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Peculiarities of Quantum Mechanics: Origins and Meaning

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Abstract

The most peculiar, specifically quantum, features of quantum mechanics — quantum nonlocality, indeterminism, interference of probabilities, quantization, wave function collapse during measurement — are explained on a logical-geometrical basis. It is shown that truths of logical statements about numerical values of quantum observables are quantum observables themselves and are represented in quantum mechanics by density matrices of pure states. Structurally, quantum mechanics is a result of applying non-Abelian symmetries to truth operators and their eigenvectors — wave functions. Wave functions contain information about conditional truths of all possible logical statements about physical observables and their correlations in a given physical system. These correlations are logical, hence nonlocal, and exist when the system is not observed. We analyze the physical conditions and logical and decision-making operations involved in the phenomena of wave function collapse and unpredictability of the results of measurements. Consistent explanations of the Stern-Gerlach and EPR-Bohm experiments are presented.

CONTENTS

Introduction

This paper analyzes only a small fragment of quantum mechanics, namely, quantum mechanics without second quantization and quantum field theory. However, most strange quantum properties — quantum nonlocality, indeterminism, interference of probabilities, quantization, collapse of wave functions during measurements, are exposed perfectly well in this fragment. Our analysis is not a new version of quantum mechanics, and not even a new interpretation; it provides only a consistent explanation of and a logical "picture" for the basic ideas of the Copenhagen interpretation.

It is shown here that strange quantum properties appear because, in quantum mechanics, non-Abelian symmetries are applied to truth operators of logical statements about numerical values of physical observables, while in classical mechanics, symmetries are applied to numerical values of observables themselves. These truth operators (as shown in Sec. 1) are also quantum observables, nonlocal by nature, and are represented in quantum mechanics by density matrices of pure states. The inherent contradiction between quantum logical nonlocality and classical locality of measurements leads to the appearance of noncomputable functions (see Secs. 3 and 5), and hence to indeterminism, probabilistic behavior of quantum systems, and wave function collapse. This does not mean, however, that we cannot measure nonlocal observables (as shown in Sec. 4).

The concept of probability makes sense only in connection with measurement, since truth operators — density matrices — develop deterministically between measurements, obeying quantum equations.

A given state vector of a physical system contains information about the conditional truths of all possible logical statements describing physical observables relevant to the system, provided that this system is in the given quantum state (see Sec. 2). And since such a state can be destroyed by a measurement, the concept of conditional truth applies only between measurements. So, all possible local and nonlocal correlations are present in the logic of the physical system before the system is observed. This logic, by its nature, does not need to be localized. The physical system itself may be placed somewhere, but in most cases a true statement about its position is not expressible (see below).

Although, numerically, conditional truths are defined in such a way that they are equal to the corresponding probabilities, this does not mean that nonlocal correlations observed in measurements are results of mutual influences of spatially separated measuring procedures. These correlations were born together with the initial state of the physical system. (The problem of the origins of nonlocal correlations has created a huge literature; we refer readers only to [1]. We analyze the EPR-Bohm experiments (Sec. 6) in connection with this problem.)

It follows from the foregoing that a quantum Hilbert space appears as a result of applying symmetry transformations to a full set of eigenvectors belonging to originally commuting truth operators of statements describing a given physical system. The result is a quantum logic of noncommuting truth operators. In this logic, symmetry transformations are merely redefinitions of meanings of "yes's" and "no's". From this point of view, the strangest quantum features are more linguistic than physical problems. Structurally, any quantum Hilbert space is a continuum of mutually noncommuting (though equivalent) classical logics and languages interconnected with each other by a set of conditional truths. By definition, conditional truths vary between 0 ("false") and 1 ("true"). The conditional truth of a complex statement containing simpler constitutent statements makes sense only if it does not depend on the order of those constituent statements. It is important that such independence exists — at least under special conditions. (This is discussed in Sec. 2.)

The question arises why logic and language play such a fundamental role in quantum mechanics, while in classical mechanics they play only an auxiliary one. The following is a qualitative explanation.

Though undescribed Nature certainly exists, scientific knowledge of Nature exists only in the form of logically organized descriptions. When these descriptions become "too precise" at some level of accuracy, the fundamental features of logic and language acquire the same importance as the features of what is being described. At this "micro"-level, we cannot separate the features of "matter per se" from the features of the logic and language used to describe it. In particular, two properties of classical mathematical logic are potentially "quantum" and become crucial at the micro-level of accuracy:

(a) The truth values of classical logic are naturally quantized.

(b) There exists a hidden, unformalized symmetry in classical logic — namely, any logical tautology remains the same tautology, regardless of how we change the meaning of the truth values of its constituent statements. The only requirement is that every newly redefined "no" be the negation of a correspondingly redefined "yes."

So we immediately get a quantum-like logical system when we formalize these properties (as done in [2]) using the assumptions that symmetries are linear, continuous and non-

Abelian.

The known features of quantum measurements are consistent with this logical picture. In quantum mechanics, when our measurements become "too precise," i.e., when they reach a quantum scale of accuracy, we cannot exclude the influence of measuring processes on measuring systems. As a result, we lose the possibility of unconditionally defining truth: the definition of truth now depends on how we observe the physical system, on our choice of apparatus. This logical relativism does not exist in classical mechanics, where logical statements are precise logically but not physically because they describe huge intervals of quantum numerical values, $\Delta S/\hbar \gg 1$; as a result, all truth operators of logical statements about observables commute and truth values are never redefined (transformed). Not being transformed, classical truth values do not depend on choice of apparatus.

In quantum mechanics, truths of logical statements about dynamic variables become dynamic variables themselves, because they depend on parameters of symmetry transformations that redefine truth values.

If the truths of statements become dynamic variables, then whose statements are they? The answer is that abstract sets of all possible languages and statements describing physical observables exist objectively, as do sets of the conditional truths of those statements, whereas the choice of a language (= quantum representation) and questions to be answered, as well as the formulation of statements describing the results of measurements, belong to the researcher.

Another question arises about whether we can verbally express statements when their truths are quantum operators. The answer is that since statements themselves are not operators, they can be (and always really are) expressed verbally in an ordinary way. That, say, p and q are noncommuting observables does not limit our formulations of any statements about exact numerical values of both noncommuting p and q. In fact, when describing any observable informally, we always use its own representation. The truths of such statements, however, cannot be expressed explicitly and simultaneously for both p and q. If, for example, it is true that $p = p_0$, then nothing precise can be said about the truth of the statement $q = q_0$. Still, we can describe numerically the conditional truth of this statement, under the condition $p = p_0$.

We should note that in [2] and [3], logical statements are not distinguished from their truth, so truth operators are simultaneously operators of statements themselves. This representation of statements by their truth values can be formalized (see, for example, [4], §45); however, in practice, such mixing can create ambiguity. In this paper, we avoid such mixing.

1. Truths of statements about observables, as observables. The meaning of density matrix and wave function.

Let K be an observable with a set of possible numerical values (quantum numbers), $\{k_1, k_2, \ldots\}$, and let a physical system be in state $\mid k_i >$. The logical statement Λ_{k_i},

$$\Lambda_{k_i}: \text{ "The system is in state } \mid k_i > \text{"} \quad , \tag{1a}$$

or, equivalently,

$$\Lambda_{k_i}: \quad \text{"}K = k_i\text{"} \quad , \tag{1b}$$

describes the real situation in this case and therefore is true.

We will prefer to evaluate truths of statements numerically; let the truth value "true" be assigned the number 1, and the truth value "false" the number 0. In our case, the truth of Λ_{k_i} is equal to 1.

This truth value can be confirmed by measurement. Our apparatus should measure K; if after (theoretically infinitely) many repetitions of the same experiment, we get the same number, $K = k_i$, then the truth of Λ_{k_i} is equal to 1, while all statements Λ_{k_j}, $k_j \neq k_i$, are false and their truths are equal to zero. Thus, the truth of Λ_{k_i} is measured simultaneously with K and is an observable which can be called a logical observable. We can represent this observable by a Hermitian "truth operator", $\hat{\Lambda}_{k_i}$, commuting with the operator $\hat{K}$ representing observable K.

Since any truth operator, $\hat{M}$, possesses only two exact numerical values, 1 and 0, it is a projector:

$$\hat{M}^2 = \hat{M} \quad . \tag{2}$$

Since all $\hat{\Lambda}_{k_i}$, $i = 1, 2, \ldots$, commute with $\hat{K}$, the eigenvectors of $\hat{\Lambda}_{k_i}$ and $\hat{K}$ are the same, so $\hat{\Lambda}_{k_i} \mid k_j >= \delta_{k_i k_j} \mid k_j >$, or, in matrix form,

$$\sum_{k'} \Lambda_{k_i}(k, k') \mid k_j(k') >= \delta_{k_i k_j} \mid k_j(k) > \quad ; \quad \delta_{k_i k_j} = \begin{cases} 1, & k_j = k_i \\ \\ 0, & k_j \neq k_i \end{cases} \quad . \tag{3a}$$

Here $\delta_{k_i k_j}$ is an eigenvalue of $\hat{\Lambda}_{k_i}$. It follows from (3a) that for any $\mid \psi >= \sum a_j \mid k_j >$,

$$\sum_i \Lambda_{k_i} \mid \psi >=\mid \psi > \quad . \tag{3b}$$

In the diagonal K-representation, $\mid k_j(k) >= \delta_{k k_j}$; using this in (3a) we get $\hat{\Lambda}_{k_i}$ in the diagonal K-representation:

$$\Lambda_{k_i}(k, k') = \delta_{k k_i} \delta_{k' k_i} \quad ; \tag{4}$$

in this representation, all matrix elements of $\hat{\Lambda}_{k_i}$ equal 0 except for a single 1 at the i-place on the main diagonal. This means that the truth operators are density matrices of pure states; in the diagonal representation,

$$\Lambda_{k_i}(k, k') =\mid k_i(k) >< k_i(k') \mid \quad . \tag{5}$$

From (4) and (5), we get:

$$tr \hat{\Lambda}_{k_i} = 1 \quad , \tag{6}$$

$$\begin{aligned} \hat{\Lambda}_{k_i} \hat{\Lambda}_{k_j} &= \textstyle\sum_{k'} \mid k_i(k) >< k_i(k') \mid k_j(k') >< k_j(k'') \mid = \\ &= \delta_{k_i k_j} \hat{\Lambda}_{k_i} \quad , \end{aligned} \tag{7}$$

where $\hat{\Lambda}_{k_i} \hat{\Lambda}_{k_j}$ is the matrix product. And

$$\hat{K} = \sum_i k_i \hat{\Lambda}_{k_i} \quad . \tag{8}$$

In Sec. 2 we will investigate complex statements composed of elementary statements represented in quantum mechanics by density matrices of pure states. Equation (6) is valid only for elementary statements. Equation (7) describes the features of logical conjunction, "Λ_{k_i} and Λ_{k_j}", composed of two elementary statements. Repetition of the same statement, Λ_{k_i}, i.e., "Λ_{k_i} and Λ_{k_i}" is logically equal to the same statement; and in such a case $\delta_{k_i k_i} = 1$, $\hat{\Lambda}_{k_i}\hat{\Lambda}_{k_i} = \hat{\Lambda}^2_{k_i}$, and (7) coincides with (2). If $k_i \neq k_j$, then Λ_{k_i} and Λ_{k_j} are mutually exclusive statements; therefore, "Λ_{k_i} and Λ_{k_j}" should be false and, correspondingly, $\hat{\Lambda}_{k_i}\hat{\Lambda}_{k_j} = 0$. Equation (8) gives a logical meaning for quantum operators representing physical observables.

Let a physical system be in state $\mid \psi >$, which may be an eigenstate of some physical observable, Q, so $\mid \psi >\equiv\mid q >$, where q is a numerical value of Q in this state. According to (8), the average value of K in state $\mid q >$, $< q \mid \hat{K} \mid q >$, is equal to

$$< q \mid \hat{K} \mid q >\equiv tr\left(\hat{\Lambda}_q \hat{K}\right) = \sum_i k_i tr\left(\hat{\Lambda}_q \hat{\Lambda}_{k_i}\right) \quad , \tag{9}$$

where $tr(\hat{\Lambda}_q\hat{\Lambda}_{k_i}) \equiv < q \mid \hat{\Lambda}_{k_i} \mid q > = \mid < q \mid k_i > \mid^2$,

$$\hat{\Lambda}_q = \mid q >< q \mid \quad , \quad \Lambda_q \ : \quad \text{“}Q = q\text{”} \ . \tag{10}$$

$tr(\hat{\Lambda}_q\hat{\Lambda}_{k_i})$ is the average truth of Λ_{k_i} when Λ_q is true; but we can say as well that it is the conditional truth of Λ_{k_i}, $Tr(\Lambda_q \mid \Lambda_{k_i})$, under the condition that Λ_q is true; it would also be correct to say that $tr(\hat{\Lambda}_q\hat{\Lambda}_{k_i})$ is the level of confidence in the value $K = k_i$ when the physical system is in state $\mid q >$. Thus,

$$Tr(\Lambda_q \mid \Lambda_{k_i}) = tr\left(\hat{\Lambda}_q \hat{\Lambda}_{k_i}\right) \quad . \tag{11}$$

(Note that for two elementary statements, Λ_q and Λ_{k_i}, $Tr(\Lambda_q \mid \Lambda_{k_i}) = Tr(\Lambda_{k_i} \mid \Lambda_q)$.)

The logical meaning of formula (9) is the following: when calculating the average K in state $\mid q >$, we need to take every k_i with its level of confidence, and then summarize the terms.

As a rule, the density matrices $\hat{\Lambda}_q$ and $\hat{\Lambda}_{k_i}$ in (11) should be taken at the same time, although there are cases in which it is not essential. What is essential, however, is that <u>both</u> Λ_q and Λ_{k_i} describe possible physical situations <u>between</u> measurements, i.e., when the physical system is not observed. If Λ_q describes quantum state $\mid q >$ before the measurement of observable K, which does not commute with Q, and Λ_{k_i} describes a state created by a single measurement, then it makes no sense to talk about the conditional truth of Λ_{k_i} in the state $\mid q >$ that has already been destroyed by the measurement. In this case $tr(\hat{\Lambda}_q\hat{\Lambda}_{k_i})$ equals probability $w(\Lambda_q \mid \Lambda_{k_i})$ that $K = k_i$ will be the result of the measurement, if before this measurement the system was in state $\mid q >$. Although the formulae are the same

$$Tr(\Lambda_q \mid \Lambda_{k_i}) = w(\Lambda_q \mid \Lambda_{k_i}) = tr(\hat{\Lambda}_q\hat{\Lambda}_{k_i}) \equiv \mid < q \mid k_i > \mid^2 \quad , \tag{12}$$

the physical situations and hence the meanings are different. This difference is essential in explaining the main peculiarity of the EPR-Bohm experiments: the allegedly "faster-than-light" mutual influence of two spatially separated actions of measurements.

(Note that Tr and w in (12) are equal only because we chose truth values 1 for "true" and 0 for "false". If we followed the choice given, say, in [4], equation (12) would be different.)

We see from this analysis that quantum density matrices of pure states represent logical truths of statements describing those states. And both density matrices and wave functions — the eigenvectors of density matrices — contain information about conditional truths of all possible logical statements about observables of a given physical system. If the system wave function is $\mid \psi >$, then the conditional truth of a statement M is equal to $Tr(\Lambda_\psi \mid M) = tr(\hat{\Lambda}_\psi \hat{M})$, where Λ_ψ describes the condition: "The system is in state $\mid \psi >$". Although $tr(\hat{\Lambda}_\psi \hat{M})$ does not depend on the order of cofactors, the order in $Tr(\Lambda_\psi \mid M)$ can be important: Λ_ψ is always an elementary statement, $tr\hat{\Lambda}_\psi = 1$, while $\hat{M}$ can be any complex statement. For any statement M,

$$0 \leq Tr(\Lambda_\psi \mid M) \leq 1 \quad . \tag{13}$$

M and Λ_ψ need not be the same when $Tr = 1$, since M can be a disjunction containing Λ_ψ; they are mutually exclusive when $Tr = 0$.

One type of complex statement plays an important role in the transition from quantum to classical descriptions:

$$\Delta \hat{I}_k = \sum_{i_1}^{i_2 = i_1 + N} \hat{\Lambda}_{k_i} \quad , \quad N \gg 1 \quad , \tag{14}$$

$$\Delta I_k \quad : \quad \text{“}\Lambda_{k_1} \text{ or } \Lambda_{k_2} \text{ or } \ldots \text{or } \Lambda_{k_{1+N}}\text{”} \quad . \tag{15}$$

(See Sec. 2 for proof that truth operator (14) represents statement (15).) (15) describes a big interval of neighboring quantum numbers of observable K. Consider ΔI_p and ΔI_q, where p and q are generalized momentum and coordinate. Defining p as a translational invariant in q-space (see Sec. 3) we can find $\hat{\Lambda}_{p_i}$ and $\hat{\Lambda}_{q_j}$ in the same representation. It is then easy to see that if intervals Δq, Δp in (14) are such that $\Delta S/\hbar \gg 1$, then $\Delta \hat{I}_p$ and $\Delta \hat{I}_q$ commute. Obviously, classical trajectories are distinguished from each other, if $\Delta S/\hbar \ll S/\hbar$, where S is a typical action of a given physical system. Under these conditions, the description becomes classical. Thus, the classical picture of the world appears when physical differences between micro-states inside macro-intervals (15) either are not essential or cannot be resolved, and $1 \ll \Delta S/\hbar \ll S/\hbar$.

It follows from the above that quantum Hilbert space is an information-space in the sense explained earlier.

2. Representations of complex statements. Logical correlations between noncommuting observables.

The simplest complex statement is a mixture of mutually exclusive statements. If wave functions $\mid k_j >, j = 1, 2, \ldots$, represent a full set of orthonormalized states, then a mixture of Λ_{k_j}'s is, by definition, a probabilistic distribution of statements $\Lambda_{k_j}, j = 1, 2, \ldots$; w_j's are classical probabilities in the sense that they result from lack of knowledge. We will denote the mixture of N statements with probabilities $w_1, \ldots, w_N$ by $\rho_{w_1,w_2,\ldots,w_N}$. The corresponding truth operator is a mixed density matrix (which can also be considered a probability operator):

$$\hat{\rho}_{w_1,w_2,\ldots,w_N}(x, x') = \sum_j w_j \hat{\Lambda}_{k_j}(x_1 x') \quad . \tag{16}$$

Our next aim is to find the truth operators of complex statements composed of more simple statements, by using logical connectives; negation of a statement M (denoted by $\bar{M}$); "and" (denoted by $\wedge$); "or" ($\vee$, inclusive); implication ($\rightarrow$); etc. Quantifiers $\forall$, $\exists$ are not discussed.

The formulae for the truth operators of complex statements are well known [2,3]; they coincide with the formulae for the probabilities of complex events. However, these formulae are not formally deduced in [2,3]; moreover, it is assumed in [3] that <u>all</u> projectors can be interpreted as propositions. Thus far we have proved only that density matrices of pure states can be interpreted as truth operators of corresponding propositions (statements). In this section, which relates to noncommuting truth operators, it will be shown that when the necessary conditions (24) are met, the formulae for complex statements are valid for noncommuting observables.

Let M_1, M_2 be two logical statements about quantum observables of a given physical system. We will first assume that their truth operators commute, $[\hat{M}_1, \hat{M}_2] \equiv \hat{M}_1 \hat{M}_2 - \hat{M}_2 \hat{M}_1 = 0$, and then find the conditions under which the formulae for commuting truth operators are also valid for noncommuting $\hat{M}_1$ and $\hat{M}_2$.

If N mutually orthogonal states, $\mid \psi_n >$, $n = 1, 2, \ldots, N$, represent a basis of an N-dimensional complex vector space, then there also exist N mutually exclusive elementary statements, $\Lambda_{\psi_n}, \hat{\Lambda}_{\psi_n} = \mid \psi_n >< \psi_n \mid$, $tr\hat{\Lambda}_{\psi_n} = 1$. They constitute a basic language for constructing 2^N different statements, M_i, $i = 1, 2, \ldots, 2^N$, with the help of logical connectives. All M_i's commute, $\hat{M}_i^2 = \hat{M}_i$. (There is a continuum of such statements when $N = \infty$, but we will have no problems with it, if we do not operate with the full set of these statements.) 2^N is the number of different distributions of 0's and 1's along the main diagonals of density matrices representing different M_i's in their common diagonal representation. The full set of these 2^N statements forms what we can call a classical logic of quantum propositions in a given representation. Then a transition to another quantum representation (which is some unitary transformation in the vector space) will provide us with another classical logic having its own 2^N mutually commuting different statements. Most of the truth operators of these statements, however, will not commute with the truth operators of the first representation; important exceptions are logical tautologies, which are invariant in all representations.

Thus, in any quantum vector space with $N > 1$, there is an infinite number of mutually noncommuting classical logics that are transformed into each other by symmetry transformations. These quantum transformations can be interpreted as redefinitions of logical truth values. Assuming $[\hat{M}_1, \hat{M}_2] = 0$ we consider only one of such logics.

Any nonelementary truth-operator containing $\hat{M}_1$ and $\hat{M}_2$, $\hat{M}_i^2 = \hat{M}_i$, can be written as

$$\hat{M} = a + b\hat{M}_1 + c\hat{M}_2 + d\hat{M}_1\hat{M}_2 \quad . \tag{17}$$

Indeed, all higher terms for commuting operators obeying (2) can be reduced to (17). Using (2), we get from (17) the equation $a^2 + b(b+2a)\hat{M}_1 + c(c+2a)\hat{M}_2 +$ $+ [d^2 + 2(ad + bc + bd + cd)]\, \hat{M}_1\hat{M}_2 \; = \; a + b\hat{M}_1 + c\hat{M}_2 + d\hat{M}_1\hat{M}_2$. If $\hat{M}_{1,2} \neq \hat{0}, \hat{1}$ (these are zero and identity operators), then

$$a^2 = a,\ b^2 + 2ab = b,\ c^2 + 2ac = c,\ d^2 + 2(ad + bc + bd + cd) = d \quad . \tag{18}$$

Different solutions of these equations give us truth operators for different logical connectives. Identification of these connectives is based on the conventional logical truth tables given in textbooks. We will demonstrate the process of identification in cases of negation and conjunction.

The symbol $\Rightarrow$ means here "is represented by the truth operator." For example: $M \Rightarrow \hat{M}$ means "M is represented by the truth operator $\hat{M}$."

Negation. $a = 1, b = -1, c = d = 0$ (negation of M_1 denoted by $\bar{M}_1$).

$$M = \bar{M}_1 \Rightarrow \hat{1} - \hat{M}_1 \quad . \tag{19}$$

According to the truth table defining negation, if M is true, then $\bar{M}$ is false, and vice versa. In quantum mechanics, "M_1 is true" means that our physical system is in such a state $\mid \psi >$ that $\hat{M}_1 \mid \psi >=\mid \psi >$. Then $\hat{\bar{M}}_1 \mid \psi >= (1 - \hat{M}_1) \mid \psi >= 0$, which means that formula (19) is correct.

When $N > 2$ and $M_1 = \Lambda_{\varphi_n}$ (an elementary statement), $\bar{M}_1$ is not elementary since there is more than one state orthogonal to $\mid \varphi >$. Every Λ_{φ_k}, $k \neq n$, is a "partial negation" of Λ_{φ_n}, since Λ_{φ_k} declares that the system is in state $\mid \varphi_k >$, which excludes the presence of state $\mid \varphi_n >$. In (19), $\hat{\bar{M}}_1$ represents "total negation" (or, simply, negation): if $M_1 = \Lambda_{\varphi_n}$, then

$$\bar{M}_1 = \text{“}\Lambda_{\varphi_1} \vee \Lambda_{\varphi_2} \vee \ldots \vee \Lambda_{\varphi_{n-1}} \vee \Lambda_{\varphi_{n+1}} \ldots \text{”} \quad , \quad \bar{M}_1 \Rightarrow \hat{\bar{M}}_1 = \sum_{k \neq n} \hat{\Lambda}_{\varphi_k} \quad ; \qquad (20)$$

see the explanation of "disjunction" below.

From (19) we get the conditional truth for negation when the physical system is in state $\mid \psi >$:

$$Tr(\Lambda_\psi \mid \bar{M}) = 1 - Tr(\Lambda_\psi \mid M) = 1 - tr(\hat{\Lambda}_\psi \hat{M}) \quad . \qquad (21)$$

It is interesting to note that M and $\bar{M}$ may have the same conditional truth, 0.5.

Conjunction. $a = b = c = 0, d = 1$.

$$M = M_1 \wedge M_2 \Rightarrow \hat{M}_1 \hat{M}_2 \ ; \ \ Tr\left(\Lambda_\varphi \mid M_1 \wedge M_2\right) =< \varphi \mid \hat{M}_1 \hat{M}_2 \mid \varphi >= tr\left(\hat{\Lambda}_\varphi \hat{M}_1 \hat{M}_2\right) \quad . \ (22)$$

M is true in state $\mid \varphi >$ if both M_1 and M_2 are true in that state, and false if at least one of them is false. This corresponds to the conventional truth table for "and." In (22), M_1, M_2 can be any statements about two coexisting quantum numbers, and $\mid \varphi >$ is an arbitrary state; therefore, the commuting $\hat{M}_1, \hat{M}_2$ may or may not commute with $\hat{\Lambda}_\varphi$.

If $\hat{M}_1, \hat{M}_2$ do not commute, then $\hat{M}_1 \hat{M}_2$ does not represent any statement, since $(\hat{M}_1 \hat{M}_2)^2 \neq \hat{M}_1 \hat{M}_2$. The average, $< \varphi \mid \hat{M}_1 \hat{M}_2 \mid \varphi >$, where $\hat{M}_1$, $\hat{M}_2$ are placed in a certain order, can of course exist. And again, this average <u>can</u> be interpreted as the conditional truth of the statement "M_1 and M_2" in state $\mid \varphi >$, if at least one of the truth operators, $\hat{M}_1$ or

$\hat{M}_2$, commutes with Λ_φ. If, say, $\hat{M}_1\hat{\Lambda}_\varphi = \hat{\Lambda}_\varphi\hat{M}_1$, then $\mid \varphi >$ is their common state and $\hat{M}_1 \mid \varphi >= \lambda \mid \varphi >, \quad \lambda = 0, 1$. Therefore,

$$Tr\left(\Lambda_\varphi \mid M_1 \wedge M_2\right) = \lambda < \varphi \mid \hat{M}_2 \mid \varphi > \ ; \quad \hat{M}_1 \mid \varphi >= \lambda \mid \varphi > \ , \quad \hat{M}_1\hat{M}_2 \neq \hat{M}_2\hat{M}_1 \ . \quad (23)$$

The case is clear: when, for example, $\lambda = 1$, and $\hat{\Lambda}_\varphi\hat{M}_1 = \hat{M}_1\hat{\Lambda}_\varphi$, operator $\hat{M}_1$ does not influence state $\mid \varphi >$ and therefore can be ignored. When $\lambda = 0$, the statement "$M_1 \wedge M_2$" is false because M_1 is false in state $\mid \varphi >$, in which Λ_φ is true; therefore, $Tr(\Lambda_\varphi \mid M_1 \wedge M_2) = 0$. In such cases the order of cofactors in (23) can be arbitrary. It is easy to see that the most general conditions under which the order of $\hat{M}_1$ and $\hat{M}_2$ is not important and, therefore, the statement "M_1 and M_2" *effectively* makes sense, are described by the equations

$$Tr\left(\hat{\Lambda}_\varphi\left[\hat{M}_k\hat{M}_\ell\right]\right) \equiv Tr\left(\left[\hat{\Lambda}_\varphi\hat{M}_k\right]\hat{M}_\ell\right) \equiv Tr\left(\hat{M}_k\left[\hat{M}_\ell\hat{\Lambda}_\varphi\right]\right) = 0 \quad , \quad k \neq \ell = 1 \text{ or } 2 \ . \quad (24)$$

(24) provides the only logical restriction on meaningful logical correlations between any two noncommuting logical observables, $\hat{M}_1$ and $\hat{M}_2$ (as well as between physical observables $K_1, \ [\hat{K}_1\hat{M}_1] = 0$, and $K_2, \ [\hat{K}_2\hat{M}_2] = 0$), when the physical system is in state $\mid \psi >$.

Let state $\mid \psi >$ of a system of two identical particles consist of two macroscopically separated branches. So Λ_ψ is "nonlocal" in the sense that it describes both branches simultaneously. While every particle may be localized somewhere (see the discussion about this in Sec. 4), we cannot formulate <u>true</u> statements about their localization when the nonlocal Λ_ψ is true. But we can formulate hypothetical statements, M_1 and M_2, about physically possible states in which M_1 and M_2 are localized in different branches. If M_1 and M_2 relate to relativistically nonoverlapping places, $\hat{M}_1$ and $\hat{M}_2$ commute. Therefore, the statement "M_1 and M_2" makes sense, and we can calculate its conditional truth (22). The value we will get belongs to the nonobservable physical system. (For more about this, see Sec. 6.)

It can be proved, however, that if a system is in state $\mid \psi >$ before our observation, and the result of the observation is described by statement M, then the probability of such a result is equal to $tr(\hat{\Lambda}_\psi \hat{M})$ [3], i.e., equal to the conditional truth of M in state $\mid \psi >$. Our interpretation of this equality is that conditional truth is measurable and $\hat{M}$ is an observable. Now, if $M =$ "M_1 and M_2", then the measurement of $\hat{M}$ means the measurement of the correlation between $\hat{M}_1$ and $\hat{M}_2$, which is a typical EPR situation. Can the measured correlation be a result of a long-range, faster-than-light interaction between two spatially separated measuring processes? Of course not: these long-distance correlations were born together with state $\mid \psi >$, existed before the measurement, and were only confirmed by measurement. (Nevertheless, the phenomenon of the faster-than-light disappearance of the initial information contained in the pre-measured state $\mid \psi >$ exists. It will be explained in Sec. 4.)

In other complex statements investigated below we will meet the same restriction (24) on "M_1 and $\hat{M}_2$" when $\hat{M}_1$ and $\hat{M}_2$ do not commute.

Disjunction. $a = 0, \; b = c = 1, \; d = -1.$

$$M = M_1 \vee M_2 \Rightarrow \hat{M}_1 + \hat{M}_2 - \hat{M}_1 \hat{M}_2 \quad . \tag{25}$$

If M_1 and M_2 are mutually exclusive, then $\hat{M}$ degenerates into the sum, $\hat{M} = \hat{M}_1 + \hat{M}_2$; this explains formula (20). Since the disjunction of a full set of elementary statements Λ_{ψ_i}, $\hat{\Lambda}_{\psi_i} =\mid \psi_i >< \psi_i \mid$, $i = 1, 2, \ldots, N$, is merely the enumeration of all mutually exclusive quantum states, such a disjunction is an invariant: $\sum_{i=1}^{N} \hat{\Lambda}_{\psi_i} = \hat{1}$. In the diagonal representation, any

$\hat{M}$ can be represented as some distribution of 1's and 0's along the main diagonal of the matrix representing M. The "total negation" of M, $\bar{M} \Rightarrow \hat{\bar{M}}$, contains 0's instead of 1's and 1's instead of 0's; so $\hat{\bar{M}} = \hat{1} - \hat{M}$.

The identity matrix, $\hat{1}$, is a purely logical invariant. Other invariants are defined by irreducible representations of physical symmetries of a given physical system. For example, the statement Λ_{p_k},

$$\Lambda_{p_k}\text{: "The momentum of the physical system is equal to } p_k\text{"} \quad ,$$

is a translational invariant. The truth operator $\hat{\Lambda}_{p_k}(q_1 q')$ in the coordinate q-representation is not diagonal, that is, Λ_{p_k} is not local. Finding this matrix is one of the methods of quantization which will be demonstrated in the next section.

Exclusive "or". $a = 0, b = c = 1, d = -2$.

$$M = M_1 \text{ or } M_2 \text{ but not both } \Rightarrow \hat{M}_1 + \hat{M}_2 - 2\hat{M}_1\hat{M}_2 \quad . \tag{26}$$

Implication. $a = 1, b = -1, c = 0, d = 1$.

$$M = M_1 \to M_2 \Rightarrow \hat{1} - \hat{M}_1 + \hat{M}_1\hat{M}_2 \quad . \tag{27}$$

By definition,

$$Tr(\Lambda_\varphi \mid M_1 \to M_2) \;\text{ and }\; w\left(\Lambda_\varphi \mid M_1 \to M_2\right) = 1 \text{ , if} \begin{cases} \hat{M}_1 \mid \varphi >= 0 \text{ ,} & \text{or} \\ \hat{M}_2 \mid \varphi >=\mid \varphi > & . \end{cases} \tag{28}$$

In (28), the commutation of $\hat{M}_1$ and $\hat{M}_2$ is not required, since (24) is fulfilled. In general,

$$w\left(\Lambda_\varphi \mid M_1 \to M_2\right) = 1 - w\left(\Lambda_\varphi \mid M_1\right) + w\left(\Lambda_\varphi \mid M_1 \wedge M_2\right) \quad . \tag{29}$$

So if, for example, $\hat{M}_1\hat{\Lambda}_\varphi = \hat{\Lambda}_\varphi\hat{M}_1$, then $w(\Lambda_\varphi \mid M_1 \to M_2) = 1 - \lambda + \lambda w(\Lambda_\varphi \mid M_2)$, $\hat{M}_1 \mid \varphi >= \lambda \mid \varphi >$. If M_1 is true in state $\mid \varphi >$, then $w(\Lambda_\varphi \mid M_1 \to M_2) =< \varphi \mid \hat{M}_2 \mid \varphi >$, the same as for the conjunction $M_1 \wedge M_2$, and for measuring M_2 in state $\mid \varphi >$. Thus, if $M_1 \to M_2$ is true and M_1 is true, then M_2 is true (modus ponens).

Equivalence. $a = 1, b = c = -1, d = 2$.

$$M = M_1 \longleftrightarrow M_2 \Rightarrow \hat{1} - \hat{M}_1 - \hat{M}_2 + 2\hat{M}_1\hat{M}_2 \quad . \tag{30}$$

If M_1 is true in state $\mid \psi >$, then, again, $w(\Lambda_\psi \mid M_1 \leftrightarrow M_2) =< \psi \mid \hat{M}_2 \mid \psi >$.

Other choices of a, b, c, d correspond to the substitutions $M_{1,2} \to M_{2,1}$, or $M_{1,2} \to \bar{M}_{1,2}$.

3. Origins of quantization. Appearance of noncomputable functions. Quantum Hilbert space.

Here we will explain why the unification of a purely geometrical concept of symmetry with a purely logical concept of truth is inconsistent with the classical concept of trajectories and, together with the assumption of linearity, leads to quantization.

Consider a pair of canonically conjugate observables, generalized momentum p and coordinate q. In classical mechanics, there are two definitions of p: (1) $p = \dot{q}$ $(m = 1)$, and (2) p is an integral of motion in a homogeneous space. These are two dynamic definitions of p; however, the second definition is geometrical as well, since the homogeneity of space means its translational symmetry. Let us now discard the dynamic part of the second definition, and introduce a purely geometrical concept of momentum:

Momentum p is an invariant of translational symmetry.

Another conceptual departure from classical mechanics follows from a realistic look at scientific theory. We have nothing to say about undescribed matter besides the fact that it exists; our scientific theories are about our own logically organized descriptions of observed and nonobserved subjects, and our predictions about results of future observations of the latter. Quantum mechanics is a partial realization of the above geometrical and logical concepts.

Let us introduce a logical statement: Λ_{p_i} : "$p = p_i$", where p_i is a possible numerical value of p, one of the values which can appear in our observations. We will analyze the logical truth of Λ_{p_i}, $\hat{\Lambda}_{p_i}$, as a basic variable.

Since p_i is a translational invariant, the truth of the statement "$p = p_i$" is also a translational invariant. Since we accept two possible numerical values for truth, 1 and 0, we may express the truth of Λ_{p_i}, $\hat{\Lambda}_{p_i}$, as a matrix diagonal operator, $\Lambda_{p_i}(p, p') = \delta_{pp_i}\delta_{p'p_i}$, that should not depend on q. $\hat{\Lambda}_p$ is defined in p-space. In classical mechanics we have, however, another definition of p: $p_i = dq/dt = \lim(q_{k+1} - q_k)/(t_{k+1} - t_k)$, where p is defined in q-space. We should now reject this definition, assuming instead that, in q-space, the truth of Λ_{p_i}, $\hat{\Lambda}_{p_i}$ must not depend on translations of coordinates, $q_k \to q_{k+1} = q_k + \delta q$. Calculation of matrix $\Lambda_{p_i}(q, q')$ under this condition, plus two other conditions, $\hat{\Lambda}^2_{p_i} = \hat{\Lambda}_{p_i}$ and $tr\hat{\Lambda}_{p_i} = 1$, gives:

$$\Lambda_{p_i}(q, q') = A \exp\{2\pi i(q' - q)/\lambda_i\} \quad , \tag{31}$$

where λ_i is a wave length depending on p_i and A is a normalization constant. On the basis of an analogy between p and q we conclude that $\lambda_i \propto 1/p_i$; and from experiment, $2\pi/\lambda_i = p_i/\hbar$. Indeed, q is a translational invariant in p-space, like p in q-space. In classical mechanics, at a turning point, $\partial q/\partial p = 0$ along the trajectory in phase space. And in nonrelativistic quantum mechanics, in order to measure a particle coordinate q, we need to stop this particle for an instant, thus creating a turning point and an instantaneous translational invariance along the p-axis.

Since $\hat{\Lambda}_{p_i}(p)$, with matrix elements $\Lambda_{p_i}(p, p')$, and $\hat{\Lambda}_{p_i}(q)$, with matrix elements $\Lambda_{p_i}(q, q')$, both represent the truth of statement "$p = p_i$" in different spaces, there should be a connection between them. The connection is:

$$\Lambda_{p_i}(q, q') = \sum_{p,p'} S(q, p)\Lambda_{p_i}(p, p')S^{+}(q', p') \quad , \tag{32}$$

$$S(q, p) = \sqrt{A} \exp iqp/\hbar \ ; \quad S^{+} = S^{-1} \quad . \tag{33}$$

Note that the unitarity (33) is derived from (31), and not postulated.

The eigenvectors of $\hat{\Lambda}_{p_i}(p)$ and $\hat{\Lambda}_{p_i}(q)$, with the eigenvalue $+1$, are p- and q-representations of the same wave function, ψ_{p_i}, corresponding to momentum $p = p_i$:

$$\psi_{p_k}(q) = S\psi_{p_k}(p) = \sqrt{A}\exp ip_k q/\hbar \ ; \quad \psi_{p_k}(p) = \delta_{pp_k} \quad . \tag{34}$$

The quantization is virtually completed. Using, now, the definition of p,

$$\hat{p} = \sum p_i \hat{\Lambda}_{p_i} \quad , \tag{35}$$

we will find that $\hat{p} = -i\hbar\partial/\partial q$. We have also got a complex, infinite (hence Hilbert) vector space with unitary transformations in it. As noted in the Introduction, the very possibility of forming such spaces is contained in the classical logic of propositions. Indeed, if we represent truths of propositions as matrices, then we can immediately conclude that geometrical transformations $\hat{M}' = S\hat{M}S^{-1}$ give $(\hat{M}')^2 = \hat{M}^2$; $tr\hat{M}' = tr\hat{M}$, and $\hat{I}' = \hat{I}$, where $\hat{I}$ represents a logical tautology. Also, using the formulae for complex statements deduced in Sec. 2, and the standard textbook tables of logical tautologies, we can confirm that every logical tautology is true in every state $\mid \psi >$. Or, using the technique of forming full sets of mutually exclusive statements described in [3], we will see that all tautologies can be represented as identity matrices.

A crucial point in our deduction of quantization has been the separation of a purely geometrical symmetry from particle dynamics, i.e., from symmetry and the very existence of a Hamiltonian. Using as an example angular momentum, (M_x, M_y, M_z) and rotational SU_2 symmetry, we will now show how noncomputable functions appear in such mechanics. These functions are crucial to explaining the phenomenon of quantum indeterminism (Sec. 5).

M_z is defined now only as an invariant of axial symmetry about the z-axis; M_x and M_y are defined correspondingly. Let $M_z = m$. Then m is an invariant of rotations around the z-axis, as is the truth of the statement Λ_m, Λ_m: "$M_z = m$". Using the same equations for the truth of Λ_m, $\hat{\Lambda}_m$, that we used for Λ_{p_i} in the case of momentum $p = p_i$ and applying the condition of periodicity in this case, we get the matrix $\Lambda_m(\varphi, \varphi')$, where φ is the angle of rotation around the z-axis,

$$\Lambda_m(\varphi, \varphi') = \frac{1}{2\pi} e^{im(\varphi' - \varphi)} = \mid m >< m \mid \quad , \tag{36}$$

$$\mid m >= \frac{1}{\sqrt{2\pi}} e^{im\varphi} \quad , \quad m \text{ is integer.} \tag{37}$$

Since angle φ is uncertain (truth operator $\hat{\Lambda}_m$ is not diagonal), the directions of the x- and y-axes are also uncertain; therefore, M_x and M_y cannot have exact numerical values simultaneously with M_z.

Let us now measure $M_z/\hbar$, at first about one axis, z_1, and then about another, z_2. Let the maximal number of possible M_z-projections equal some N. In SU_2, there exists a rotation G (around the axis perpendicular to both z_1 and z_2) that transforms rotations R_{z_1} into rotations R_{z_2},

$$R_{z_2} = G R_{z_1} G^{-1} \ ; \quad R_{z_2} R_{z_1} - R_{z_1} R_{z_2} \neq 0 \quad , \tag{38}$$

and G can be represented in the form

$$G = G(\alpha_{z_1 z_2}) = \exp(i \alpha_{z_1 z_2} \hat{g}) \quad , \tag{39}$$

$\hat{g}$ is a Hermitian operator.

The symmetry in this case means that the number and values of the angular momentum projections onto axis z_1 are the same as the number and values of projections onto the equivalent axis, z_2. (This is radically different from classical mechanics.) Suppose there exists a computable function which maps one-to-one $(M_{z_1})_i \to (M_{z_2})_j$, i , $j = 1, 2, \ldots, N$. There are $N!$ versions of such mappings; every one is a permutation, $i \to j; i, j = 1, 2, \ldots, N$. Computability means that for every $\alpha_{z_1 z_2}$ we can calculate the corresponding permutation.

Our proof of noncomputability will be rather informal, and will not address the theory of recursive functions or Turing machines. Suppose that at least for some $\alpha_{z_1 z_2}$ the permutation is not the identity permutation, i.e., not $i \to i, i = 1, \ldots, N$. Let us divide $\alpha_{z_1 z_2}$ into $N!$ intervals, $\Delta\alpha = \alpha_{z_1 z_2}/N!$. So,

$$G(\alpha_{z_1 z_2}) = (G(\Delta\alpha))^{N!} \quad .$$

Every small rotation $G(\Delta\alpha)$ corresponds to some permutation (depending only on $\Delta\alpha$), $P(\Delta\alpha)$, and the full rotation to the full permutation, i.e.,

$$P^{N!}(\Delta\alpha) = P(\alpha_{z_1 z_2}) \quad .$$

However, $P^{N!}(\Delta\alpha) = \hat{1}$, identity transformation. *Contradiction.*

Thus, there does not exist a one-to-one computable function that translates certain M_{z_1} projections into certain M_{z_2} projections. (The simplest case $M_z = \pm\frac{1}{2}$ is analyzed in Sec. 5.)

It does not follow immediately from this result that quantum mechanics is indeterministic. We need to show why and how, when measuring the z_2-projection of angular momentum initially directed along the z_1-axis, we should and can exclude the possibility of quantum superpositions of different z_2-projections. We need to explain the difference between an apparatus as a measuring device, and as a physical target. This will be done in Sec. 5.

4. Stern-Gerlach experiment. Quantum nonlocality (uncertainty). Measurement of nonlocal observables.

In the Stern-Gerlach experiment (SG), two branches of the final state of a heavy atom A having 1/2-spin S are spatially separated. This is a necessary condition for measuring the probability of the S_Z-projection. Axis $\vec{Z}$ in Fig. 1 is perpendicular to the central trajectory. Atoms are polarized along $\vec{Z}_1$ or $\vec{Z}_2$. In Fig. 2 we can see an original result of the beam splitting [5].

The initial wave packet before entering the magnet is

$$\psi_0 = u^0 \left(\vec{R}t\right) \sum_m c_m \phi_m \left(\vec{r}\right) \ , \quad m = \pm 1/2 \ , \tag{40}$$

and after exiting from it,

$$\psi = \sum_m c_m u_m \left(\vec{R}t\right) \phi_m \left(\vec{r}\right) \ , \tag{41}$$

where $\phi_m(\vec{r})$ describes the internal atom motion, and $u_m(\vec{R}t)$ the center-of-mass motion; $u_{1/2}$ and $u_{-1/2}$ do not overlap spatially:

$$u_m \left(\vec{R}t\right) u^*_{m'} \left(\vec{R}t\right) = u_m \left(\vec{R}'t\right) u^*_{m'} \left(\vec{R}'t\right) = 0 \ , \quad m \neq m' \ . \tag{42}$$

As pointed out in [6], if we measure only local observables, then density matrix $\hat{\rho}$, corresponding to (41), can be effectively replaced by classical density matrix ρ_c, in which off-diagonal terms of $\hat{\rho}$ are omitted. This can explain the collapse of quantum states in most cases: during measurement of local observables, information about phases is lost. Before discussing measurement of nonlocal observables like $\hat{\rho}$, namely of their off-diagonal terms, we will first discuss the definition of quantum nonlocality.

We will call a set of eigenvalues of an observable K, $\{k_1, k_2, k_3, \ldots\}$, a K-space. We will call K *local in a given K-space*, or simply local, if K is equal to one of its eigenvalues from this space, $K = k_j$, $k_j \epsilon \{k_1, k_2, \ldots\}$. If K is local in a K-space, Q is local in a Q-space, and there exists a one-to-one correspondence, $q_j \leftrightarrow k_i$ for every q_j and every k_j, then we will say that both K and Q are local in both K- and Q-spaces. For example, in SG, in state (41), S_Z is local in $\vec{R}$-space. Obviously, K is local if it belongs to a physical system whose quantum state is an eigenvector of K, $\mid k_j >$, $k_j \epsilon \{k_1, \ldots\}$. The question is whether K is local in superposition $\mid \psi >= \sum a_j \mid k_j >$, $a_j \neq 0, 1$.

We may suggest that despite the quantum uncertainty ΔK in such a state, K <u>has</u> a certain (although unknown) numerical value, and therefore that the physical system, though in a state of a superposition, nevertheless occupies some (unknown to us) point in K-space. However, there is no way either to prove or disprove such a suggestion. Let $\mid \psi >$ be an eigenstate of an observable P not commuting with K, $\hat{P} \mid \psi >= p_i \mid \psi >$, $[\hat{P}, \hat{K}] \neq 0$. As shown in the previous section, there does not exist a computable function mapping P-space one-to-one onto K-space. Noncomputable functions (see [7], for example) cannot be described algorithmically. We have a typical quantum effect: quantum nonlocality. This nonlocality does not mean that a physical system is "smeared" in a given space; it means only that there are no algorithms to prove or disprove the statement, "The system is located at a certain point." On one hand we know, after Gödel's work [8], (see also [4], [7]), that such a statement in principle can be true; on the other hand, the question, "Where is the system located?" makes no sense.

Nevertheless, it is reasonable to call K *nonlocal in K-space* in state $\mid \psi >$, if $\hat{K}\hat{P} - \hat{P}\hat{K} \neq 0$

and $P \mid \psi >= p_j \mid \psi >$. Thus, if K is nonlocal, the quantum state is the superposition $\sum a_j \mid k_j >$, $a_j \neq 0, 1$, and vice versa.

But quantum nonlocality so defined is merely the familiar quantum uncertainty, ΔK, which depends on the state of the system. This clarification of the term will help us understand how to measure off-diagonal terms of density matrix $\hat{\rho}$. We need to clarify also the concept of measurement. There are two radically different cases of the interaction of a physical system with a detector designed to measure K.

1. The result of the interaction can be formulated as "$K = k_j$", where k_j is an exact numerical value. In such a case the detector is a part of a measuring apparatus; but this requires that there be a special logical procedure, used by the researcher, that permits him/her to distinguish different numerical values of K, k_i, k_j, $k_j \neq k_i$. In this case K is local (as are the truths of all statements Λ_{k_j}, $j = 1, 2, \ldots$, $\hat{\Lambda}_{k_j} = \mid k_j >< k_j \mid$), for all states $\mid k_j >$, $i = 1, 2, \ldots$, created by the measuring apparatus.

   And if a pre-measured state $\mid \psi >$ is an eigenstate of Q, and $[\hat{Q}\hat{K}] \neq 0$, as is usual in measurements, then neither K nor any Λ_{k_j} is local in the K-space of the pre-measured system. Let $\hat{Q} \mid \psi >= q_i \mid \psi >$. As was explained, there is no computable function for one-to-one mapping of Q-space onto K-space; therefore, there are no logical connections between statements $\Lambda_\psi \equiv \Lambda_{q_i}$ and Λ_{k_j}. This means that transition $\hat{\Lambda}_\psi \to \hat{\Lambda}_{k_j}$, caused by a single measurement, is not deterministic. The probability is equal to $tr(\hat{\Lambda}_\psi \hat{\Lambda}_{k_j})$. If (and only if) $\hat{\Lambda}_\psi$ is expressed in the K-representation, where $\hat{\Lambda}_{k_j}$ is diagonal, then the off-diagonal terms of $\hat{\Lambda}_\psi$, $(\Lambda_\psi)_{ik} = a_i a_k^*$, $i \neq k$, $\mid \psi >= \sum a_j \mid k_j >$ play no role and can be omitted. So in our predictions of the results of this measurement, $\hat{\rho}_\psi \equiv \hat{\Lambda}_\psi$ can indeed be replaced by the classical $\rho_c : (\hat{\rho})_{ik} \to (\rho_c)_{ik} = \delta_{ik} a_i a_k^*$. However, we will see that in some important cases we should express $\hat{\rho}_\psi$ in a representation different from the K-representation; in such cases the off-diagonal terms of $\hat{\rho}_\psi$ cannot be omitted.

(The logical procedure which permits us to distinguish between k_i and k_j, $i \neq j$, emerging from the measurement, and therefore forbids interferences, will be discussed in the next section.)

2. If such a logical procedure is not used, then the detector is not a part of the measuring apparatus, but is a target. Despite the fact that the apparatus is constructed to measure observable K, the quantum state emerging after the interaction of the system with the detector is not in general an eigenstate of K, i.e., K is not local either in the initial or in the final quantum states.

We now represent two methods of measuring nonlocal observables, i.e., their off-diagonal terms, using the SG example.

In the first method, to measure off-diagonal terms of $\hat{\rho}$ we need simply compare the truths of two statements, Λ_1 and Λ_2,

$$\Lambda_1 : \text{“}S_{Z_1} = \frac{1}{2}\text{”} \Rightarrow \hat{\rho}^{(1)} \ ; \ \ \Lambda_2 : \text{“}S_{Z_2} = \frac{1}{2}\text{”} \Rightarrow \hat{\rho}^{(2)} \quad , \tag{44}$$

$\vec{Z}_1$ and $\vec{Z}_2$ are shown in Fig. 1. The probability that Λ_2 is true when Λ_1 is true (or vice versa),

$$w\left(\Lambda_1 \mid \Lambda_2\right) = tr\hat{\rho}^{(1)}\hat{\rho}^{(2)} = \int d^3r d^3r' d^3R d^3R' \times$$

$$\sum_{mm'} c_m^{(1)} c_{m'}^{*(1)} u_m\left(\vec{R}t\right) \phi_m\left(\vec{r}\right) u_{m'}^*\left(\vec{R}'t\right) \phi_{m'}^*\left(\vec{r}'\right) \sum_{nn'} c_n^{(2)} c_{n'}^{*(2)} u_n\left(\vec{R}'t\right) \phi_n\left(\vec{r}'\right) u_{n'}^*\left(\vec{R}t\right) \phi_{n'}^*\left(\vec{r}\right) =$$

$$= \mid c_{1/2}^{(1)} \mid^2 \mid c_{1/2}^{(2)} \mid^2 + \mid c_{-1/2}^{(1)} \mid^2 \mid c_{-1/2}^{(2)} \mid^2 + \text{ Interference term } , \tag{45}$$

$$\text{Interference term } = c_{1/2}^{(1)} c_{-1/2}^{*(1)} c_{1/2}^{*(2)} c_{-1/2}^{(2)} + c.c. \tag{46}$$

In (45), (46) $c_{\pm 1/2}^{(k)}$ denotes the amplitude of the $S_Z = \pm 1/2$ component of wave function $\psi^{(k)}$ that corresponds to the initial polarization of spin along $\vec{Z}_k$. In Fig. 1 $(\vec{Z}_k\vec{Z}) = \cos\theta_k$. In SG, $c_{1/2}^{(k)} = e^{i\varphi_k/2} \cdot \cos\theta_k/2$, $c_{1/2}^{(k)} = \sin\theta_k/2$. The interference term is equal to $\frac{1}{2}\sin\theta_1 \sin\theta_2 \cdot \cos\frac{\varphi_2-\varphi_1}{2}$. We can measure it by comparing different probabilities $w(\theta_k) = \mid c_{1/2}^{(k)} \mid^2$ in experiments with three different initial polarizations: θ_1, θ_2, $\theta_3 = (\theta_2 - \theta_1)$, see Fig. 2. Then

$$\text{Interference term } = w\left(\theta_2 - \theta_1\right) - w\left(\theta_1\right) w\left(\theta_2\right) - \left[1 - w\left(\theta_1\right)\right]\left[1 - w\left(\theta_2\right)\right] \quad . \tag{47}$$

In these three measurements, we could keep the initial polarization fixed along the $\vec{Z}_1$-axis, and rotate only the SG-magnet, making three different angles between the $\vec{Z}_1$-axis and $\vec{Z}$-axis of the magnet; let the three directions of the Z-axis be $\vec{Z}^{(1)}(\theta^{(1)} = \theta_1)$; $\vec{Z}^{(2)}(\theta^{(2)} = \theta_2)$; $\vec{Z}^{(3)}(\theta^{(3)} = \theta_2 - \theta_1)$; $\cos\theta^{(k)} = (\vec{Z}_1\vec{Z}^{(k)})$. We always measure the probability of the $S_Z = \frac{1}{2}$ projection. So we have three $\hat{S}_Z$-operators, $\hat{S}_Z^{(1)}$, $\hat{S}_Z^{(2)}$, and $\hat{S}_Z^{(3)}$, which do not commute. Therefore, each $S_Z^{(k)}$ is local in states emerging after its own measurement, but nonlocal in states emerging after the measurement of other S_Z's. The initial pre-measurement state (which is always the same) collapses into three types of final, post-measurement states, which are described by three types of statements: $\Lambda_{\frac{1}{2}}^{(1)}$, $\Lambda_{-\frac{1}{2}}^{(1)}$; $\Lambda_{\frac{1}{2}}^{(2)}$, $\Lambda_{-\frac{1}{2}}^{(2)}$; $\Lambda_{\frac{1}{2}}^{(3)}$, $\Lambda_{-\frac{1}{2}}^{(3)}$; their truth operators do not commute.

$$\hat{\Lambda}_m^{(k)}\hat{\Lambda}_{m'}^{(k')} - \hat{\Lambda}_{m'}^{(k')}\hat{\Lambda}_m^{(k)} = \begin{cases} 0\ , & k = k' \\ \text{something}, & k \neq k' \end{cases} \quad . \tag{48}$$

The initial density matrix, $\mid \psi >< \psi \mid$, collapses all three times but into different final density matrices $\hat{\Lambda}_m^{(k)}$, $k = 1, 2, 3, m = \pm\frac{1}{2}$. Each time, the off-diagonal terms play no role, but being expressed in the same representation, they are partially restored in (47).

In the second method the final space of numerical values emerging from the measurement is not changed, but it is neither the S_Z-space of the first SG-magnet (see Fig. 3), nor the S_{Z_2}-space of the second SG-magnet; it is the space resulting from the interference of $\rho^{(1)}$ and $\rho^{(2)}$, see below. None of the S_Z's corresponding to the two magnets are local in the final states.

In Fig. 4 the atom polarized along the $\vec{Z}_1$-axis is moving toward the reader. After passing the first SG magnet with symmetry axis $\vec{Z}$, the wave function is split into upper and lower branches along the $\vec{Z}$-axis with amplitudes a and b, so the density matrix — the truth operator of logical statement about the atom after the first SG,

$$\hat{\rho}^{(1)} = \begin{pmatrix} \mid a \mid^2 & ab^* \\ a^*b & \mid b \mid^2 \end{pmatrix} \quad . \tag{49}$$

After passing the second magnet, both a- and b-branches are split again into two branches, along the $\vec{Z}_2$-axis, with the final amplitudes shown in Fig. 4, where the density matrix

$$\hat{\rho}^{(2)} = \begin{pmatrix} \mid c \mid^2 & cd^* \\ c^*d & \mid d \mid^2 \end{pmatrix} \ , \hat{\rho}^{(1)}\hat{\rho}^{(2)} - \hat{\rho}^{(2)}\hat{\rho}^{(1)} \neq 0 \quad , \tag{50}$$

and, by definition, corresponds to the transition from $\vec{Z}_2$ to $\vec{Z}$.

If, now, detectors D_A, D_B register superpositions of the upper and lower branches inside the circles of Fig. 4, then we observe

$$\mid A \mid^2 = tr\left(\hat{\rho}^{(1)}\hat{\rho}^{(2)}\right) \quad . \tag{51}$$

If $a = e^{i\psi_1/2} \cdot \cos\theta_1/2$, $c = e^{i\psi_2/2} \cdot \cos\theta_2/2$, then

$$\mid A \mid^2 = \cos^2\frac{\theta_1}{2}\cos^2\frac{\theta_2}{2} + \sin^2\frac{\theta_2}{2}\sin^2\frac{\theta_2}{2} + \text{ Interference term } \quad . \tag{52}$$

Here we have interference between upper and lower spots:

$$\text{Interference term} = \frac{1}{2}\sin\theta_1\sin\theta_2\cos\frac{\psi_2-\psi_1}{2} \quad . \tag{53}$$

We see that the interference of probabilities appears in (45), (47), and (52) if (and only if) the experiment is constructed such that neither $\hat{\rho}^{(1)}$ nor $\hat{\rho}^{(2)}$ in $tr(\hat{\rho}^{(1)}\hat{\rho}^{(2)})$ is expressed in its own diagonal representation.

5. Measurement. Indeterminism. Collapse of wave functions. Quantum → classical transition.

The Schrödinger equation describes a smooth deterministic development of wave functions and density matrices. Indeterminism and sudden collapse of wave functions are observed only in measurements. Why does this happen? What is the difference between a detector as a physical target and a detector as part of a measuring apparatus?

A logical system processing the result of a measurement must, inter alia, use a macroscopic scale whose ordered marks satisfy the following conditions:

(a) They are separable;

(b) They are equal, in the sense that they do not provide any service but marking;

(c) They have no inner structure; and

(d) There exists a one-to-one correspondence between them and numerical values of a measured observable when the representation of the measured states corresponds to the choice of apparatus.

Exactly to satisfy conditions (a) and (d), branches $S_Z = \frac{1}{2}$ and $S_Z = -\frac{1}{2}$ of the SG experiment are macroscopically separated in space.

The macroscopic separation of two wave packets in an SG magnet becomes possible only because the following quasiclassical condition is met:

$$\left| \frac{d\lambda_z}{dz} \right| = \frac{\hbar m_A}{F^2 T^3} \ll 1 \ , \quad \text{or} \quad \frac{S}{\hbar} \sim \frac{z p_z}{\hbar} \sim \frac{F^2 T^3}{\hbar m_A} \gg 1 \ , \tag{54}$$

where λ_z is the de Broglie wave length, $\lambda_z = \hbar/p_z$, $F = \mid \mu_e \frac{\partial B_Z}{\partial Z} \mid$, μ_e is the electron magnetic moment, B_Z is the Z-component of the magnetic field; m_A is the mass of the atom; and T is flight time through the magnet. Condition (54) permits us not only to separate two branches of the same wave function, but also to observe macroscopic intervals (spots) (see Fig. 2) in both branches, Δz, Δp_z, such that

$$1 \ll \frac{\Delta S}{\hbar} \ll \frac{S}{\hbar} \quad , \tag{55}$$

without distinguishing among different wave functions of different atoms inside ΔS. The logical system of the measuring procedure does not distinguish among them, in order to satisfy conditions (b) and (c).

Consider an interaction between an atom and detectors in the SG experiment, without taking into account detector efficiency. After an interaction occurs, but before it is registered by the logical system of the measurement procedure, a common (atom + detector) wave function is a superposition,

$$\psi_{com} = \sum_{m=\pm\frac{1}{2}} c_m \sum_{k}^{N} \alpha_{mk} \psi^a_{mk} \phi^d_{mk} \quad , \tag{56}$$

where c_m is taken from (41), $\sum_k \mid \alpha_{mk} \mid^2 = 1$, ψ^a_{mk} is one of many possible upper ($m = \frac{1}{2}$) or lower ($m = -\frac{1}{2}$) atom eigenstates after an interaction with a detector, and ϕ^d_{mk} is one of the detector eigenstates; $N \gg 1$. A change in the state of the detectors is observed during measurement. Without such an observation, (56) is a pure quantum state. Leaning on the examples given in the previous section we can claim that, at least in principle, some experiments can give us information about phases of $c_m \alpha_{mk}$, even if there are many unknown phases. This information is irreversibly lost only when we directly count atom-detector interactions.

When events are counted, the logical system of the measurement procedure deliberately does not distinguish among different events inside the same (upper or lower) spots, $\Delta S_{\pm\frac{1}{2}}$, which obey (55); S is the action related to the event. This means that instead of considering precise statements about final states, Λ_{mk}, with their truth operators,

$$\hat{\Lambda}_{mk} = \mid \psi^a_{mk}\phi^d_{mk} >< \psi^a_{mk}\phi^d_{mk} \mid \ , \quad m = \pm\frac{1}{2} \ , \quad k = 1, 2, \ldots \quad , \tag{57}$$

the logical system considers only complex statements, which we can call in the SG case "Spin up" and "Spin down":

$$\text{Spin}^{\text{up}}_{\text{down}} : \quad \text{"Either } \Lambda_{\pm\frac{1}{2},1} \ , \quad \text{or } \Lambda_{\pm\frac{1}{2},2} \ , \quad \text{or } \Lambda_{\pm\frac{1}{2},3} \ , \ldots \text{ "} \tag{58}$$

These statements are represented by truth operators (see formula (25) for the case of mutually exclusive statements):

$$\text{Spin}^{\text{up}}_{\text{down}} \Rightarrow \sum_k^N \mid \psi^a_{\pm\frac{1}{2}k}\phi^d_{\frac{1}{2}k} >< \psi^a_{\pm\frac{1}{2}k}\phi^d_{\frac{1}{2}k} \mid \equiv \sum_k^N \hat{\Lambda}_{\pm\frac{1}{2}k} \quad , \tag{59}$$

If we now calculate the eigenvectors of these "spin up" and "spin down" truth operators, V^{up} and V_{down}, we realize that these eigenvectors <u>cannot interfere</u>.

$$\text{Spin}^{\text{up}}_{\text{down}} = \sum_k A_{\pm\frac{1}{2}k} \mid \psi^a_{\pm\frac{1}{2}k}\phi^d_{\pm\frac{1}{2}k} > \quad , \tag{60}$$

where $A_{\frac{1}{2}k}$, $A_{-\frac{1}{2}k}$ are completely arbitrary, so their phases are not defined.

Thus, conditions (a), (b), (c), and (d) are satisfied. The transition from a "micro" to a "macro" description, essential for measurement, includes fulfillment of physical condition (55); choice of an apparatus; a decision to ignore details inside ΔS (or not to have physical tools resolve them); assigning distinguishing names to different ΔS-spots (like those in Fig. 2) separated in some not always coordinate space; and a logical system of counting events.

The unpredictability of results of measurement can be seen now even from the comparison of dimensions. We have only two possible measured values, $S_Z = \frac{1}{2}$, $S_Z = -\frac{1}{2}$, with no superpositions, while the set of possible initial states contains in addition two phases which appear in quantum mechanics in return for the lost classical projections S_X and S_Y. The phase values belong to the continuum, and there is no way to get a deterministic mapping of all possible initial states onto two states, $S_Z = \pm\frac{1}{2}$. (A simple, more formal proof is given below.)

We know, after all, that some mapping occurs since we do get results of our measurements, though unpredictable ones. The explanation is that the functions performing such mapping are noncomputable.

Suppose the opposite, namely, that there exists a computable one-to-one function in the SG case, i.e., a function mapping $S_{Z(\theta)}$ onto S_Z, where S_Z is represented by the pointer positions:

$$S_Z = f\left(\theta\ ,\quad S_{Z(\theta)}\right) \quad . \tag{61}$$

Here we use $Z(\theta)$ instead of the $\vec{Z}_K$ of Fig. 1, $\vec{Z}_K \equiv Z(\theta_K)$, $Z \equiv Z(0)$. $S_{Z(\theta)}$ is the spin projection along axis $Z(\theta)$. Only two (invariant) eigenvalues are permitted, because $N = 2$, $S_{Z(\theta)} = \pm 1/2$; therefore, there are only two possible rules for every θ:

$$f\left(\theta\ ,\quad S_{Z(\theta)}\right) = \pm S_{Z(\theta)} \quad , \tag{62}$$

and we need simply determine the correct sign. It is obvious (and this is our postulate) that for $\theta = \pi$,

$$f\left(\pi\ ,\ S_{Z(\pi)}\right) = -S_{Z(\pi)} \quad . \tag{63}$$

Another sign would have made no sense. Let $\theta_1 = \pi/2$ and $\theta_2 - \theta_1 = \pi/2$ correspond to two consecutive mappings. Rotation symmetry (about the axis perpendicular to Fig. 1) demands that only relative directions of axes are important, not their absolute directions in space. Thus, function f can depend only on the angle between axes. Taking into account that the initial projection $S^{in}_{Z(\frac{\pi}{2})}$ relative to axis $Z(\frac{\pi}{2})$ is equal to $S_{Z(\pi)}$ relative to Z, this gives:

$$S_{Z(\pi/2)} = f\left(\frac{\pi}{2}\ ,\quad S_{Z(\pi)}\right) = \pm S_{Z(\pi)} \quad , \tag{64}$$

$$S_{Z(0)} = f\left(\frac{\pi}{2}\ ,\quad S_{Z(\frac{\pi}{2})}\right) = \pm S_{Z(\pi/2)} = +S_{Z(\pi)} \quad . \tag{65}$$

But (65) contradicts (63).

Therefore, there does not exist a computable function that can perform the one-to-one mapping. We can say also that there is no computable one-to-one mutual translation of statements describing different directions of a spin (with two obvious exceptions, $\theta = 0, \pi$).

Three things were crucial for the above result: mapping being permitted only onto $S_Z = \pm 1/2$ and not onto superpositions; the existence of rotation symmetry; and the invariance of eigenvalues, that is, a property of symmetry in a complex vector space.

We see that the phenomenon of noncomputability relates only to measurements, and leads to two effects: (1) unpredictability of results of measurements, and (2) collapse of measured states.

The effect of unpredictability (indeterminisim) of measurements gives the basis for the statistical interpretation of wave functions. The formula for the probability of a result $K = k_j$, when an initial state is $\mid \psi >$, $w(\Lambda_\psi \mid \Lambda_{k_j}) = tr(\hat{\Lambda}_\psi \hat{\Lambda}_{k_j}) \equiv \mid < \psi \mid k_j > \mid^2$, can be derived using the basic properties of probabilities, plus the assumption that w should depend on both $\hat{\Lambda}_\psi$ and $\hat{\Lambda}_{k_j}$, and only on them [3]. Since the properties of truth values are similar to the properties of probabilities (for example, if both Λ_ψ and Λ_{k_j} are true in some state $\mid \varphi >$, then "Λ_ψ and Λ_{k_j}" is true, i.e., $\hat{\Lambda}_\psi \hat{\Lambda}_{k_j} = 1$; and if Λ_ψ and Λ_{k_j} are mutually exclusive, then $\hat{\Lambda}_\psi \hat{\Lambda}_{k_j} = 0$), the formula for the truth of "Λ_ψ and Λ_{k_j}" coincides with the formula for the corresponding probability. However, the concept of probability is not valid for nonobserved systems, and the concept of truth is purely logical and does not depend on measuring procedures.

As for the collapse of a state $\mid \psi >$, $\mid \psi > \rightarrow \mid k_j >$, the question arises of when and where this collapse occurs. The answer is the following. Since the collapse is merely the creation of an unpredictable new description of a measured system, it can occur only when and where the logical system processing a given individual measurement formulates the result "$K = k_j$." Only after that can the information (in Shannon's sense) about this result be transmitted to other places.

6. EPR-Bohm Experiment. The main questions and answers.

The principal setup of one of the EPR-Bohm experiments is shown in Fig. 5. (In real experiments, Stern-Gerlach magnets have never been used as analyzers. The principal setup used here for the sake of simplicity is taken from the original paper [9].) The orientations of the two analyzers, $\vec{Z}_a$, $\vec{Z}_b$, can be changed during the flight time of two wave packets which are short enough to distinguish times t_a, t_b. The detectors are widely separated, so the collisions of the wave packets with the detectors lie outside the light cones of each other. Experiments have strongly confirmed the absence of local hidden parameters, since Bell's inequalities [10] are violated. It is not clear how to check experimentally the existence or nonexistence of nonlocal physical influence [1], a problem connected with the EPR "paradox" [11]. However, we will show here that there is no need to invoke nonlocal influence to explain the results of EPR experiments.

In [8] the authors used elastic scattering of very slow protons, $p + p \to p + p$ in order to get the singlet state of a pp pair (that is, before interaction with the detectors):

$$| \psi >= \frac{1}{\sqrt{2}} \left\{ |\uparrow (\vec{Z})\vec{R}_a t_a >|\downarrow (\vec{Z})\vec{R}_b t_b > - |\downarrow (\vec{Z})\vec{R}_a t_a >|\uparrow (Z)\vec{R}_b t_b > \right\} \ , \tag{66}$$

Since $| \psi >$ in this case is a rotation invariant, axis $\vec{Z}$ in the superposition can be directed arbitrarily.

Question 1. (66) is a macroscopically nonlocal state. $\vec{R}_a$, $\vec{R}_b$ are widely separated but every particle is "present" in <u>both</u> branches. What does this mean?

The answer, according to Secs. 1 and 2 is that wave functions contain information about physical systems. Although particles are more or less located, information about them can be nonlocal. *Information, and not a particle, is present simultaneously in both branches.*

Question 2. If $\mid \psi >$ is a container of information, then it can be described verbally. But how?

A language which can be *explicitly* used to describe a quantum state can be defined by a measuring apparatus corresponding exactly to that state. In case (66) this can be an apparatus measuring observable Σ,

$$\hat{\Sigma} = \frac{1}{2}\left(1 + \vec{\sigma}_a \vec{\sigma}_b\right) \quad , \tag{67}$$

where σ_x, σ_y, σ_z are Pauli matrices. Such an apparatus would have two pointer positions, $+1$ for the triplet (pp) state, and -1 for the singlet (pp) state. The latter is our $\mid \psi >$ state:

$$\hat{\Sigma} \mid \psi >= - \mid \psi > \quad . \tag{68}$$

$(+1, -1)$ form the language corresponding to such an apparatus. "-1" explicitly expresses $\mid \psi >$, and nothing forbids us to describe $\mid \psi >$ in terms of that apparatus (i.e., in the Σ-representation). However, we cannot express $\mid \psi >$ explicitly in the language corresponding to the apparatus of Fig. 5, which is measuring $\sigma_{Z_a} \cdot \sigma_{Z_b}$, because $\sigma_{Z_a}\sigma_{Z_b}$ does not commute with $\hat{\Sigma}$. Therefore, there is no computable translation between the truth values of Λ_ψ and the truth values of statements describing $\sigma_{Z_a}\sigma_{Z_b}$.

Question 3. According to (66), the spin of particle a is completely <u>uncertain</u>. However, by measuring S_{Z_a} we get a certain S_{Z_a}-projection along a certain axis. How does this happen?

The answer is a corollary of the answer to the previous question. Since there are no computable functions translating language explicitly corresponding to physical state $\mid \psi >$

into language explicitly corresponding to the pointer positions of the d_a, d_b detectors, there are no logical connections between state $\mid \psi >$ and the result of its measurements by the apparatus of Fig. 5. (Logical connections mean, in particular, the existence of an algorithm connecting the states before and after the measurement.) Therefore, every individual measurement in this case gives us completely new information, unconnected with the previous information. The new information is contained in the new state created by the measurement; it can, for example, be $|\uparrow (\vec{Z}_a)\vec{R}_a t >|\downarrow (\vec{Z}_b)\vec{R}_b t >$, if the result of the measurement is $S_a = \frac{1}{2}$, $S_b = -\frac{1}{2}$. The old information is destroyed at that moment when, and at the place where, the logical system processing the measurement has formulated the result of the individual measurement.

Question 4. When measuring S_{Z_a} in the a-channel, we have chosen axis Z_a completely arbitrarily. How does particle b know that its spin must be directed against axis $\vec{Z}_a$?

The answer is that it does not. All information about the logical correlations between the two branches, a and b, of the common state $\mid \psi >$, was created simultaneously with this state, i.e., well before the measurement.

Let us calculate the correlation mentioned in the question, using the results of Sec. 2.

Let statement $M_a(\vec{Z}_a)$ be

$$M_a\left(\vec{Z}_a\right) = \text{“}S_{Z_a}(a) = +\frac{1}{2}\text{”};\ M_a(\vec{Z}_a) \Rightarrow \hat{M}_a(\vec{Z}_a) = |\uparrow (\vec{Z}_a)\vec{R}_a t_a >< \uparrow (\vec{Z}_a)\vec{R}_a t_a \mid \quad , \tag{69}$$

and statement $M_b(\vec{Z}_b)$ be

$$M_b\left(\vec{Z}_b\right) = \text{“}S_{Z_b}(b) = -\frac{1}{2}\text{”};\ M_b(\vec{Z}_b) \Rightarrow \hat{M}_b(\vec{Z}_b) = |\downarrow (\vec{Z}_b)\vec{R}_b t_b >< \downarrow (\vec{Z}_b)\vec{R}_b t_b \mid \quad . \tag{70}$$

Here axes $\vec{Z}_a$ and $\vec{Z}_b$ can be different.

If $\vec{Z}_a = \vec{Z}_b = \vec{Z}$, then the logical equivalence,

$$M_a\left(\vec{Z}\right) \longleftrightarrow M_b\left(\vec{Z}\right) \quad , \tag{71}$$

is true in state $\mid \psi >$ throughout the history of the particles' movement. Indeed, from (30), (66), (69), (70),

$$M = M_a \longleftrightarrow M_b \Rightarrow \hat{M} = \hat{1} - \hat{M}_a - \hat{M}_b + 2\hat{M}_a\hat{M}_b \; ; \; Tr\left(\Lambda_\psi \mid M\right) = < \psi \mid \hat{M} \mid \psi > = 1 \; . \tag{72}$$

Thus, if spin $S(a)$ is up (down), then spin $S(b)$ is down (up). (In the case of 1/2 spin, the negations of the statements "$S_Z = \pm 1/2$" are the statements "$S_Z = \mp 1/2$".) It is extremely important to note that statement (71) does not specify whether any spin is really directed along or against any given axis. It is a purely logical assumption.

The existence of a logical correlation (71) between a and b branches does not mean that there is any nonlocal mutual influence; in quantum mechanics, according to Sec. 1, almost all wave functions — the containers of information — are "nonlocal".

The logical correlation between $M_a(\vec{Z}_a)$ and $M_b(\vec{Z}_b)$ in state $\mid \psi >$, when $\vec{Z}_b \neq \vec{Z}_a = \vec{Z}$, is equal to the truth of $M_a \wedge M_b$. Using $< \psi \mid \hat{M}_a \mid \psi > = < \psi \mid \hat{M}_b \mid \psi > = \frac{1}{2}$, we get

$$\begin{aligned} Tr\left(\Lambda_\psi \mid M_a \wedge M_b\right) &= < \psi \mid \hat{M}_a\hat{M}_b \mid \psi > = \mid < \uparrow (\vec{Z})\vec{R}_b t_b \mid \uparrow (\vec{Z}_b)\vec{R}_b t_b > \mid^2 = \\ &= \tfrac{1}{2}\cos^2\theta/2 \quad , \end{aligned} \tag{73}$$

and

$$Tr(\Lambda_\psi \mid M_a \leftrightarrow M_b) = \cos^2\theta/2 \quad , \tag{74}$$

(see Fig. 5). The result does not depend on the order of the cofactors since M_a and M_b describe different channels, and therefore commute.

Question 5. Let axes $\vec{Z}_a$ and $\vec{Z}_b$ be not parallel. We will measure simultaneously $S_{Z_a}(a)$ and $S_{Z_b}(b)$. Suppose the results are $S_{Z_a}(a) = +\frac{1}{2}$ and $S_{Z_b}(b) = -\frac{1}{2}$. According to (69), (70), (71) and (72), if $S_{Z_a}(a) = +\frac{1}{2}$, then $S_{Z_a}(b) = -\frac{1}{2}$. ($S_{Z_a}(b)$ means that, in channel b, $\vec{Z}_b = \vec{Z}_a$.) However, we have measured $S_{Z_b}(b) = -\frac{1}{2}$. This means that we have measured simultaneously the noncommuting S-projections along two non-parallel axes. Is this permitted by quantum mechanics?

The answer is that nothing in quantum mechanics forbids us to measure two noncommuting observables at <u>different</u> times and/or places. The logical correlation between the two results, $S_{Z_a}(a) = +\frac{1}{2}$ and $S_{Z_b}(b) = -\frac{1}{2}$, see formula (73), existed before our measurement. Since the theory is correct, the measurement should confirm this formula. Note that when (73) is an evaluation of truth, it needs only logical confirmation, not an experimental one. When we use (73) to evaluate probability, we need the measurement on an ensemble (since $\frac{1}{2}\cos^2\theta/2 < 1$, and $\theta \neq 0$).

Question 6. (A formulation of the EPR "paradox".)

Let consecutive measurements of S_Z-projections of two different particles onto two different axes, $\vec{Z}_a$ and $\vec{Z}'_a$, in the same a-channel result in $S_{Z_a}(a) = \frac{1}{2}$ and $S_{Z'_a}(a) = \frac{1}{2}$. In the first case we predict that $S_{Z_a}(b) = -\frac{1}{2}$, in the second case that $S_{Z'_a}(b) = -\frac{1}{2}$. Therefore, we predict with certainty the values of $S_{Z_a}(b)$ and $S_{Z'_a}(b)$, represented by <u>noncommuting</u> operators without any interaction with the particle in the b-channel. As formulated in [11], "If without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity), the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." Which two elements of physical reality in channel-b

correspond to the seeming appearance there of two numerical values of two noncommuting observables (at different times)?

The answer is that there are no such elements of physical reality in the b-channel. The information about all logical correlations between the two channels is the only physical reality available to science; and this reality, information, is nonlocal in the sense explained in Sec. 4 and existed before the measurements verifying logical correlations. In this case, we have two correlations (71) with two different axes $\vec{Z}$. They belong to the same vector (66) but to different pairs of particles. This assumes that wave functions describe individual systems, not ensembles (although ensembles of identical systems can be prepared).

Indeed, wave functions are collapsed and created only individually; there is therefore no reason to interpret them as describing only ensembles of identical systems. Moreover, since conditional truths of statements about a system, whose wave function is a packet depend on the shape of the packet, and this shape may be unique, we can apply wave functions to individual systems first, and then to ensembles if ensembles is prepared.

Conclusion

The mysteries of quantum mechanics can be understood if we recognize that at the quantum level of accuracy we enter a world in which logic and language become parts of nature, and where we have to deal with noncomputable functions. However, full understanding of these mysteries will be reached only when a similar approach is developed to quantum field theory.

In a radically different direction, this understanding of quantum phenomena can be useful in developing a theory of mind. After all, quantum nonlocality is nonlocality of logic and language.

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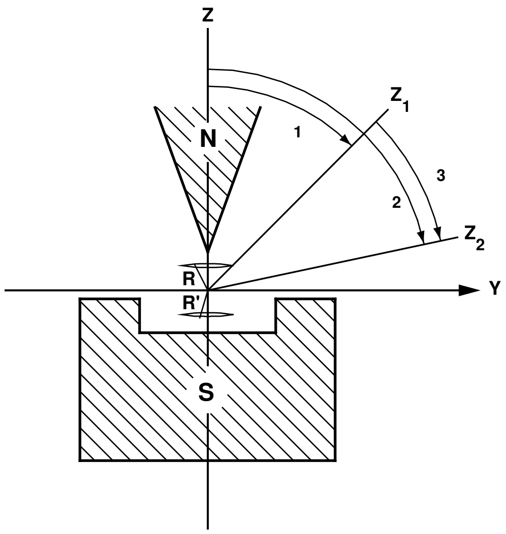
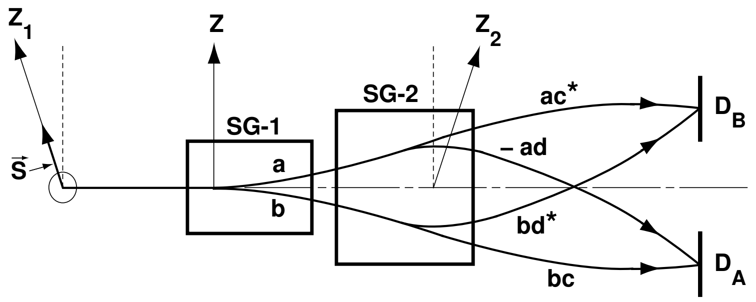


Fig. 1

See external graphics file.

Fig. 2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Z
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SG-1
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S⃗
a
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DB
DA

Fig. 3

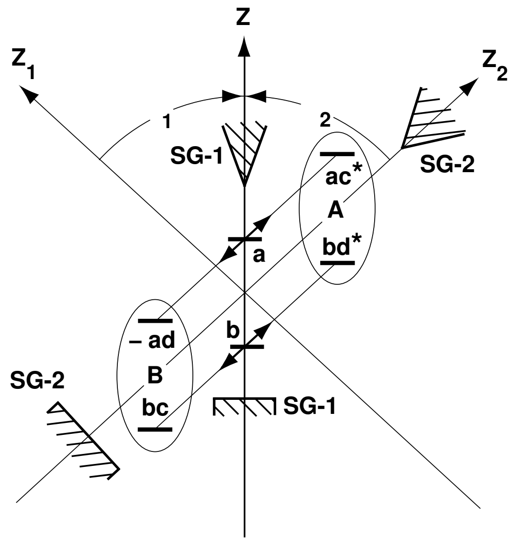
Z
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Fig. 4

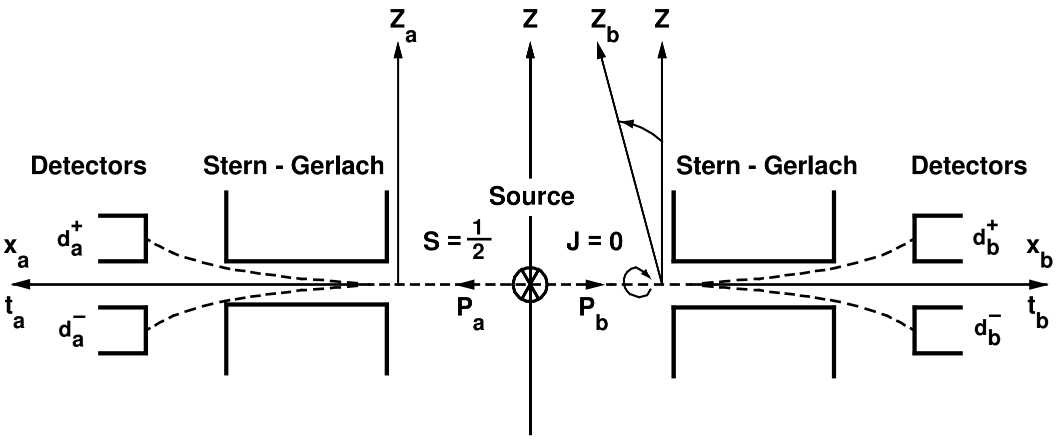
Za
Z
Zb
Z
Detectors
Stern - Gerlach
Source
Stern - Gerlach
Detectors
xa
da+
S = 1/2
J = 0
db+
xb
ta
da−
Pa
Pb
db−
tb

Fig. 5