy-to-light" ones like $\Lambda_c \to \Lambda$. The latter case is particularly interesting, as in this case the large N_c reduction of former factors is more powerful than that in the $N_c = 3$ heavy quark limit. (One form factor in the former case, in contrast to two for the latter.) In the notation of Eq. (2), we have $\beta(w) = 0$ in the large N_c limit. More will be said about the interpretation of this statement.

Further reduction of form factors is still possible. In Ref. [8], it has been proved that in the large N_c limit, the weak form factors in the Λ_Q and Σ_Q sectors are related. (This result first appeared in Ref. [9] in the context of the chiral soliton model.) In the notation above, we get

$$\zeta_{1QQ'}(w) = -(1+w)\zeta_{2QQ'}(w) = \eta_{QQ'}(w). \tag{10}$$

Hence we have reached the main result of this article: *in the leading order of the large N_c limit, all baryon transitions involving the same quark level diagram $Q \to Q'$ are described by a single form factor, in regardless of the masses of Q and Q'.* It should not be a surprising result. It is well known that, in the large N_c limit, the static properties of the tower states are closely related. They all have the same mass in the first two orders in the $1/N_c$ expansion, and their axial current couplings are simply interrelated by Clebsch–Gordan coefficients. Our study just shows that such interrelations also hold for weak form factors.

On the other hand, some important implications in the heavy quark limit cannot be reproduced here. Here we do not have the heavy quark *flavor* symmetry, and the $\eta_{QQ'}(w)$ for different Q and Q' are in general unrelated. Moreover, the weak form factor in the large N_c limit is in general not normalized. Of course, if one in addition assume some flavor symmetry between the initial and final quarks Q and Q', the form factor will be normalized at certain kinematic points. For example, when both Q and Q' are heavy, the form factor is normalized at the point of zero recoil $w = 1$. Unfortunately, for a "heavy-to-light" decay, no flavor symmetry is applicable and the form factor is not normalized throughout the whole kinematic range.

Several points of discussion are in place here. Firstly, we have restricted ourselves to weak transitions between s, c and b quarks. It is natural to ask if our results can be applied to weak transitions producing u and d quarks, like $c \to d$ or $b \to u$. The answer is affirmative. Under SU(3) flavor symmetry, the same form factor describes, say, $c \to s$ and $c \to d$ transitions. So the applicability of our results to the former case warrants that to the latter. So our results are in fact not only valid for hyperons but all (orbitally unexcited) baryons.

Another point of interest is the asymptotic behavior of these form factors is the large N_c limit. For $w \neq 1$, i.e., when the initial and final baryons have different velocities, the weak transition involves changing the momenta of all the quarks inside the baryons[2]. As a result, one expect the transition amplitude to be highly suppressed when N_c is large. This expectation is indeed verified in the special cases of $\Lambda_b \to \Lambda_c$ [10] and $\Sigma_b^{(*)} \to \Sigma_c^{(*)}$ [9], where the form factors scales like $\exp(-N_c^{3/2})$ in the large N_c limit. So, when we claim above in Eq. (10) $\eta_{QQ'}(w) = \zeta_{1QQ'}(w)$, that should be read as

$$\frac{\zeta_{1QQ'}(w)}{\eta_{QQ'}(w)} = 1 + O(1/N_c), \tag{11}$$

with both form factors vanishing in the large N_c limit when $w \neq 1$. Similarly, when we assert that for a "heavy-to-light" decay $\beta(w)$ vanishes in the large N_c limit, it should be read as

$$\frac{\beta(w)}{\eta(w)} = O(1/N_c). \tag{12}$$

As mentioned before, the lack of any absolute normalization of the form factors may limit the usefulness of these results. Hence it is tempting to assume heavy quark symmetry from the outset and perform a double expansion of $1/N_c$ and $1/m_Q$. This, however, depends on the commutativity of the heavy quark and the large N_c limit, which is a highly nontrivial assumption. It has been shown that the chiral limit and large N_c limit does not always commute [11–13], and the non-commutativity is embodied in this ratio:

$$d = \frac{m_\pi}{m_\Delta - m_N}, \tag{13}$$

the numerator and the denominator measuring the deviations from the chiral and large N_c limits respectively. So going to the chiral limit amounts to setting $d = 0$, while in the large N_c limit we have $d \to \infty$. Clearly these two conditions cannot be both satisfied in the same time. Experimentally $d \sim 0.5$, so the real world does not resemble any of these two

[2] I would like to thanks M. Lu for a discussion on this point.

limiting cases very closely. We will spend the rest of the letter discussing the similar non-commutativity issue between the heavy quark and the large N_c limit, and use the leading order corrections to the $\Lambda_b \to \Lambda_c$ decay as our prime example.

It has been proved that, when $m_b \to \infty$ and arbitrary m_c, the $\Lambda_b \to \Lambda_c$ decay is controlled by two form factors, which are denoted by $\eta(w)$ and $\beta(w)$ is Eq. (2). Then one can expand these form factors in orders of $1/m_c$.

$$\eta(w) = \eta_0(w) + \tfrac{1}{m_c}\eta_1(w) + \tfrac{1}{m_c^2}\eta_2(w) + \dots \ , \tag{14a}$$

$$\beta(w) = \beta_0(w) + \tfrac{1}{m_c}\eta_1(w) + \tfrac{1}{m_c^2}\beta_2(w) + \dots \ . \tag{14b}$$

In leading order of $1/m_c$, i.e., when the c quark is also heavy, we have $\beta_0(w) = 0$. The leading term of $\beta(w)$ appears at the first order of $1/m_c$, which is calculated in Ref. [14].

$$\beta_1(w) = -\eta_0(w)\delta(1+w)^{-1}, \tag{15}$$

where

$$\delta = \frac{\bar{\Lambda}}{m_c} = \frac{m_{\Lambda_c} - m_c}{m_c}. \tag{16}$$

Since the numerator $\bar{\Lambda} \sim N_c$, it seems that the first order correction diverges in the large N_c limit, in contradiction of our claim above that $\beta(w) = 0$ when N_c is large.

To get a better understanding of this paradox, one needs to retrace the derivation in Ref. [14], be careful to keep all the terms of positive powers of δ, which are legitimately discarded in Ref. [14] as $N_c = 3$ in their work. The $1/m_c$ correction arises from the matching of current in full QCD $\bar{c}\Gamma b$ to that in the effective theory $\bar{h}_c\Gamma h_b$. The effective field h_c is related to the quark field c in full QCD by

$$c = \exp(-im_c v' \cdot x)\left(1 - \frac{i\not{D}}{m_c}\right)^{-1} h_c. \tag{17a}$$

in the "Harvard formulation" of heavy quark effective theory, or alternatively

$$c = \exp(-im_c v' \cdot x)\exp\left(\frac{i\not{D}}{m_c}\right) h_c. \tag{17b}$$

in the "Mainz formulation"[3]. When acting on a hadron state, the derivative operator in the effective theory measures the residual momentum, which is typically of the order of the mass of the light degrees of freedom.

$$\frac{i\not{D}}{m_c} \sim \frac{\bar{\Lambda}}{m_c} = \delta. \tag{18}$$

It is clear that the heavy quark expansion is an expansion in δ. When $N_c = 3$, δ is formally small and Eqs. (17) reduces to the normal relation

$$c = \exp(-im_c v' \cdot x)\left(1 + \frac{i\not{D}}{m_c}\right) h_c. \tag{19}$$

When $N_c \to \infty$, however, the expansion scheme becomes ambiguous. The size of the correction term depends on how we approach the double limits of heavy quark and large N_c; the relative size of the two expansion parameters is described by the quantity δ. In the heavy quark limit one have $\delta = 0$, while with large N_c we get $\delta \to \infty$. The situation is analogous to that in Ref. [11–13], where the non-commutativity between the chiral and large N_c limits are characterized by the parameter d. Experimentally $\bar{\Lambda} \sim 700$ MeV while $m_c \sim 1500$ MeV, so $\delta \sim 0.5$ and the real world is not especially close to both limiting cases. Our studies suggested that the apparent divergence of $\beta(w)$ in the large N_c limit is an artifact of the truncation to the first term in the $1/m_c$ expansion, while the whole $\beta(w)$, with the contributions from all orders of $1/m_c$ summed, should be at most of order N_c^{-1}.

The discussion above shows that an $1/N_c$ expansion in the heavy quark limit is more subtle than one might naively expect. Since the heavy quark symmetry is more predictive than the large N_c limit for "heavy-to-heavy" decays, and since $\delta < 1$ suggested that the real world has more resemblance to the heavy quark limit than to the large N_c limit, we do not

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[3]See Ref. [15,16] for a comparison between the two formulations of heavy quark effective theory. The two different formulations give identical predictions to physical quantities like scattering matrix elements, so the choice of formulation should be immaterial.

expect that our results will be highly important to the understanding of these decays[4]. In our opinion, our results are best applied to "heavy-to-light" decays, where the large N_c result is more predictive than the heavy quark counterparts. Moreover, it is known that nine different form factors, four local and five non-local, are needed to parametrize the $1/m_Q$ corrections to the "heavy-to-light" decay form factors [17]. Since only six form factors are necessary in the most general formulation, it means that the heavy quark limit loses all predictive power for such transitions at order $1/m_Q$. Therefore it is interesting to study the $1/N_c$ corrections for such decays. If our scheme can retain its predictive power at $1/N_c$, it will be a practical alternative expansion scheme to study the "heavy-to-light" or even "light-to-light" transitions. So we conclude by stating that the large N_c limit simplifies weak transition matrix elements dramatically, but the $1/N_c$ corrections are not well understood yet and deserves further investigation.

ACKNOWLEDGMENTS

I am grateful to Ming Lu, Dan Pirjol and Tung–Mow Yan for discussions. This work is supported in part by the National Science Foundation.

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[4]One can certainly not exclude the possibility that a careful reformulation of the heavy quark effective theory may allow such a double expansion. Such pursuit, however, is beyond the scope of this letter.

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