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CKM Favored Semileptonic Decays of Heavy Hadrons at Zero Recoil

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Abstract

We study the properties of Cabibbo-Kobayashi-Maskawa (CKM) favored semileptonic decays of mesons and baryons containing a heavy quark at the point of no recoil. We first use a diagrammatic analysis to rederive the result observed by earlier authors that at this kinematic point the B meson decays via $b \to c$ transitions can only produce a D or D^* meson. The result is generalized to include photon emissions which violate heavy quark flavor symmetry. We show that photons emitted by the heavy quarks and the charged lepton are the only light particles that can decorate the decays $\bar{B} \to D(D^*) + \ell\nu$ at zero recoil, and the similar processes of heavy baryons. Implications for the determinations of the CKM parameter V_{cb} are discussed. Also studied in this paper is the connection between our diagrammatic analysis of suppression of particle emission and the formal observation based on weak currents at zero recoil being generators of heavy quark symmetry. We show that the two approaches can be unified by considering the Isgur-Wise function in the presence of an external source.

I. Introduction

In semileptonic decays of heavy mesons and baryons containing a b quark, a large phase space is available for emission of many light particles such as pions and photons. Synthesis of spin and flavor symmetry of heavy quarks and chiral symmetry of light quarks provides a natural framework to describe these processes involving soft pions and photons [1,2,3]. At zero recoil, the Cabibbo-Kobayashi-Maskawa (CKM) favored semileptonic decays of heavy mesons and baryons exhibit some remarkable properties. For instance, it has been pointed out [4,5] that within strong interactions at the point of no recoil, the CKM favored semileptonic decays of a B meson in the heavy quark limit can only produce a D or D^* meson. This follows from the observation that when the initial and final heavy quarks have the same velocity, the weak currents become generators of heavy quark symmetry and their action on a B meson can only give rise to a linear combination of D and D^* mesons. At this kinematic point, the amplitudes for CKM favored semileptonic decays of a B meson accompanied by emission of light hadrons are of order $1/m_Q$ and the rates are suppressed by $1/m_Q^2$.

The importance of the above result is beyond the obvious theoretical interest of being a precise statement. It implies that the Isgur-Wise function measured experimentally at the point of no recoil is not contaminated by corrections of order 1 or order $1/m_Q$ due to emissions of pions. When combined with the Luke's theorem [6], the above result would imply that the corrections to the Isgur-Wise function at the point of no recoil are at least of order $1/m_Q^2$. Motivated by these observations, we devote this paper to a detailed stuty of the properties of CKM favored semileptonic decays of heavy hadrons at zero recoil, including a brief discussion in the last section of measuring the CKM parameter V_{cb} in $\bar{B} \to D^* + \ell\nu$. Although the formal argument of refs.[4,5] is as simple as it is, it is desirable and instructive to have an explicit derivation of this result based on an analysis of the multiparticle amplitudes. The reason is obvious. The amplitudes for $\bar{B} \to D(D^*) + \pi + \ell\nu$ vanish at $v = v'$ due to cancellation between emissions of the pion before and after the weak vertex and cooperation of pion emission from different intermediate states. As the number of pions increases, the cancellation becomes increasingly complex. One of the purposes of the present paper is to show how to organize the diagrams systematically to establish the general results to all orders in perturbation theory, not only for emission of pions and other Goldstone bosons, but also for emission of any other light hadrons. Another purpose of our study is to examine if the result still holds in the presence of electromagnetic interactions. First of all, photon emission from the heavy quarks violates the heavy flavor symmetry since the b and c quarks have different electric charge. Secondly, the strong interaction coupling constant g responsible for pion emission is rather small, $g \sim 0.3-0.5$. Photon emission could, in principle, compete favorably with pion emission. We will show that photon emission is forbidden if it is accompanied by any other light particles. Only photons emitted by the heavy quarks and the charged lepton need to be considered and they are to accompany the semileptonic decays $\bar{B} \to D(D^*)\ell\nu$. Moreover, these photon emissions can be calculated by the standard QED technique. However, we must hasten to add that photons treated here are not the "soft photons" in the conventional sense; their momenta can be comparable with those of pions. Of course, in order for us to make use of the framework of heavy quark effective theory, the momenta of all light particles must be small compared with the heavy quark masses. In principle, we can control such a kinematic region by selecting leptons with appropriate momenta.

Finally, we offer a proof of the general result which relates the formal argument to our diagrammatic analysis. Specifically, we consider the Isgur-Wise function in the presence of an external source which couples only to the light quark's degrees of freedom. The Isgur-Wise function has still the normalization unity at the point of no recoil. This is the statement that the weak current at this kinematic point can only change a B meson into a D or D^* meson. But now, all of its functional derivatives evaluated at zero source must vanish. These functional derivatives reproduce the pion emission amplitudes at zero recoil.

We should point out that the diagrammatic analysis reveals a feature which is not apparent from the formal argument. Consider the amplitude for emission of n pions in addition to the D or D^* meson. There are $n!$ sets of diagrams which differ only in permutations of the pions. Our analysis shows that each of the $n!$ sets vanishes by itself. An interesting and obvious question is why not all the $n!$ sets are needed to make the amplitudes vanish at $v = v'$. Is there any deep physics hidden here ? At the moment, we do not have an answer.

Once we have seen how the suppression of particle emissions works at the point of no recoil for the CKM favored semileptonic decays of heavy mesons, it is straightfoward to extend the same analysis to the corresponding case of heavy baryons.

II. Suppression of a Single Soft Pion Emission at Zero Recoil

To gain some insight from simple examples and to prepare for a general treatment in the next section, we shall recall in this section the explicit calculations in [1,7] for the CKM favored semileptonic weak decays of heavy hadrons involving one soft pion emission: $\bar{B} \to D(D^*) + \pi + \ell\nu$, $\Sigma_b \to \Sigma_c(\Sigma_c^*) + \pi + \ell\nu$ and $\Sigma_b \to \Lambda_c + \pi + \ell\nu$. The coupling strength for a single pion emission is proportional to q/f_π where q is the pion momentum, so the semileptonic decay amplitude for producing a pion is naively expected not to be suppressed when $q \sim f_\pi$. However, we shall see that the amplitudes for pion emission from the initial hadron and final hadron conspire to give a zero sum at zero recoil in the heavy quark limit.

Figs. 1 and 2 show the Feynamn diagrams for $\bar{B} \to D(D^*)\pi\ell\nu$. The amplitudes have been evaluated in [1,7] with $v' \neq v$ (see Eqs. (4.27-4.29) of [1] and Eqs.(3.3-3.8) as well as Eqs.(4.1-4.4) of [7]). At zero recoil $v \cdot v' = 1$, the amplitudes are given by

$$\begin{aligned}
\langle D(v)\pi^a(q)|J_\mu^W|\bar{B}(v)\rangle &= F[(q\cdot v)v_\mu - q_\mu]\left(\frac{1}{v\cdot q+\Delta_B} - \frac{1}{v\cdot q-\Delta_D}\right), && (1)\\
\langle D^*(v,\varepsilon')\pi^a(q)|J_\mu^W|\bar{B}(v)\rangle &= F\Big[\frac{1}{-v\cdot q-\Delta_B}[i\epsilon_{\mu\nu\lambda\kappa}\varepsilon'^{\nu}q^\lambda v^\kappa - (\varepsilon'\cdot q)v_\mu] \\
&\quad + \frac{1}{v\cdot q} i\epsilon_{\mu\nu\kappa\lambda}\varepsilon'^{\nu}q^\lambda v^\kappa - \frac{1}{v\cdot q+\Delta_D}(\varepsilon'\cdot q)v_\mu\Big]. && (2)
\end{aligned}$$

In above equations, $\Delta_B = M_{B^*} - M_B$, $\Delta_D = M_{D^*} - M_D$, and

$$F = iu(\bar{B})^\dagger \frac{1}{2}\tau_a u(D^*)\sqrt{M_B M_{D^*}}\,\frac{f}{f_\pi}C_{cb}\xi(1), \tag{3}$$

where $u(P)$ is the isospin wave function of the heavy meson P, C_{cb} is a QCD correction factor, and $\xi(1)$ is the Isgur-Wise function evaluated at $v \cdot v' = 1$. In the heavy quark limit, $\Delta_B = \Delta_D = \mathcal{O}(1/m_b)$ or $\mathcal{O}(1/m_c)$. Therefore, we see that the $\bar{B} \to D(D^*)\pi\ell\nu$ amplitudes vanish at zero recoil.

There are three baryonic Isgur-Wise form factors for weak transitions of heavy baryons: $\zeta(v\cdot v')$ for antitriplet-antitriple transition, e.g., $\Lambda_b \to \Lambda_c$, and $\xi_1(v\cdot v')$, $\xi_2(v\cdot v')$ for sextet-sextet transition with the normalization $\zeta(1) = \xi_1(1) = 1$. The form factor ξ_2 drops out at zero recoil. The amplitude deduced from the Feynman diagrams in Fig. 3 at $v\cdot v'=1$ reads (see Eqs. (4.42) and (4.43) of [1])

$$\langle\Sigma_c^f(v,s')\pi^d(q)|J_\mu^W|\Sigma_b^e(v,s)\rangle=\varepsilon_{def}\frac{g_1}{2f_\pi}C_{cb}\xi_1(1)\overline{u}(v,s')[C_{a\mu}+C_{b\mu}+C_{c\mu}+C_{d\mu}]u(v,s), \quad (4)$$

where ε_{def} is the totally antisymmetric symbol associated with the isospin of the particles involved, g_1 is one of the unknown coupling constants given in Eq.(40) below, and $C_{a\mu}\cdots C_{d\mu}$ correspond to the contributions from Fig. 3(a) $\cdots$ 3(d) respectively:

$$\begin{aligned}
C_{a\mu} &= \frac{1}{3}\cdot\frac{1}{-v\cdot q}[\gamma_\mu(1-\gamma_5)+4v_\mu\gamma_5](\not{q}-q\cdot v), && (5)\\
C_{b\mu} &= \frac{1}{v\cdot q+\Delta_{\Sigma_b}}(1-\gamma_5)[-q_\mu+\frac{1}{3}\gamma_\mu(\not{q}-q\cdot v)+\frac{1}{3}v_\mu(\not{q}+2q\cdot v)], && (6)\\
C_{c\mu} &= \frac{1}{3}\cdot\frac{1}{v\cdot q}(\not{q}-q\cdot v)[\gamma_\mu(1-\gamma_5)-4v_\mu\gamma_5], && (7)\\
C_{d\mu} &= -\frac{1}{v\cdot q-\Delta_{\Sigma_c}}[-q_\mu+\frac{1}{3}(\not{q}-q\cdot v)\gamma_\mu+\frac{1}{3}(\not{q}+2q\cdot v)v_\mu](1+\gamma_5), && (8)
\end{aligned}$$

where $\Delta_{\Sigma_{b(c)}}=M_{\Sigma^*_{b(c)}}-M_{\Sigma_{b(c)}}$. Using the fact that $\bar{u}(v)\gamma_5u(v)=0$ and $\Delta_{\Sigma_{b(c)}}=\mathcal{O}(1/m_{b(c)})$ in the heavy quark limit, we find that $C_{a\mu}+C_{c\mu}=(\not{q}\gamma_\mu-\gamma_\mu\not{q})/(3v\cdot q)$ is exactly canceled out by $C_{b\mu}+C_{d\mu}$.

The next example is the semileptonic decay $\Sigma_b\to\Sigma_c^*\pi\ell\nu$ with the amplitude, for $v\cdot v'=1$ (see Eqs. (4.44) and (4.45) of [1])

$$\langle\Sigma_c^{*f}(v,s')\pi^d(q)|J_\mu^W|\Sigma_b^e(v,s)\rangle=\varepsilon_{def}\frac{g_1}{2f_\pi}C_{cb}\xi_1(1)\overline{u}_\lambda(v,s')[D^\lambda_{a\mu}+D^\lambda_{b\mu}+D^\lambda_{c\mu}+D^\lambda_{d\mu}]u(v,s), \quad (9)$$

where $u_\lambda(v,s)$ is a Rarita-Schwinger vector spinor, and $D^\lambda_{a\mu}\cdots D^\lambda_{d\mu}$ are the contributions from each of the Feynman diagrams in Fig. 4 respectively,

$$\begin{aligned}
D^\lambda_{a\mu} &= \frac{2}{\sqrt{3}}\cdot\frac{1}{-v\cdot q}g^\lambda_\mu(1+\gamma_5)(\not{q}-q\cdot v), && (10)\\
D^\lambda_{b\mu} &= \frac{\sqrt{3}}{2}\cdot\frac{1}{-v\cdot q-\Delta_{\Sigma_b}}[-\gamma_\mu(1-\gamma_5)q^\lambda+\frac{2}{3}g^\lambda_\mu(1+\gamma_5)(\not{q}-q\cdot v)], && (11)\\
D^\lambda_{c\mu} &= \frac{1}{2\sqrt{3}}\cdot\frac{1}{v\cdot q+\Delta_{\Sigma_c}}q^\lambda[\gamma_\mu(1-\gamma_5)-4v_\mu], && (12)\\
D^\lambda_{d\mu} &= \sqrt{3}\frac{1}{v\cdot q}[-g^\lambda_\mu(\not{q}-q\cdot v)+\frac{2}{3}q^\lambda(\gamma_\mu-v_\mu)](1+\gamma_5). && (13)
\end{aligned}$$

Noting that $\bar{u}_\lambda(v)\gamma_5u(v)=0$, $\bar{u}_\lambda(v)(\gamma_\mu-v_\mu)u(v)=0$, and $\bar{u}_\lambda(v)(\not{q}-q\cdot v)u(v)=0$, we find a zero total amplitude.

The last example is the decay $\Sigma_b \to \Lambda_c \pi \ell \nu$, whose amplitude at zero recoil is given by (see Eqs. (4.46) and (4.47) of [1])

$$\langle \Lambda_c(v,s')\pi^d(q)|J_\mu^W|\Sigma_b^e(v,s)\rangle = i\frac{g_2}{\sqrt{2}f_\pi}\delta_{ed}C_{cb}\overline{u}(v,s')[E_{a\mu}+E_{b\mu}+E_{c\mu}]u(v,s). \tag{14}$$

Again, $E_{a\mu}$, $E_{b\mu}$ and $E_{c\mu}$ are the contributions from Fig. 5(a), 5(b) and 5(c) respectively:

$$\begin{aligned} E_{a\mu} &= \zeta(1)\frac{1}{-v\cdot q+M_{\Sigma_b}-M_{\Lambda_b}}\gamma_\mu(1-\gamma_5)(\not{q}-q\cdot v), & (15)\\ E_{b\mu} &= -\frac{1}{3}\xi_1(1)\frac{1}{v\cdot q+M_{\Lambda_c}-M_{\Sigma_c}}(\not{q}-q\cdot v)[\gamma_\mu(1-\gamma_5)-4v_\mu\gamma_5], & (16)\\ E_{c\mu} &= -2\xi_1(1)\frac{1}{v\cdot q+M_{\Lambda_c}-M_{\Sigma_c^*}}[-q_\mu+\frac{1}{3}(\not{q}-q\cdot v)\gamma_\mu+\frac{1}{3}(\not{q}+2q\cdot v)v_\mu](1+\gamma_5). & (17)\end{aligned}$$

Using the identity

$$\bar{u}(v)\not{q}\gamma_\mu\gamma_5 u(v) = \bar{u}(v)(q\cdot v\gamma_\mu - \not{q}v_\mu)\gamma_5 u(v) \tag{18}$$

together with $\bar{u}(v)\gamma_5 u(v)=0$, and noting that $M_{\Sigma_b}-M_{\Lambda_b}=M_{\Sigma_c}-M_{\Lambda_c}=M_{\Sigma_c^*}-M_{\Lambda_c}$ in the heavy quark limit, we find again a vanishing total amplitude. The equality of the mass differences noted above and the relation $\zeta(1)=\xi_1(1)=1$ are crucial to obtaining a null amplitude. Both features are very general and are essential to the general treatment given in next section.

We remark that all the above calculations can be greatly simplified in the "superfield" framework in which a pseudoscalar and a vector meson field are combined into a meson superfield [2], and likewise a baryon superfield [8] for spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ sextet baryon fields. Feynman rules in terms of superfields become much simpler, and there are fewer Feynman diagrams to evaluate. Moreover, this method has the advantage that, as we shell see in the next section, the numerator is the same for all the diagrams with a fixed number of soft particles emitted at zero recoil as long as the particles are emitted in the same sequence. This feature enables us to generalize the analysis in this section to an arbitrary number of light hadron emissions and to include the excited heavy hadrons in the intermediate and final states, as elucidated in the next section.

III. Suppression of Light Meson Emissions at Zero Recoil

In this section we will establish within the superfield framework that the amplitudes for $\bar{B} \to D(D^*) + n\pi + \ell\nu$ with $n \geq 1$ vanish at zero recoil. Here, a pion is used generically to denote any light hadron. We will proceed in several steps. First, we will only include the ground states $\bar{B}$, $\bar{B}^*$, D, D^* and Goldstone bosons in our discussion. Once we have established the result for this simple system, we will generalize it to include the excited heavy mesons in the intermediate and final states, and light particles other than the Goldstone bosons. Finally, we outline briefly a similar analysis for the heavy baryons.

The octet of Goldstone bosons is represented by the matrix

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}. \tag{19}$$

The ground states of the heavy mesons are the pseudoscalar $P(v)$ and vector $P^*_\mu(v)$ with the quark content $Q\bar{q}$. The spin symmetry of the heavy quark is incorporated automatically by the use of a superfield matrix, which combines $P(v)$ and $P^*_\mu(v)$ [2]

$$H^a_{Ii}(v) = \left\{ \frac{1+\not{v}}{2}[-P^a(v)\gamma_5 + P^{a*}_\mu(v)\gamma^\mu] \right\}_{Ii}, \tag{20}$$

$$\overline{H}^a = \gamma^0 H^\dagger \gamma^0, \tag{21}$$

where the indices I and i refer to the heavy quark and light antiquark, respectively; the label a indicates the SU(3) flavor of the light antiquark.

The Lagrangian under our consideration is

$$\mathcal{L} = \frac{f_\pi^2}{4} \mathrm{tr}\left(\partial_\mu \Sigma^\dagger \partial^\mu \Sigma\right) + \sum_{Q=c,b} \left\{ i\mathrm{tr}(H_Q v \cdot D\overline{H}_Q) + g\,\mathrm{tr}(\overline{H}_Q H_Q \not{\mathcal{A}}\gamma_5) \right\}, \tag{22}$$

where $f_\pi = 94$ MeV,

$$\Sigma = \exp\left(\frac{2iM}{\sqrt{2}f_\pi}\right), \tag{23}$$

$$D\overline{H} = (\partial_\mu + \mathcal{V}_\mu)\overline{H}, \tag{24}$$

and

$$\mathcal{V}_\mu = \frac{1}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger) = \frac{1}{f_\pi^2}[M,\ \partial_\mu M] + \cdots, \tag{25}$$

$$\mathcal{A}_\mu = \frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) = -\frac{\sqrt{2}}{f_\pi}\partial_\mu M + \cdots, \tag{26}$$

$$\xi = \Sigma^{1/2}. \tag{27}$$

In (22) we have used a dimension $\frac{3}{2}$ field H to absorb all the heavy mass factors. The structure of the interaction vertices in Eq.(22) shows that the light mesons are coupled to light quark's degrees of freedom and the interactions are independent of the spin and flavor of the heavy quark. The heavy meson's propagator plays a crucial role in our discussion. It is given by

$$\int d^4x e^{iq\cdot x}\langle 0|T H^a_{Ii}(x)\overline{H}^b_{jJ}(0)|0\rangle = -\frac{i}{v\cdot q}\left(\frac{1+\not v}{2}\right)_{IJ}\left(\frac{1-\not v}{2}\right)_{ji}\delta_{ab}. \tag{28}$$

It should be noted that the Dirac indices for the heavy quark (IJ) and the light antiquark (ji) decouple. The weak currents for $b\to c$ transitions in terms of the heavy meson fields are

$$J^W_\mu = -C_{cb}\xi(v\cdot v')\,\mathrm{tr}\left[\overline{H}_c(v')\gamma_\mu(1-\gamma_5)H_b(v)\right], \tag{29}$$

where C_{cb} is a QCD correction factor and $\xi(v\cdot v')$ is the Isgur-Wise function.

Let us start with the diagram in which all the n pions are emitted before the weak vertex. We now move the pions one by one through the weak vertex without changing their quantum numbers (momentum and SU(3) flavor), as depicted in Fig. 6. Because the initial and final heavy mesons must have the same velocity v, the weak vertex acts only on the heavy quarks, and the pions are emitted from the light quarks, all the $(n+1)$ diagrams described above have a common numerator (see Fig. 7)

$$N_n = -g^n\mathrm{tr}\left[\overline{H}'\gamma_\mu(1-\gamma_5)H\not{\mathcal{A}}_1\gamma_5\frac{1-\not v}{2}\not{\mathcal{A}}_2\gamma_5\cdots\frac{1-\not v}{2}\not{\mathcal{A}}_n\gamma_5\right]. \tag{30}$$

For a single pion emission $\mathcal{A}^\mu_i \sim \frac{1}{2}\lambda^{a_i}q^\mu_i$. The wave functions for the initial and final mesons are denoted by H and $\overline{H}'$, respectively; for a pseudoscalar meson $H = -\frac{(1+\not v)}{2}\gamma_5$, and for a vector meson $H = \frac{(1+\not v)}{2}\not\varepsilon$, where ε^μ is a polarization vector. We have not explicitly showed the SU(3) flavor wave functions. We have ignored the QCD factor C_{cb} and the Isgur-Wise function is unity. The amplitude for the $(n+1)$ diagrams is given by

$$M(q_1,\cdots,q_n) = N_n A(q_1,\cdots,q_n), \tag{31}$$

with

$$A(q_1,\cdots,q_n) = \sum_{i=0}^{n}(-1)^{n-i}\frac{1}{v\cdot q_1}\cdot\frac{1}{v\cdot(q_1+q_2)}\cdots\frac{1}{v\cdot(q_1+q_2+\cdots+q_{n-i})} \tag{32}$$

$$\cdot \frac{1}{v\cdot(q_{n-i+1}+\cdots+q_n)}\cdot\frac{1}{v\cdot(q_{n-i+2}+\cdots+q_n)}\cdots\frac{1}{v\cdot(q_{n-1}+q_n)}\cdot\frac{1}{v\cdot q_n}.$$

For $1 \le i \le n-1$, the two propagators adjacent to the weak vertex can be rearranged as follows:

$$\begin{aligned}
&\frac{1}{v\cdot(q_1+q_2+\cdots+q_{n-i})}\cdot\frac{1}{v\cdot(q_{n-i+1}+q_{n-i+2}+\cdots+q_n)} \\
&=\left[\frac{1}{v\cdot(q_1+q_2+\cdots+q_{n-i})}+\frac{1}{v\cdot(q_{n-i+1}+q_{n-i+2}+\cdots+q_n)}\right]\cdot\frac{1}{v\cdot(q_1+q_2+\cdots+q_n)}.
\end{aligned} \tag{33}$$

The summation over $i=1$ to $i=n$ is now broken into two series. We note that the two series almost cancel each other except for the first term of the first series and the last term of the second series. These two terms in turn cancel exactly the $i=0$ and $i=n$ terms in the original series. Therefore,

$$A(q_1,\cdots,q_n)=0. \tag{34}$$

This is the central result of our paper. In what follows we make a series of generalizations:

(A). In writing down the amplitudes, we have implicitly assumed that a single particle is emitted at each vertex, but this is not necessary. If more than one particles are emitted at a particular vertex, e.g., in seagull diagrams, the momentum q_i is simply the sum of the total momenta of emitted particles. The proof still goes through. Thus, the multiparticle vertices in $\mathcal{A}_\mu$ and $\mathcal{V}_\mu$ of Eqs.(25-27) can be included. Another class of diagrams can be incorporated in a similar fashion: a single pion can turn into multipions via the nonlinear interactions among the Goldstone bosons.

(B). It is easy to include emissions of light particles other than the Goldstone bosons. All it requires is that their interactions with the heavy hadrons be independent of the flavors and spin of the heavy quarks. Then the numerators for all the diagrams in Fig. 6 will be the same, and the same result will follow.

(C). We now consider a somewhat nontrivial generalization to include excited states in the intermediate states and final states. As in all previous cases, the interactions between the light particles and heavy mesons are independent of heavy quark's spin and flavor. At the point of no recoil, the weak vertex is nonzero only if both sides belong to the same supermultiplet. (States in a supermultiplet differ only in how the heavy quark spin is combined with light antiquark states of a definite angular momentum and parity.) Moreover, there is only one Isgur-Wise function contributing and it has the value unity; the weak vertex has the standard $(V-A)$ combination acting on the heavy quarks. Thus, the numerator factor for the diagrams similar to Fig. 6 is of the same form as in (30) with a different and possibly more complicated matrix $\mathcal{A}$ at each vertex. The excited states and the ground states in general have mass differences of order 1 due to excitations of the light quark system; they may have an order 1 decay widths. All these mass differences and decay widths respect heavy quark symmetry. (In contrast, the mass differences and decay widths within a supermultiplet are of order $1/m_Q$ or smaller and they violate heavy quark symmetry.) To account for these we assign a complex mass M_i to the intermediate state which follows the emission of the ith pion (see Fig. 8). More precisely, we will use the notation $M_i(Q)$ if the intermediate state appears before the weak vertex, and $M_i(Q')$ if it appears after the weak vertex. Let us define

$$\begin{aligned}\Delta_i &= M(Q) - M_i(Q) \\ &= M(Q') - M_i(Q'), \qquad i = 1, 2, \cdots, n,\end{aligned} \tag{35}$$

where $M(Q)$ is the mass of the initial state, and $M_n(Q')$ is the mass of the final state. The masses $M(Q')$ and $M_n(Q)$ are, respectively, those of the corresponding states with the heavy quarks Q and Q' interchanged. The two forms of (35) follow from the heavy flavor independence of the mass differences. Consider two examples. When all the pions are emitted after the weak vertex, the intermediate state after the weak vertex has the same quantum numbers as the initial state except the heavy quark Q is replaced by Q'. According to (35), the mass of this intermediate state is denoted by $M(Q')$. Likewise, when all the pions are emitted before the weak vertex, the intermediate state before the weak vertex has the same quantum numbers as the final state except its heavy quark and its mass is denoted by $M_n(Q)$. The amplitude corresponding to (32) becomes

$$\begin{aligned}A'(q_1, \cdots, q_n) &= \sum_{i=0}^{n} (-1)^{n-i} \frac{1}{v\cdot q_1 - \Delta_1} \cdot \frac{1}{v\cdot (q_1+q_2) - \Delta_2} \cdots \frac{1}{v\cdot (q_1+q_2+\cdots+q_{n-i}) - \Delta_{n-i}} \\ &\cdot \frac{1}{v\cdot (q_{n-i+1}+\cdots+q_n) + \Delta_{n-i} - \Delta_n} \cdots \frac{1}{v\cdot (q_{n-1}+q_n) + \Delta_{n-2} - \Delta_n} \\ &\cdot \frac{1}{v\cdot q_n + \Delta_{n-1} - \Delta_n}.\end{aligned} \tag{36}$$

One may repeat the same analysis following steps similar to (33), or one may simply observe that (36) can be reproduced from (32) by the substitutions

$$
\begin{aligned}
& v\cdot q_1 \to v\cdot q_1 - \Delta_1, \\
& v\cdot q_2 \to v\cdot q_2 + \Delta_1 - \Delta_2, \\
& \cdots \\
& v\cdot q_i \to v\cdot q_i + \Delta_{i-1} - \Delta_i, \\
& \cdots \\
& v\cdot q_n \to v\cdot q_n + \Delta_{n-1} - \Delta_n.
\end{aligned} \tag{37}
$$

In the rest frame of v, we have $v\cdot q_i = q_i^0$, and the substitutions become simple shifts in q_i^0's. Since the result (34) holds for arbitrary values of v, and $q_1, \cdots, q_n$, we conclude that

$$
A'(q_1, \cdots, q_n) = 0. \tag{38}
$$

(D). We now consider how to include closed loops. A closed loop can be produced by setting $q_i = -q_j$ and supplying the appropriate propagator for a light particle of momentum q_i and finally integrating over the loop momentum q_i. Since the amplitude vanishes for arbitrary q_i and q_j, this procedure will not alter the conclusion (34). There is only a minor complication when the closed loops form self energy parts of the external legs. In these cases, the sum of the momenta $q_i + \cdots + q_l$ in the loop(s) vanishes and some of the propagators become singular. We can regulate these singularities by supplying temporarily small, but nonzero, mass differences to these lines similar to the case of excited states. This procedure is equivalent to lifting the external legs slightly off the mass shell.

(E). It is straightforward to generalize our discussion to heavy baryons. Introducing the superfields

$$
\begin{aligned}
S^\mu &= B_6^{*\mu} - \frac{1}{\sqrt{3}}(\gamma^\mu + v^\mu)\gamma_5 B_6, \\
\bar{S}^\mu &= \bar{B}_6^{*\mu} + \frac{1}{\sqrt{3}}\bar{B}_6\gamma_5(\gamma^\mu + v^\mu), \\
T &= B_{\bar{3}},
\end{aligned} \tag{39}
$$

where $B_{\bar{3}}$, B_6 and B_6^* are baryon component fields for spin-$\frac{1}{2}$ antitriplet, sextet, and spin-$\frac{3}{2}$ sextet baryon states in SU(3) flavor representations, respectively (see ref.[1] for details). The relevant chiral Lagrangian and weak Lagrangian are [8,9]

$$
\begin{aligned}
\mathcal{L}_B &= -i\mathrm{tr}(\bar{S}^\mu v \cdot DS_\mu) + \frac{i}{2}\mathrm{tr}(\overline{T} v \cdot DT) + \Delta\,\mathrm{tr}(\bar{S}^\mu S_\mu) \\
&\quad + i\frac{3}{2} g_1 \epsilon_{\mu\alpha\beta\nu}\mathrm{tr}(\bar{S}^\mu v^\alpha \mathcal{A}^\beta S^\nu) - \sqrt{3}\, g_2 \mathrm{tr}(\bar{S}^\mu \mathcal{A}_\mu T) + h.c., \qquad (40) \\
\mathcal{L}_W &= C_{ji}\Big\{ \overline{S}'^\lambda(v')\gamma_\mu(1-\gamma_5)S^\kappa(v)[-g_{\lambda\kappa}\xi_1(v\cdot v') + v_\lambda v'_\kappa \xi_2(v\cdot v')] \\
&\quad + \overline{T}'(v')\gamma_\mu(1-\gamma_5)T(v)\zeta(v\cdot v')\Big\}, \qquad (41)
\end{aligned}
$$

where $\Delta = m_S - m_T$ is the mass splitting between the sextet and antitriplet baryon multiplets. As mentioned before, $\zeta(1) = \xi_1(1) = 1$ and $\xi_2(1)$ drops out in the amplitude. From the examples of Eqs.(40) and (41) it is clear that because of the spin and flavor symmetry of the heavy quarks, it is also true in the heavy baryon sector that (i) the coupling constants and structure of interactions between light particles and heavy baryons are the same for baryons containing a different heavy quark, and (ii) the numerator for the matrix element of emission of n light particles is the same regardless of the number of particles emitted before and after the weak vertex as long as the n particles are emitted in the same sequence. This is because the Dirac matrices only appear at the weak vertex which are coupled to the heavy quark indices. Moreover, the only contributing Isgur-Wise function has the universal value unity. For example, the numerator for the amplitude of $S \to S' + n\pi + \ell\nu$ is of the form $\bar{S}'^\beta\gamma_\mu(1-\gamma_5)\cdots S_\alpha$, where the expression represented by the ellipses $\cdots$ depends on the intermediate states involved but are free of Dirac matrices. The light quarks in a heavy baryon behave as a whole like a Bose system and the vertices for light particle emissions involve the same string of vectorial indices for all the diagrams in Fig. 8 with heavy mesons replaced by heavy baryons. Therefore, these diagrams will lead to a partial amplitude proportional to $A(q_1, \cdots, q_n)$ of Eq.(34). Thus, light particle emissions are suppressed at zero recoil.

IV. Photon Emissions

As pointed out in the Introduction, photon emissions are interesting because the heavy quark's electromagnetic interactions violate heavy quark's flavor symmetry , and the coupling strengths for pion and photon emission are comparable (the former is given by $\frac{q}{f_\pi}g$ with

q being the pion momentum, and the latter is given by e.) It is therefore important to investigate whether light particle emissions are still suppressed at the point of no recoil in the presence of electromagnetic interactions.

However, photons come from several different sources. Some of these photons preserve heavy quark symmetry. Among these are the photons emitted by the usual convection currents and magnetic moment (Pauli terms) couplings of the light quarks in the heavy hadrons. These photons can be included in the category of other light particles discussed in the last section. Photons can also be emitted by the charged light particles themselves, such as pions and ρ mesons. These interactions belong to another category already treated in the last section in which many particles are emitted at a single vertex. Finally, photons can be emitted from the heavy quarks. A heavy quark's magnetic moment is of order $1/m_Q$ and hence it can be neglected in our present discussion. Thus, we only have to deal with the photons emitted by the heavy quark's convection currents.

We will show in this section that the suppression of light hadron emissions at the point of no recoil still presisits when photon emissions from the heavy quarks are included. The reason is that a heavy quark's convection current is effectively an identity operator provided the emitted photon's momentum is much smaller than the heavy quark mass m_Q. In terms of Feynman diagrams, the above statement translates into a factorization of the photon emission amplitude and the light hadron emission amplitude. At the end of this section, we will conclude that photon emissions accompanied by any light hadron emission is forbidden at the point of no recoil. The only photons emitted are those from the heavy quarks and the charged lepton and are to accompany the basic processes $\bar{B} \to D(D^*) + \ell\nu$.

We should emphasize the distinction between the factorization of photon emissions and the light hadron emissions here and the well-known result in soft photon physics. The photons emitted here are soft compared with the heavy quark mass m_Q, but they are not soft compared with the other light hadrons emitted. Consequently, we have to include photons emitted from the interior of a Feynman diagram, while the really soft photons can only be emitted from the external legs.

We begin by considering the part of the amplitude in which a photon of momentum k and n_1 light hadrons of momenta $q_1, \cdots, q_{n_1}$ are emitted from the initial heavy quark (see Fig. 9). The photon can be emitted anywhere on the initial heay quark line, and the light hadrons with momenta $q_1, \cdots q_{n_1}$ are emitted from the initial heavy quark in a specified sequence. This partial amplitude is proportional to

$$\begin{aligned}
A(k, q_1, \cdots, q_{n_1}) &= (-1)^{n_1+1} e_Q (v \cdot \varepsilon) \\
&\times \sum_{i=0}^{n} \frac{1}{v \cdot q_1} \cdot \frac{1}{v \cdot (q_1 + q_2)} \cdots \frac{1}{v \cdot (q_1 + q_2 + \cdots + q_i)} \cdot \frac{1}{v \cdot (q_1 + \cdots + q_i + k)} \\
&\cdot \frac{1}{v \cdot (q_1 + q_2 + \cdots + q_i + q_{i+1} + k)} \cdots \frac{1}{v \cdot (q_1 + q_2 + \cdots + q_{n_1} + k)},
\end{aligned} \tag{42}$$

where e_Q is the electric charge of the initial quark and ε is the photon's polarization vector. Adding up the terms one by one sequentially in the order of increasing i, we find the factorized form

$$A(k, q_1, \cdots, q_{n_1}) = \left(-e_Q \frac{v \cdot \varepsilon}{v \cdot k}\right) \left[(-1)^{n_1} \frac{1}{v \cdot q_1} \cdot \frac{1}{v \cdot (q_1 + q_2)} \cdots \frac{1}{v \cdot (q_1 + q_2 + \cdots + q_{n_1})}\right]. \tag{43}$$

The above result can be stated in words: insertion of a photon anywhere on the initial heavy quark line gives rise to a factorized amplitude as a product of the one photon emission amplitude and the amplitude of multiparticle emission. We can repeat the procedure by inserting the second photon everywhere, etc. We then find the amplitude for l_1 photons emitted in addition to n_1 light hadrons emitted from the initial heavy quark to be given by

$$\begin{aligned}
A^i(k_1, \cdots, k_{l_1}, q_1, \cdots q_{n_1}) &= \left(-e_Q \frac{v \cdot \varepsilon_1}{v \cdot k_1}\right) \cdots \left(-e_Q \frac{v \cdot \varepsilon_{l_1}}{v \cdot k_{l_1}}\right) \\
&\times \left[(-1)^{n_1} \frac{1}{v \cdot q_1} \cdot \frac{1}{v \cdot (q_1 + q_2)} \cdots \frac{1}{v \cdot (q_1 + \cdots + q_{n_1})}\right].
\end{aligned} \tag{44}$$

A similar result holds for emissions of l_2 photons and n_2 light hadrons from the final heavy quark of electric charge $e_{Q'}$ (Fig. 10):

$$\begin{aligned}
A^i(k_{l_1+1}, \cdots, k_{l_1+l_2}, q_{n_1+1}, \cdots q_{n_1+n_2}) &= \left(e_{Q'} \frac{v \cdot \varepsilon_{l_1+1}}{v \cdot k_{l_1+1}}\right) \cdots \left(e_{Q'} \frac{v \cdot \varepsilon_{l_1+l_2}}{v \cdot k_{l_1+l_2}}\right) \\
\times \left[\frac{1}{v \cdot (q_{n_1+1} + \cdots + q_{n_1+n_2})} \cdot \frac{1}{v \cdot (q_{n_1+2} + \cdots + q_{n_1+n_2})} \cdots \frac{1}{v \cdot q_{n_1+n_2}}\right]&.
\end{aligned} \tag{45}$$

We now combine the two results (44) and (45) by keeping fixed both the n_1 and n_2 particles emitted by the two heavy quarks, respectively, and the total number of photons emitted,

$l_1 + l_2 = l$, varying l_1 and l_2 and permuting the photons in all possible ways. The result is

$$
\begin{aligned}
& A(k_1, \cdots, k_l, q_1, \cdots, q_{n_1}, q_{n_1+1}, \cdots, q_{n_1+n_2}) \\
& = \prod_{i=1}^{l} (-e_Q + e_{Q'}) \frac{v \cdot \varepsilon_i}{v \cdot k_i} \cdot \Big[(-1)^{n_1} \frac{1}{v \cdot q_1} \cdot \frac{1}{v \cdot (q_1 + q_2)} \cdots \frac{1}{v \cdot (q_1 + \cdots + q_{n_1})} \\
& \cdot \frac{1}{v \cdot (q_{n_1+1} + \cdots + q_{n_1+n_2})} \cdot \frac{1}{v \cdot (q_{n_1+2} + \cdots + q_{n_1+n_2})} \cdots \frac{1}{v \cdot q_{n_1+n_2}} \Big]. \qquad (46)
\end{aligned}
$$

The result (46) holds for any choice of n_1 and n_2. Adding up all contributiuons with the fixed sum $n_1 + n_2 = n$ and keeping the sequence of light hadrons unchanged as they move across the weak vertex, we find the amplitude for emission of l photons and n light hadrons in a definite sequence is

$$
\begin{aligned}
A(k_1, \cdots, k_l, q_1, \cdots, q_n) &= \prod_{i=1}^{l} (-e_Q + e_{Q'}) \frac{v \cdot \varepsilon_i}{v \cdot k_i} A(q_1, \cdots, q_n) \\
&= 0, \qquad (47)
\end{aligned}
$$

where $A(q_1, \cdots, q_n)$ is given by (32) and (34).

To include the excited states in the above discussion, one proceeds exactly the same as the discussion on particle emission. The same conclusion (47) holds. To be precise, we should have included photon emissions from the charged lepton in the above amplitude. Since these contributions are already clearly in the factorized form, they will not affect the conclusion above. Eq.(47) states that at the point of no recoil, suppression of light hadron emissions still persists in the presence of photon emissions from the heavy quarks.

For completeness, we give the results for photon emissions which decorate the basic processes $\bar{B} \to D(D^*) + \ell\nu$. To arrive at a gauge invariant result, it is necessary to include photon emissions from the charged lepton. In addition, to obtain a simple closed form, we will assume that photons are soft compared with the charged lepton momentum. Under these conditions, the leading contributions from the charged lepton are due to its convection current. The treatment of summing up these photon emissions is well-known [10]. The result at zero recoil involves the basic amplitude for a single photon emission

$$
A(k) = (-e_Q + e_{Q'}) \frac{v \cdot \varepsilon}{v \cdot k} + e_\ell \frac{v_\ell \cdot \varepsilon}{v_\ell \cdot k}, \qquad (48)
$$

where e_ℓ and v_ℓ are the electric charge and velocity of the charged lepton. Eq.(48) is gauge invariant as a result of the charge conservation $-e_Q + e_{Q'} + e_\ell = 0$. Contributions from arbitrary number of virtual photons supply a multiplicative factor to the amplitude for $\bar{B} \to D(D^*) + \ell\nu$:

$$V = e^X, \tag{49}$$

with

$$X = \frac{1}{2}e_\ell^2 \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 + i\epsilon} \left(\frac{v}{v \cdot k} - \frac{v_\ell}{v_\ell \cdot k}\right)^2, \qquad \lambda < |\vec{k}| < \Lambda. \tag{50}$$

The correction to the rate due to virtual photons is then given by the factor

$$|V|^2 = e^{2\mathrm{Re}X} = \left(\frac{\lambda}{\Lambda}\right)^A, \tag{51}$$

$$A = -\frac{2\alpha}{\pi}\left\{1 - \frac{v \cdot v_\ell}{\sqrt{(v \cdot v_\ell)^2 - 1}} \ln[v \cdot v_\ell + \sqrt{(v \cdot v_\ell)^2 - 1}]\right\}. \tag{52}$$

Summing up the contributions from all real photons leads to the multiplicative factor to the rate

$$\mathcal{R} = \left(\frac{E}{\lambda}\right)^A F(\frac{E}{E_T}, A), \tag{53}$$

where

$$F(x, A) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{du}{u} \mathrm{sin}u \exp\left[A \int_0^x \frac{dk}{k}\left(e^{iku} - 1\right)\right]. \tag{54}$$

The energy parameters E and E_T are introduced in the constraints for individual photon energy and the total energy carried by the photons:

$$\lambda < k_i < E, \tag{55}$$

$$\sum_i k_i < E_T. \tag{56}$$

The virtual and real photons together produce a correction factor to the rate for $\bar{B} \to D(D^*) + \ell\nu$:

$$e^{2\mathrm{Re}X} \cdot \mathcal{R} = \left(\frac{E}{\Lambda}\right)^A F(\frac{E}{E_T}, A), \tag{57}$$

where Λ is a factorization scale for separating soft and hard photons, and E is the energy below which a photon cannot be detected. The Λ dependence in (57) is canceled by the hard photon corrections to the cross sections for $\bar{B} \to D(D^*) + \ell\nu$. A judicious choice of Λ will minimize the effects of hard photons, and (57) will be the dominant radiative correction factor. For instance, this occurs when $A \ln(W/\Lambda) << 1$, where W is the typical energy in the process. For B decays at zero recoil, there is a wide latitude for such a choice of Λ.

Identical discussions apply to the CKM favored semileptonic decays of heavy baryons. Again, the photons are the only light particles that can be emitted in this limit; and they are to accompany the simple processes $S_b \to S_c + \ell\nu$, $T_b \to T_c + \ell\nu$. The photon emissions are described by the same multiplicative factors given by (57).

V. Connection between Formal Argument and Diagrammatic Analysis

An important and interesting result such as the one discussed in this paper deserves, on the one hand, a derivation from some general principle, and, on the other hand, a deeper understanding how it actually works in a concrete framework. The original argument in [4,5] is of the first kind, while our discussion of the last two sections is of the second kind. In this section, we offer a proof of the key result (34) which provides a link between the general argument and the diagrammtic analysis. In doing so we have discovered that the general result does not require the full power of the formal argument. It is hoped that our work will stimulate more research on this subject.

The key result (34) is derived without reference to the details of the vertices $\not{A}(q_i)$ in (30) except that they are independent of heavy quark's spin and flavor. For all purposes, they can be regarded as external sources. We are motivated to consider the vertex function of the weak current

$$\mathcal{J}_\mu(v \cdot v'|K) \equiv \langle H_c(v')|J_\mu^W(0)|H_b(v)\rangle = -\xi(v \cdot v'|K)\mathrm{tr}[\overline{H}_c(v')\gamma_\mu(1-\gamma_5)H_b(v)], \tag{58}$$

where $H_Q(v)$ denotes the wave function of a heavy meson with quark content $Q\bar{q}$. Its explicit form is $\frac{1+\not{v}}{2}\not{\epsilon}$ for a vector meson and $\frac{1+\not{v}}{2}(-\gamma_5)$ for a pseudoscalar meson. The argument K in the Isgur-Wise function emphasizes the fact that ξ is a functional of the external source $K(x)$ which appears in the interaction (see Fig. 11)

$$\mathcal{L}_K = \sum_{Q=c,b} \mathrm{tr}(\overline{H}_Q H_Q K), \tag{59}$$

where

$$K(x) = \sum_i K_i(x) t_i, \tag{60}$$

and t_i is possibly a combination of Dirac and SU(3) flavor matrices. The external source respects heavy quark symmetry. We will further assume that its Fourier component $K(q)$ carries momentum much smaller than m_Q. The standard argument still applies to obtain

$$\xi(v \cdot v' = 1|K) = 1, \tag{61}$$

which states that the external source does not alter the state of a heavy quark, and the weak current at the point of no recoil can only convert a $\bar{B}$ meson into a D or D^* meson. Now, the right-hand side of (61) is independent of the external source. Consequently, its functional derivatives evaluated at zero source must vanish:

$$\frac{\delta^n}{\delta K(q_1)\delta K(q_2)\cdots\delta K(q_n)} \xi(v \cdot v' = 1|K)\bigg|_{K=0} = 0. \tag{62}$$

Or equivalently,

$$\frac{\delta^n}{\delta K(q_1)\delta K(q_2)\cdots\delta K(q_n)} \mathcal{J}_\mu(v \cdot v' = 1|K)\bigg|_{K=0} = 0. \tag{63}$$

A simple evaluation gives

$$\begin{aligned} \frac{\delta^n}{\delta K(q_1)\delta K(q_2)\cdots\delta K(q_n)} \mathcal{J}_\mu(v \cdot v' = 1|K)\bigg|_{K=0} &= N'_n A(q_1, \cdots, q_n) \\ &+ \text{ all permutations in } (1, \cdots, n), \end{aligned} \tag{64}$$

where N'_n is very similar to N_n in (30):

$$N'_n = -\mathrm{tr}\left[\overline{H}_c \gamma_\mu (1-\gamma_5) H_b t_1 \frac{1-\not{v}}{2} t_2 \cdots \frac{1-\not{v}}{2} t_n\right], \tag{65}$$

and $A(q_1, \cdots, q_n)$ is the same as (32). Eqs.(61) and (62) together establish the connection between the formal and diagrammatic appraches. However, we find it interesting that our explicit calculation in Section 3 shows that $A(q_1, \cdots, q_n) = 0$ all by itself without having to combine with similar contributions from permutations. Does this fact imply that there is more information in (34) than what we can extract so far ? On the other hand, the numerator N'_n given by (65) is not symmetric with respect to the light particles. It contains noncommuting Dirac and/or SU(3) flavor matrices, and each source may refer to different species of particles. Were it essential to include all the $n!$ sets of diagrams, the analysis in Section 3 would have been much more complicated. The utility of employing an external source allows us to, effectively, deal with all the multiparticle final states at once. The single equation (61) contains all the physics needed to derive the general result (62).

VI. Discussions and Conclusions

In this section we discuss our results and their implications. First of all, let us recapitulate what we have accomplished in this work. We have established the result of refs.[4,5] by a diagrammatic analysis, and have generalized it to include photon emissions. The diagrammatic approach is complementary to the formal arguments of refs.[4,5]. The two approaches are united by considering the Isgur-Wise function in the presence of an external source which is independent of heavy quark's spin and flavor.

Thanks to heavy quark symmetry, it is possible to make a precise statement on some properties of systems containing a heavy quark. The subject investigated in this paper is such an example. The conclusion of our work is the following exact statement: For CKM favored semileptonic decays of a $\bar{B}$ meson or a bottom baryon, the only light particles that can be emitted at the point of no recoil are photons emitted from the heavy quarks and the charged lepton. These photons are to accompany the basic decay $\bar{B} \to D(D^*)\ell\nu$. The corrections to the above statement are of order $1/m_Q^2$ in the decay rates. Similar statements apply to the heavy baryons S_b and T_b.

In the so-called Shifman-Voloshin limit [4], $(m_Q + m_{Q'})\Lambda_{\rm QCD} << (m_Q - m_{Q'})^2 << (m_Q+m_{Q'})^2$, the variable $v.v' = 1+\mathcal{O}[(m_Q-m_{Q'})/(m_Q+m_{Q'})]^2$, so light particle emissions are severely suppressed. The decays $\bar{B} \to D(D^*)+\ell\nu$ are not far from this limit. Therefore, we do not expect B decays involving additional pions or excited states of D and D^* to have significant rates.

The conclusion of this paper has at least another important practical application. The fundamental parameter V_{cb} of the Standard Model is directly related to the differential decay rate of $\bar{B} \to D^*+\ell\nu$ at $v = v'$. Since the Isgur-Wise function has the value unity at this point,

V_{cb} is the only unknown in this decay rate. At CESR the B and $\bar{B}$ mesons are produced at rest, so the decay rate for $\bar{B} \to D + \ell\nu$ at $v = v'$ is vanishingly small compared with the D^* mode due to conservation of angular momentum[1]. According to the Luke's theorem, the corrections to the Isgur-Wise function is of order $1/m_Q^2$ while corrections due to light particle emissions and excited charmed meson states are also of the same order. It appears that using $\bar{B} \to D^* + \ell\nu$ at this kinematic point to measure V_{cb} is possibly the cleanest way.

In this paper we have restricted ourselves to emissions of light particles with momenta small compared with m_Q. This is necessary in order for us to apply heavy quark symmetry. In principle, we can ensure that we are in this kinematic region for decays of B mesons at rest by selecting lepton pairs with the total lepton energy to be close to $m_B - m_{D^*}$ [11].

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[1]The ratio for the two rates near zero recoil is $\Gamma_D : \Gamma_{D^*} = v_D^2 : 12 \left(\frac{m_b - m_c}{m_b + m_c}\right)^2$, where v_D is the velocity of the D meson.

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11. We would like to thank Prof. Persis Drell for a useful conversation on how CLEO measures the missing momentum and energy of neutrinos.

FIGURE CAPTIONS

Fig. 1 Feynman diagrams contributing to the decay $\bar{B} \to D + \pi + \ell\nu$. The weak vertex is represented by a black dot. The pion momentum is q.

Fig. 2 Feynman diagrams contributing to the decay $\bar{B} \to D^* + \pi + \ell\nu$. The weak vertex is represented by a black dot. The pion momentum is q.

Fig. 3 Feynman diagrams contributing to the decay $\Sigma_b \to \Sigma_c + \pi + \ell\nu$. The weak vertex is represented by a black dot. The pion momentum is q.

Fig. 4 Feynman diagrams contributing to the decay $\Sigma_b \to \Sigma_c^* + \pi + \ell\nu$. The weak vertex is represented by a black dot. The pion momentum is q.

Fig. 5 Feynman diagrams contributing to the decay $\Sigma_b \to \Lambda_c + \pi + \ell\nu$. The weak vertex is represented by a black dot. The pion momentum is q.

Fig. 6 A typical Feynman diagram contributing to $H_Q \to H_{Q'} + \pi_1 + \pi_2 + \cdots + \pi_n + \ell + \nu$. The black dot denotes the weak vertex.

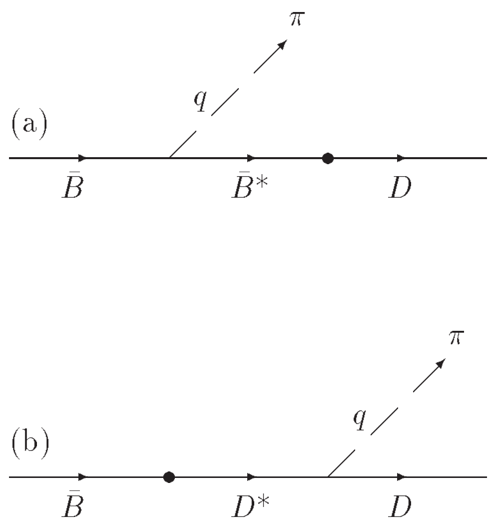
Fig. 7 Graphic representation of the numerator of the amplitudes for all diagrams of the type in Fig. 6 with the pion sequence unchanged as the index i varies from 0 to n.

Fig. 8 A typical Feynman diagram contributing to n pion emissions with excited heavy mesons as intermediate and final states. At the point of no recoil, the two states next to the weak vertex (the black dot) must have the same light quark quantum numbers but with a different heavy quark.

Fig. 9 A typical Feynman diagram contributing to the emission of a photon of momentum k and n_1 pions from the initial heavy meson H_Q.

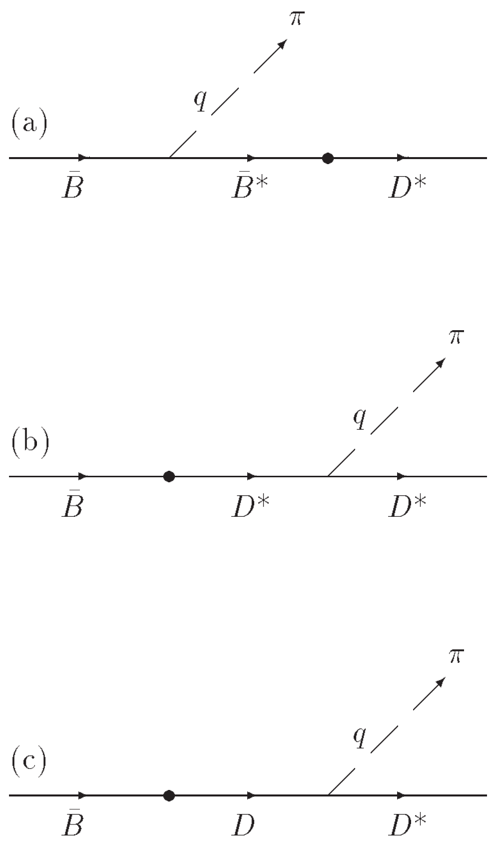
Fig. 10 A typical Feynman diagram contributing to the emission of a photon of momentum k and n_2 pions from the final heavy meson $H_{Q'}$.

Fig. 11 A typical Feynman diagram contributing to the Isgur-Wise function in the presence of an external source represented by the crosses.



(a)
π
q
$\bar{B}$
$\bar{B}*$
D
(b)
π
q
$\bar{B}$
$D*$
D

FIG. 1.



(a)
π
q
$\bar{B}$
$\bar{B}*$
$D*$
(b)
π
q
$\bar{B}$
$D*$
$D*$
(c)
π
q
$\bar{B}$
D
$D*$

FIG. 2.

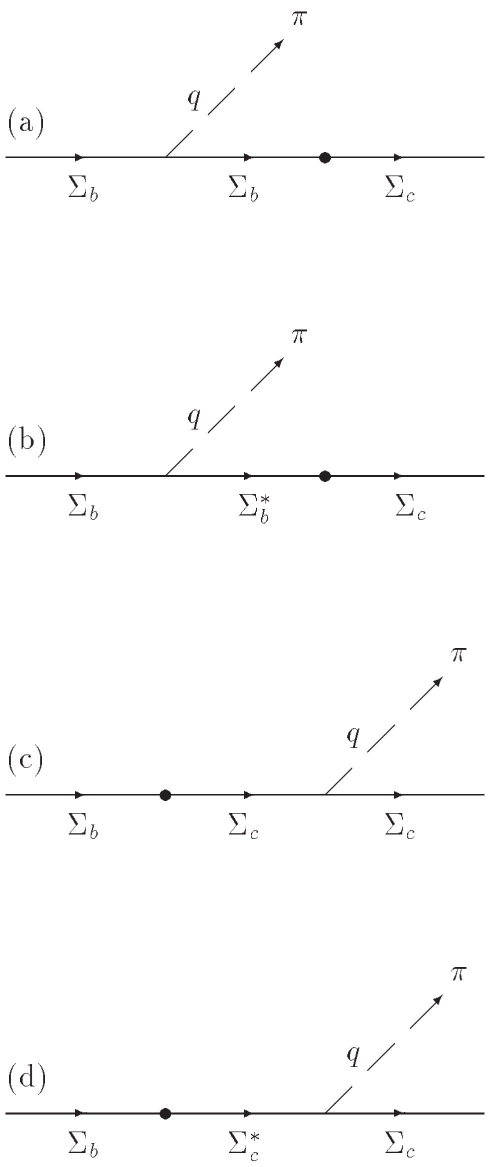

(a)
q
π
Σb
Σb
Σc
(b)
q
π
Σb
Σb*
Σc
(c)
q
π
Σb
Σc
Σc
(d)
q
π
Σb
Σc*
Σc

FIG. 3.

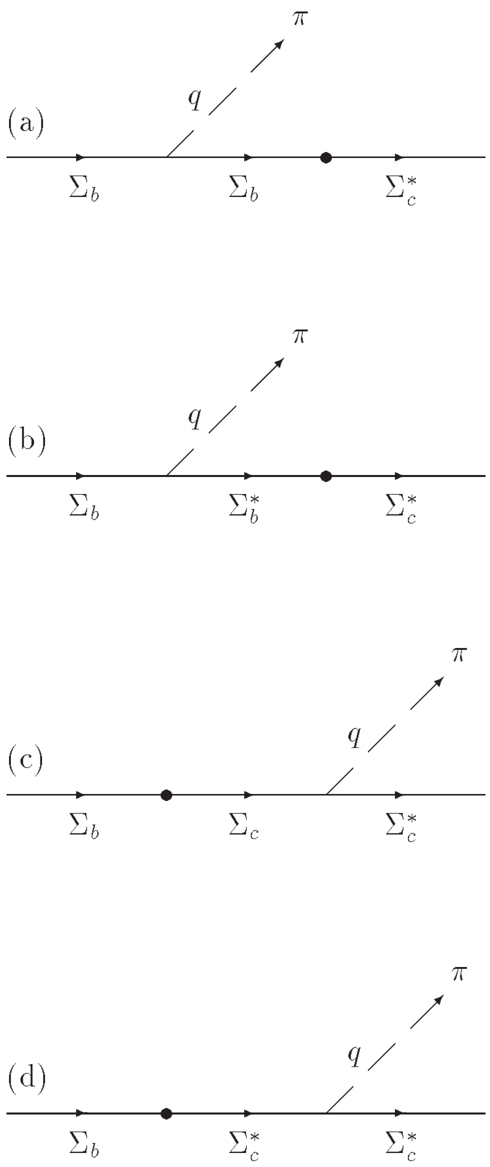
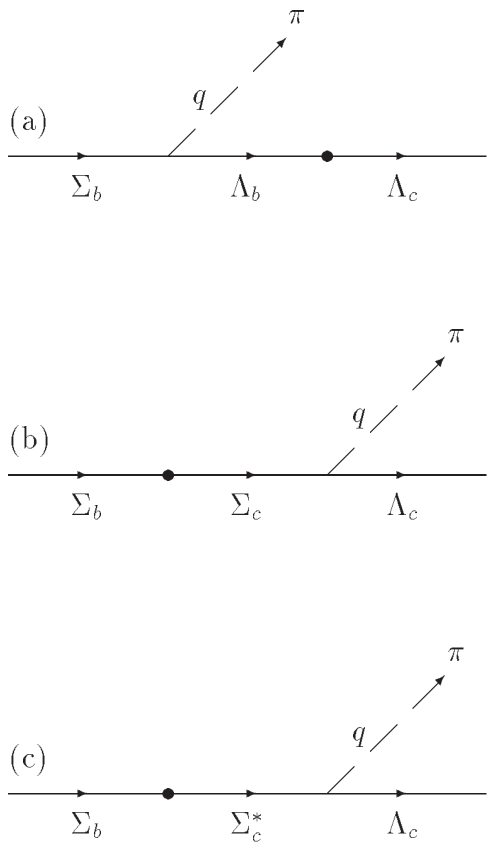

(a)
q
π
Σb
Σb
Σc*
(b)
q
π
Σb
Σb*
Σc*
(c)
q
π
Σb
Σc
Σc*
(d)
q
π
Σb
Σc*
Σc*

FIG. 4.



(a)
q
π
Σ_b
Λ_b
Λ_c
(b)
q
π
Σ_b
Σ_c
Λ_c
(c)
q
π
Σ_b
Σ_c^*
Λ_c

FIG. 5.

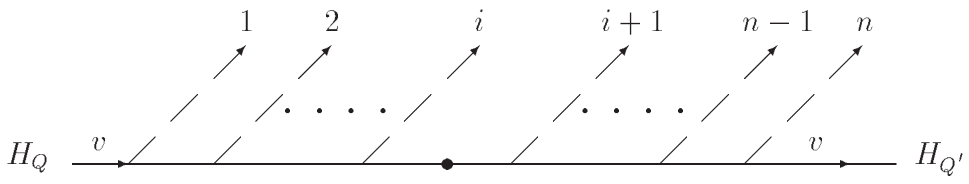


FIG. 6

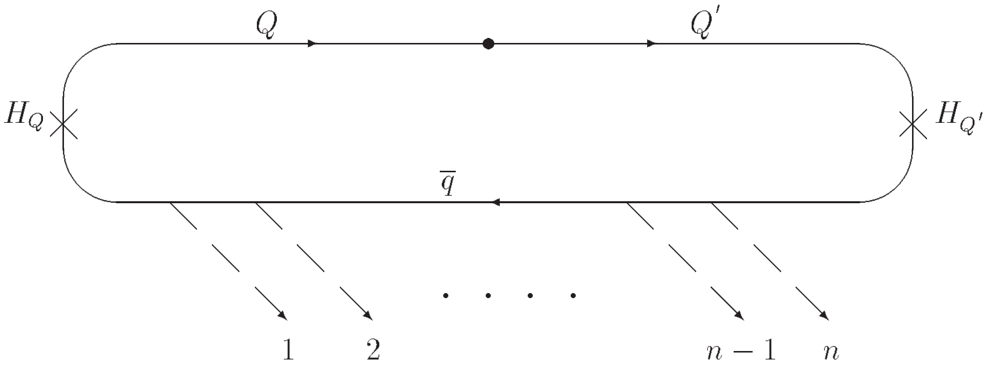


FIG. 7.

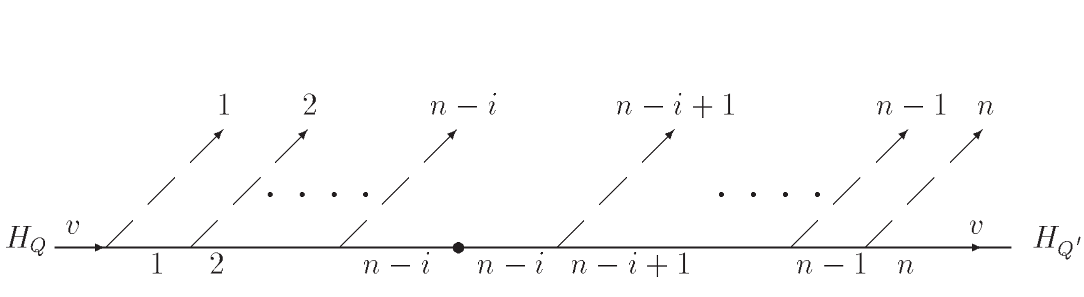


FIG. 8.

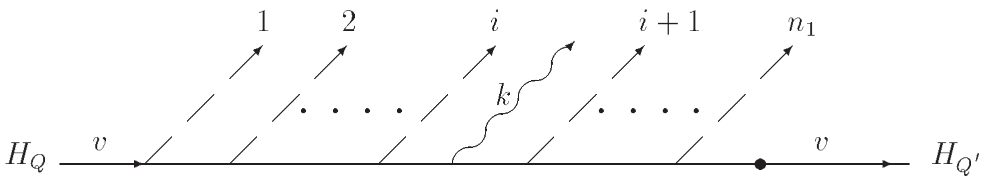


FIG. 9

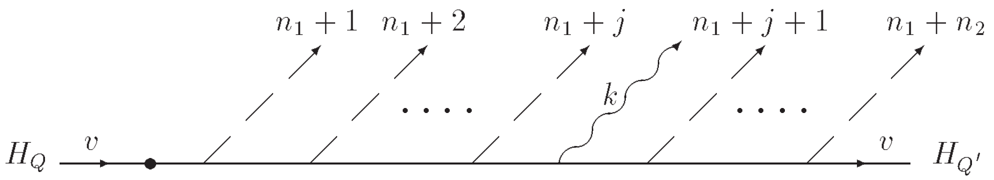


FIG. 10.

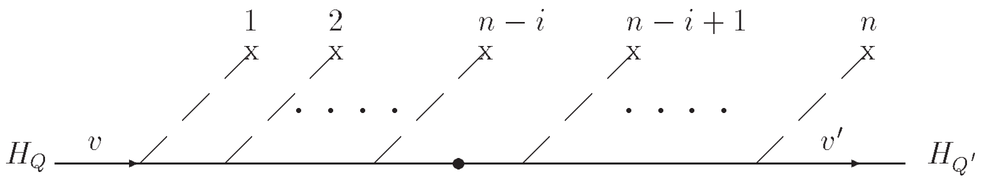


FIG. 11