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The Impossibility of Baryogenesis at a Second Order Electroweak Phase Transition

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Abstract

We investigate whether baryogenesis is possible at a second order electroweak phase transition. We find that under rather general conditions, the departure from thermal equilibrium is suppressed by the expansion rate of the Universe, and hence baryon production is also suppressed by the expansion rate. We conclude that if no additional sources of departure from thermal equilibrium such as topological defects are present, then electroweak baryogenesis is ruled out if the phase transition is second order or crossover. However, a non-vanishing net baryon to entropy ratio *is* generated, and we provide both upper and lower bounds for the result. Our technique is also applicable to other baryogenesis mechanisms taking place during and immediately after a second order phase transition. We estimate the lowest value of the transition energy scale for which the resulting baryon to entropy ratio might be large enough to explain observations.

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1 Introduction

Recently the constraints on the Higgs sector, at least in the Minimal Standard Model have improved so that the prevalent view today is that the electroweak phase transition is either very weakly first order, second order, or perhaps even crossover. The first evidence is a direct lower limit for the mass of the minimal Higgs particle $M_H > 65$GeV (95% C.L.) set in experiments at CERN [1] (for a review, see for example [2]) and based on the lack of Higgs production in hadronic decays of the Z boson. An indirect limit arises from the precision electroweak data; when the CDF and D0 measurements of the top quark mass are taken account of ($m_t \simeq 181 \pm 12$GeV), one arrives at the 1σ allowed range for the Minimal Standard Model (MSM) Higgs mass: 26GeV$< M_H < 230$GeV (68% C. L.) [4]. In addition the requirement for (meta)stability of the electroweak vacuum [5], [6] implies a lower bound of $M_H > 116$GeV, and the requirement that the Standard Model couplings remain perturbative up to a scale $\Lambda_p \sim M_{Pl} \sim 10^{19}$GeV results in a perturbative upper bound $M_H \leq 190$GeV [7]. Taking into account all of the above we see that there are strong indications that the MSM Higgs mass is between 100 and 200 GeV.

In theories with an extended Higgs sector (with two or more Higgs doublets) the experimental limits on the mass of the lightest Higgs scalar are somewhat weaker. For example, in the Minimal Supersymmetric Standard Model (MSSM) the direct limits from the LEP experiments [8] on the lightest neutral Higgs boson mass are roughly $M_h > 40$GeV. The direct constraints on the mass of the second neutral Higgs boson M_A become rather weak once the radiative corrections from other supersymmetric particles of unknown mass are taken into account. The meta-stability bounds do not apply to the MSSM vacuum, while the intrinsic upper bound on M_h is about 140GeV (for $m_t \leq 190$GeV) [9]. So, in the case of the MSSM, the bounds on the lightest Higgs mass indicate a somewhat lighter Higgs particle: 40GeV $< M_h < 140$GeV.

Next we discuss how the nature of the electroweak phase transition depend on the details of the Higgs sector, and in particular on the mass of the (lightest) Higgs particle. We now understand the nature of the phase transition in the Minimal Standard Model for a moderately light Higgs $M_H \leq m_W \simeq 81$GeV. The pictures of the phase transition emerging from two-loop calculations [10], [11] and lattice calculations [12] agree and indicate that the transition is strongly first order for small Higgs mass and becomes weaker as the mass becomes comparable to m_W. It is believed that for a sufficiently large Higgs mass the phase transition eventually becomes second order [13], [14], [15] or even a crossover, since at large Higgs masses the theory simply resembles more and more scalar theory. At what Higgs mass this exactly happens is not clear at the moment. Also how the dynamics of the phase transition is affected by the presence of more than one Higgs doublet, as is the case in the supersymmetric versions of the Standard Model, is not completely understood. It is possible, for example, that the phase transition proceeds in two stages [16], but we will not consider this possibility here. We are mainly interested in how the strength of the phase transition changes; even though there is no complete analysis of a general supersymmetric two Higgs doublet model, in the case of the MSSM it is known [17] that the stop sector tends to strengthen and a light CP odd neutral Higgs scalar tends to weaken the phase transition. We conclude that, given the above considerations, it is likely that the electroweak phase transition is second order or crossover.

In this letter we will address the question: *Is baryogenesis at a second order electroweak phase transition ruled out?* (For reviews of electroweak baryogenesis see [18], [19].) We will assume that no other sources of departure from thermal equilibrium are present except for those caused by the phase transition and the expansion rate of the Universe. This means, for example, that we will not consider the case when one has a network of cosmological defects [20] floating around that could drive the system out of equilibrium.

The letter is organized as follows. In Section 2 we derive a simple dissipative equation of motion which out of equilibrium may lead to a net baryon number production. In the subsequent section we analyze this equation, derive an upper bound on the number of baryons produced and find that it is suppressed as the expansion rate of the Universe. In Section 4 we discuss a lower bound on the strength of baryogenesis at a second order phase transition and use the result to estimate the minimal energy scale for a second order baryogenesis mechanism without extra out of equilibrium effects such as topological defects. In the final section we summarize our results and their consequences for electroweak baryogenesis.

2 Equation of motion

We start from a standard near equilibrium formula of statistical mechanics for the rate of approach to equilibrium of some charge Q

$$\dot{Q} = -\Gamma_Q \frac{\Delta F}{T} \tag{1}$$

where $\mu_Q = \Delta F$, the chemical potential for Q, measures by how much the free energy changes when Q changes by one unit, and Γ_Q is the rate of decay of Q (per unit time). This equation is a macroscopic form of the charge (non)conservation and can be thought of as an integral of the Boltzmann equation (see for example [28]).

When applied to the baryon number ($Q = B$), we get (see [21] for a similar analysis):

$$\dot{n_B} \equiv \frac{\dot{B}}{V} = -\frac{\Gamma_{sph}}{V}\frac{\Delta F_{(\Delta B=1)}}{T}\Delta B(sph)\,, \quad \Delta F_{(\Delta B=1)} = \mu_B \tag{2}$$

$\dot{B}$ is the total rate of change of baryon number, Γ_{sph}/V the sphaleron rate per unit volume, T is the temperature of the plasma, $\mu_B = \Delta F_{(\Delta B=1)}$ the chemical potential for baryon number (which we defined as the change in free energy when baryon number changes by one unit), and $\Delta B(sph)$ is the change in baryon number per sphaleron transition. In the symmetric phase the sphaleron rate is $\Gamma_{sph}/V = \kappa(\alpha_w T)^4$ with $\kappa \simeq 1.1$ [22], [24], and in the broken phase

$$\frac{\Gamma_{sph}}{V} \propto \exp -\frac{E_{sph}}{T}, \tag{3}$$

where E_{sph} is the sphaleron energy.

We will now illustrate how using Eq. (2) and a one loop formula for the free energy of fermions in a thermal plasma one can obtain an equation for near equilibrium baryon number production. The (one-loop) thermal contribution to the free energy reads:

$$F = T\sum_i \int \frac{d^3p}{(2\pi)^3} \ln\left(1 + \exp(-\beta(E_i - \mu_i))\right), \tag{4}$$

where the *sum* $\sum_i$ is over all fermionic degrees of freedom and the energy in the presence of a non-vanishing $\dot{\theta}$ is given (in the WKB limit) by [25]

$$E_i = \left[(|\vec{p}| \mp g_\theta \dot{\theta})^2 + m_i^2\right]^{1/2} \qquad \text{for} \quad \Sigma^3 = \pm 1 \tag{5}$$

where $g_\theta = -v_2^2/2(v_1^2+v_2^2)$ is the 'θ-charge,' (which is related to the axial charge), v_1 and v_2 are the vacuum expectation values (*vevs*) of the two Higgs fields, respectively, and Σ^3 is the spin of the particle. In the zero mass limit the particles with $\Sigma^3 = +1$ reduce to the left handed fermions or left handed anti-fermions (anti-particles of the right handed fermions), whilst those with $\Sigma^3 = -1$ reduce to the right handed fermions or right handed anti-fermions. Even though this relation is derived using equilibrium techniques and $\dot{\theta}$ is a time dependent quantity, we expect it to be a good approximation to description of near equilibrium processes since the typical time scale at which $\dot{\theta}$ changes is given by the expansion rate of the Universe and the processes which equilibrate the system are typically much faster.

For any particle species denoted by a subscript i, it follows from Eqs. (1) and (2) that the number density n_i obeys the following equation (*cf.* [26]):

$$\dot{n}_i = -\frac{\Gamma_{sph}}{VT}\nu_i \sum_j \nu_j \mu_j \tag{6}$$

where ν_j are the stechiometric coefficients of the reaction and μ_j are the corresponding chemical potentials. In the case of sphalerons, $\Gamma_{sph}/2V$ is the rate per unit volume for each of the following two sphaleron transitions: $t_L t_L b_L \tau_L ... \leftrightarrow 0$, $t_L b_L b_L \nu_\tau ... \leftrightarrow 0$, where *dots* denote the particles of the lighter two families. The coefficients ν_i of the first process are for example $\nu_{t_L} = 2$, $\nu_{b_L} = 1$, *etc,* so that (6) for the left-handed top quarks and left-handed bottom quarks yields:

$$\begin{aligned}
\dot{n}_{t_L} &= -\frac{\Gamma_{sph}}{2VT}[5\mu_{t_L} + 4\mu_{b_L} + 2\mu_{\tau_L} + \mu_{\nu_\tau} \qquad (7)\\
&\quad +5\mu_{c_L} + 4\mu_{s_L} + 2\mu_{\mu_L} + \mu_{\nu_\mu} + 5\mu_{u_L} + 4\mu_{d_L} + 2\mu_{e_L} + \mu_{\nu_e}]\\
\dot{n}_{b_L} &= -\frac{\Gamma_{sph}}{2VT}[4\mu_{t_L} + 5\mu_{b_L} + \mu_{\tau_L} + 2\mu_{\nu_\tau} \qquad (8)\\
&\quad +4\mu_{c_L} + 5\mu_{s_L} + \mu_{\mu_L} + 2\mu_{\nu_\mu} + 4\mu_{u_L} + 5\mu_{d_L} + \mu_{e_L} + 2\mu_{\nu_e}]
\end{aligned}$$

Having in mind that the total baryon number density B is

$$B = \frac{1}{3}\sum_{\text{quark species}} n_i, \tag{9}$$

and $\dot{n}_{i_R} = 0$ for right handed particles, one gets:

$$\dot{B} = -N_F\frac{\Gamma_{sph}}{2VT}[3\mu_{t_L} + 3\mu_{b_L} + \mu_{\tau_L} + \mu_{\nu_\tau} \tag{10}$$

$$+3\mu_{c_L} + 3\mu_{s_L} + \mu_{\mu_L} + \mu_{\nu_\mu} + 3\mu_{u_L} + 3\mu_{d_L} + \mu_{e_L} + \mu_{\nu_e}]$$

where $N_F = 3$ is the number of families. Note that we have ignored the anti-particles; it is rather trivial to include them in Eq. (11): one should just subtract the chemical potentials for the anti-particles.

The final step is to relate the chemical potentials in this relation to the number densities. The particle number density can be easily obtained from Eq. (4) as follows:

$$n_i = -\frac{\partial}{\partial E_i} F = \int \frac{d^3p}{(2\pi)^3} \frac{1}{1 + \exp[\beta(E_i - \mu_i)]} \tag{11}$$

Next we expand the number density to linear order in μ_i:

$$\begin{aligned} n_i &= n_i^0 + \beta \int \frac{d^3p}{(2\pi)^3} \frac{\exp(\beta E_i^0)}{(1 + \exp(\beta E_i^0))^2} \left[-E(p, \dot{\theta}, m_i) + E(p, 0, m_i) + \mu_i \right] \\ n_i^0 &= \int \frac{d^3p}{(2\pi)^3} \frac{1}{1 + \exp(\beta E_i^0)}, \qquad E_i^0 = (p^2 + m_i^2)^{1/2}. \end{aligned} \tag{12}$$

After some algebra (see Appendix A) we get (to leading order in $\dot{\theta}$ and in the high temperature limit $T > m_i$)

$$\mu_i = \frac{12}{T^2}[(n_i - n_i^0) \pm c(m_i^2) g_\theta \dot{\theta}], \qquad c(m_i^2) = \frac{1}{4\pi^2} m_i^2 (1 - \frac{m_i}{3T}). \tag{13}$$

We have displayed the cubic mass term just to get a feeling when the high temperature expansion breaks down; for $m_i \simeq T$ we expect it still to be reasonably accurate. Eq. (11) can now be re-written as

$$\dot{B} = -6 N_F \frac{\Gamma_{sph}}{V T^3} \sum_i \left[n_{iL} \pm c(m_i^2) g_\theta \dot{\theta} \right] \tag{14}$$

where $N_F = 3$ is the number of families, $\sum_i n_{iL} = n_L$ is the total left handed fermion number density, which can be recast in terms of baryon and lepton numbers as $3B_L + L_L$. (In the above the equilibrium contribution from particles $\sum_i n_i^0$ has been cancelled by the contribution from anti-particles $-\sum_i \bar{n}_i^0$.) We write the final form of the baryon number equilibration equation as follows:

$$\dot{B} = -6 N_F \frac{\Gamma_{sph}}{V T^3} \left[3B_L + L_L + 6c(m_t^2) g_\theta \dot{\theta} \right] \tag{15}$$

where the contribution from the light fermions to $c(m_i^2)$ has been ignored in comparison to the top quark. The coefficient $6 = 2N_c$ in front of the $\dot{\theta}$ term is the number of degrees of freedom for the left-handed top quark.

In the next section we discuss the solution to this equation assuming that the phase transition is second order or crossover.

3 An upper bound on the baryon to entropy ratio

We will now analyze Eq. (15) and, assuming that before the transition the Universe is in thermal equilibrium so that baryon number is *zero* initially, we will investigate what is the maximum baryon number produced at a second order phase transition in a two Higgs doublet model.

First we simplify Eq. (15). Since the time scale for the weak sphaleron transitions is at least $\tau_{sph} \sim 1/(\alpha_w^4 T)$, τ_{sph} is much larger than the time scale of strong sphaleron transitions $\tau_{ss} \sim 1/(\alpha_s^4 T)$, so that at any moment when discussing only the dynamics of the weak sphaleron processes, the strong sphalerons will be to a very good approximation in chemical equilibrium, which means $B_L = B_R$. We also have to make an assumption about the time scale for equilibration of the left and right lepton numbers $\tau_{LR} \sim 1/(\alpha_w y_l^2 T)$ (here $y_l \sim y_\tau \sim 10^{-2}$ is the Yukawa coupling constant). For simplicity we will assume $\tau_{LR} << \tau_{sph}$, which implies that to a good approximation $L_L = L_R$. This will certainly be overwhelmingly satisfied when the weak sphaleron rate is exponentially suppressed, the case of most interest to us. When the symmetry is restored and when the sphaleron rate is un-suppressed, $\tau_{LR} > \tau_{sph}$ may be a more reasonable approximation. We will not study this case here since, as we will see below, most of the baryons are produced at a time when the sphaleron rate is exponentially suppressed. In addition we know that $B - L$ is conserved by the the Standard Model. For definiteness we will set $B - L$ to zero, which is motivated by the symmetric initial conditions. To summarize we have:

$$\begin{aligned} B_L &= B_R \qquad && \tau_{sph} >> \tau_{ss} \\ B &= L && \text{(symmetric initial condition)} \\ L_L &= L_R && \tau_{sph} >> \tau_{LR} \end{aligned} \tag{16}$$

¿From these constraints we easily infer $B_L = L_L = B/2$ which implies $3B_L + L_L = 2B$. Eq. (15) can be now written in a simple form

$$\dot{B} = -\Gamma(B + \mathcal{C}\dot{\theta}) \ , \quad \mathcal{C} = g_\theta \frac{3m_t^2}{2\pi^2}(1 - \frac{m_t}{3T}) \ , \quad \Gamma = 12 N_F \frac{\Gamma_{sph}}{VT^3}, \tag{17}$$

where $m_t = y_t \phi$ is the top mass. Here, ϕ is the Higgs expectation value which changes during the electroweak phase transition from $\phi = 0$ to $\phi = \phi(T = 0)$.

The solution to this equation with the zero initial baryon number can be easily obtained by the Greens function method

$$B(t) = -\int_{t_{in}}^{t} dt' \, \mathcal{C}\dot{\theta}(t')\Gamma(t') \ \exp[-\int_{t'}^{t} \Gamma(t'')dt'']. \tag{18}$$

In order to derive an upper bound on $B(t)$, we write

$$B(t) \leq \mathcal{B}_0 \int_{t_{in}}^{t} dt' \Gamma(t') \exp[-\int_{t'}^{t} dt'' \Gamma(t'')] = \mathcal{B}_0 \tag{19}$$

with

$$\mathcal{B}_0 = \max_{t' \in [t_{in}, t]} |C(t')\dot{\theta}(t')| \tag{20}$$

and we have assumed that $\int_{t_{in}}^{t_f} \Gamma dt >> 1$. This formula is our upper bound for baryon number production. Next we will show that, in the context of a simple model for the effective potential near the phase transition, $\mathcal{B}_0$ is suppressed by the expansion rate of the Universe. We believe that our argument is not limited to the simple form for the potential we use:

$$V(\phi, T) = D(T^2 - T_0^2)\phi^2 + \frac{\lambda}{4}\phi^4 \tag{21}$$

Here we quote the one-loop Standard Model values for the parameters: $T_0^2 \simeq 1.39 M_H^2 + (60GeV)^2$ is the spinodal temperature, $D \simeq 0.18$, and λ depends on the Higgs mass ($M_H^2/2v_0^2$, $v_0 = 246$GeV) and, in addition, it is weakly (logarithmically) dependent on temperature. $\lambda \sim 0.05 - 0.16$, depending on the Higgs mass, the lower bound corresponding to $M_H = 80$GeV, the upper to $M_H = 140$GeV [27]. The value of λ is weakly dependent on temperature; for example when temperature decreases by about 10% (which will turn out to be the value at which (20) is maximized), the value of λ changes by not more than 10%, which we can ignore. Since we do not trust the one loop values at the above quoted Higgs masses, we will not discuss the values of these parameters in the MSSM. The reader should keep in mind that we do not know what the true form of the effective potential is near the phase transition, since we are interested in the case of a large Higgs mass for which the perturbative expansion breaks down. A more appropriate treatment may be the ϵ-expansion as advocated in [13] or some other technique based on renormalization group [15]. However, it is not clear whether the ϵ-expansion is accurate either in the case of the Standard Model or its supersymmetric version, so we do not know very well what the form of the effective potential is. The non-perturbative calculations [12], [15] are still not at the level to be able to reconstruct the form of the potential for large Higgs masses. In order to illustrate our mechanism we made a simple assumption on the form of the effective potential, which is motivated by the one-loop finite temperature effective potential (with the cubic term, which gives rise to a first order phase transition, set to zero). This form of the effective potential should give a reasonable qualitative description the phase transition since it has the desirable generic features of a second order phase transition: it is maximally flat at the critical temperature where the quadratic term vanishes; the order parameter ϕ changes continuously as temperature drops. We believe that the use of the true form of the effective potential in the analysis that follows would not alter our main observation that $\mathcal{B}_0$ is suppressed by the expansion rate of the Universe at all times.

The time dependence of $C(t)$ and $\dot{\theta}(t)$ in (20) are both determined by $\phi(t)$, the value of the Higgs scalar expectation value. In Appendix B we show that to a very good approximation $\phi(t)$ is given by the location of the minimum of the finite temperature effective potential (21):

$$\phi_0^2(T) = \frac{2D}{\lambda}[T_0^2 - T^2] \qquad \text{for} \quad T < T_0. \tag{22}$$

The relative departure of the true *vev* from this value, as is shown in Appendix B, is suppressed by the expansion rate of the Universe. This remains true when the friction of the Φ field is taken into account.

In order to obtain the time dependence of θ, we make the simplified assumption

$$\theta \equiv \epsilon \frac{\phi(T)}{\phi(T=0)} = \epsilon[1 - T^2/T_0^2]^{1/2}, \tag{23}$$

where ϵ is the net change in the CP violating phase (the relative phase between the Higgs doublets in the case of the two Higgs doublet model). Cline *et al* [29] have done an extensive study of the dynamics of θ and ϕ fields neglecting friction and find that for rather large range of parameters the above linear approximation is reasonable. As the analysis of Appendix B suggests, this conclusion should not be altered even when the friction of both fields is taken into account.

In order to relate time derivatives to the derivatives with respect to temperature we will use the Friedmann equation for the radiation era:

$$H = \frac{1}{2t} = h\sqrt{g_*}\frac{T^2}{m_{Pl}}\,, \qquad h^2 = \frac{8\pi}{3}\frac{\pi^2}{30}\,, \quad G = m_{Pl}^{-2}\,, \quad g_* = 106.75 \tag{24}$$

so that $tT^2 = const$ and $d/dt = -HTd/dT$, or $Hdt = -dT/T$.

¿From Eqs. (23) and (24) it then follows that

$$\dot\theta = H\theta\frac{1}{(T_0/T)^2 - 1} = \epsilon H\frac{T}{T_0}\left[T_0^2/T^2 - 1\right]^{-1/2}\,, \tag{25}$$

and thus, maximizing $\mathcal{C}\dot\theta$, gives

$$\mathcal{B}_0 = \frac{24}{25\sqrt{5}\pi^2}|g_\theta\epsilon|y_t^2\frac{2D}{\lambda}H(T_0)T_0^2. \tag{26}$$

Using the equation

$$s = \frac{2\pi^2}{45}g_*T_0^3 \tag{27}$$

for the entropy density, the baryon to entropy ratio is bounded from above by

$$\frac{B}{s} \le \frac{108}{5\sqrt{5}\pi^4}\frac{y_t^2}{g_*}|g_\theta\epsilon|\frac{2D}{\lambda}\frac{H(T_0)}{T_0}. \tag{28}$$

For $T_0 = 110\text{GeV}$ ($M_H = 80\text{GeV}$), the value of H/T_0 is about $1.6 \cdot 10^{-16}$, and for $T_0 = 175\text{GeV}$ ($M_H = 140\text{GeV}$), $H/T_0 \simeq 2.5 \cdot 10^{-16}$; $g_\theta \simeq -1/4$ (we have assumed here that $v_1 = v_2$), $y_t \simeq 1/\sqrt{2}$. $2D/\lambda$ is bounded from above by about 7 when $M_H = 80\text{GeV}$. (For a larger value $M_H = 140\text{GeV}$, $2D/\lambda \simeq 2.5$.) Our final bound for the baryon to entropy ratio is then:

$$\frac{B}{s} \le 1.5 \times 10^{-19}|\epsilon|, \tag{29}$$

which completes the proof that without additional sources of out-of-thermal equilibrium such as topological defects, electroweak baryogenesis is not possible if the electroweak phase transition is not first order.

4 A lower bound on the baryon to entropy ratio

We are now interested in obtaining an approximate lower bound on the baryon number today (which corresponds to $B = B(\infty)$). It proves convenient to split the integral in Eq. (18) into two parts: the first for which the sphaleron rate Γ is large so that the time integral in the exponent is much larger than one, and the second part for which this integral is smaller than one. The time t_H which separates the two regimes is defined by

$$\int_{t_H}^{\infty} \Gamma(t'')dt'' = 1. \tag{30}$$

On the first interval $[t_{in}, t_H]$ we can perform a partial integration to obtain

$$B(\infty) = -\mathcal{C}\dot{\theta}(t_H)\mathrm{e}^{-1} + \mathcal{C}\dot{\theta}(t_{in})\mathrm{e}^{-\int_{t_{in}}^{t_H}\Gamma(t)dt}\mathrm{e}^{-1} \tag{31}$$

$$+\int_{t_{in}}^{t_H} dt'\frac{d(\mathcal{C}\dot{\theta})}{dt'}\mathrm{e}^{-\int_{t'}^{t_H}\Gamma(t'')dt''}\mathrm{e}^{-1} - \int_{t_H}^{\infty} dt'\,\mathcal{C}\dot{\theta}(t')\Gamma(t')\,\mathrm{e}^{-\int_{t'}^{\infty}\Gamma(t'')dt''} \tag{32}$$

We can immediately see that the second term vanishes if t_{in} is chosen such that the Universe is initially in the symmetric phase.

The idea behind the derivation of the lower bound is the following: the first and fourth terms have a negative sign, the third term is positive. We will show that the third term is smaller than the first in absolute value, and that hence the absolute value of the baryon number can be bounded from below by the absolute value of the final term.

To find an upper bound on the third term, we replace the exponential factor by 1. The remaining integral exactly cancels the first term. Thus, the absolute value of $B(\infty)$ can be bounded from below by a lower bound on the absolute value of the last term.

In order to bound the last term, it is necessary to know the exact temperature dependence of Γ (recall that for obtaining the upper bound on $B(t)$

this was not required). In the case of sphaleron-induced baryogenesis at the electroweak scale, we have:

$$\Gamma(T) = \gamma T y^7 e^{-By}, \quad y = \sqrt{\frac{4\pi}{\alpha_w}}\frac{\phi}{T}, \tag{33}$$

where $B \sim 1.5 - 2.7$ is a slowly varying function of the coupling constant ratio, and $\gamma \sim 10^{-4}\kappa_1$, where κ_1 is the one loop fluctuation determinant. Before we discuss the last term in (32), we will evaluate the integral in Eq. (30). We can solve this integral approximately by using the equation (33) for Γ and expressing the time differential in terms of the temperature differential with the help of the Friedmann equation (24). The result is

$$\begin{aligned} \frac{\Gamma(x_H)}{T(x_H)} &\simeq \frac{2D}{\lambda}\frac{B}{x_H}\sqrt{\frac{4\pi}{\alpha_w}}\frac{H(T_0)}{T_0} \\ x_H \equiv \frac{\phi_H}{T} &= \frac{1}{B(g^2/\lambda)}\sqrt{\frac{\alpha_w}{4\pi}}\ln\left[\gamma\frac{\lambda}{2D}\left(\frac{4\pi}{\alpha_w}\right)^3\frac{1}{B(g^2/\lambda)}\frac{T_0}{H(T_0)}x_H^8\right] \end{aligned} \tag{34}$$

which is solved for $x_H = \phi_H/T \simeq 1.2$. This is nothing but the condition for the sphaleron erasure bound of Ref. [33] (see below).

With the help of Eqs. (13), (25) and (30), the last term of Eq. (32) can now be estimated as follows:

$$\begin{aligned} &|\int_{t_H}^{\infty} dt'\, \mathcal{C}(t')\dot{\theta}(t')\Gamma(t')\, \mathrm{e}^{-\int_{t'}^{\infty}\Gamma(t'')dt''}| > \mathrm{e}^{-1}|\int_{t_H}^{\infty} dt'\, \mathcal{C}(t')\dot{\theta}(t')\Gamma(t')| \\ &\simeq \mathrm{e}^{-1}\frac{3}{2\pi^2}\left(\frac{2D}{\lambda}\right)^{1/2}|\epsilon\, g_\theta|y_t^2\frac{x_H}{[x_H^2(\lambda/2D)+1]^3}H(T_0)T_0^2 \simeq \frac{\mathcal{B}_0}{\mathrm{e}}\,. \end{aligned} \tag{35}$$

Comparing with Eq. (26) it follows that the lower bound on the baryon to entropy ratio produced in the second order phase transition is only suppressed by a factor of e^{-1} compared to the upper bound.

5 Conclusions

In this paper we have shown that without additional sources which drive the system out of thermal equilibrium (such as a network of topological defects), baryogenesis during a second order electroweak phase transition is much too weak to be able to produce the observed baryon to entropy ratio.

We have shown that rather generically baryon number production in this case is suppressed by the expansion rate of the Universe. The case studied was that of a two Higgs doublet model with CP violation in the Higgs sector with the assumption that both Higgs fields couple the same way to the plasma and hence the phase transition occurs simultaneously for both Higgs fields. Even though in principle it would be interesting to study a more generic situation of a two stage phase transition, we believe that the main conclusions of our letter still hold.

In general, the source of CP violation may not be in the Yukawa terms, but in the neutralino and chargino mass matrices [31]. In this case the effective shift in free energy should be proportional to a time derivative of the imaginary part of the masses, *i.e.* a gauge or Higgs coupling times a time derivative of the relative Higgs phase, so that the analysis is very similar to the one presented in this letter and again baryon production is suppressed by H.

The second main result is that a non-vanishing baryon asymmetry *is* generated during such a phase transition. We have derived upper and lower bounds for the strength of this mechanism. They are of the same order of magnitude and suppressed by a factor H/T_0, where H is the Hubble expansion rate and T_0 is the temperature of the phase transition. Note that the baryon production mechanism presented in this paper does not suffer from the sphaleron erasure problem [33] since the bulk of baryons is produced when the sphaleron rate is switching off and becomes smaller than the expansion rate of the Universe.

We have illustrated our mechanism for the effective potential which describes a second order phase transition, such that the order parameter changes continuously at the phase transition but its (time) derivative is discontinuous, as opposed to crossover when the order parameter is a smooth function of time. However, from the point of view of baryon production in our mechanism, it is irrelevant whether the transition is second order or crossover since the bulk of baryon production occurs sufficiently below the phase transition when the difference between these two types of phase transition becomes immaterial.

The formalism illustrated in this paper may be applicable to other baryogenesis mechanisms taking place in a second order phase transition. Provided that a formula similar to Eq. (15) holds, then an analysis similar to what was done in this paper would be applicable. The most optimistic value of the resulting B/s is $\epsilon H/T_0\sqrt{g_*} \sim \epsilon T_0/m_{pl}$, where as before ϵ is a constant parameterizing the strength of CP violation in the particle physics sector determining the phase transition. Hence, provided that the scale of symmetry breaking is higher than about 10^9GeV [4] from the lower bound formula, it is possible to imagine a sufficiently powerful baryogenesis scenario involving only second order phase transitions.

6 Appendix A: Derivation of the expression for the chemical potential

In this Appendix we outline the derivation of Eq. (13) starting from Eq. (11) in the limit $\dot{\theta} \to 0$ and $m_i < T$.

Linearizing in $\dot{\theta}$, Eq. (11) gives

$$n_i = n_i^0 + \frac{\mu T^2}{2\pi^2} I_\mu \mp \frac{g_\theta \dot{\theta} T^2}{2\pi^2} I \tag{36}$$

where

$$I_\mu = \int_0^\infty x^2 dx \frac{\exp\sqrt{x^2+m_T^2}}{(1+\exp\sqrt{x^2+m_T^2})^2} \tag{37}$$

and

$$I = \int_0^\infty \frac{x^3 dx}{\sqrt{x^2+m_T^2}} \frac{\exp\sqrt{x^2+m_T^2}}{(1+\exp\sqrt{x^2+m_T^2})^2}. \tag{38}$$

The notation $m_T = m_i/T$ and $x = p/T$ has been used.

The integral I_μ can be performed explicitly in the limit $m_T \to 0$, yielding $\zeta_2 = \pi^2/6$. The second term is more tricky since the leading term (which is independent of m_T) just represents a shift in μ. The method of evaluating I consists of breaking up the integration region into two intervals, the first from 0 to y, where $m_T \ll y \ll 1$, and the second from y to ∞. In the first interval, we can set the exponential factors to 1 and evaluate the remaining integral to leading order in m_T, in the second interval we introduce a new

[4] A more realistic lower bound on the scale of symmetry breaking is about 5×10^{10}GeV; this is easily obtained from Eq. (29) and the requirement $\epsilon \leq 1$.

integration variable $z = \sqrt{x^2 + m_T^2}$, obtaining

$$I_2 = \int_{\sqrt{y^2+m_T^2}}^{\infty} dz(z^2 - m_T^2)\frac{\exp z}{(\exp z + 1)^2}. \tag{39}$$

The second integral (coefficient m_T^2) can be performed explicitly and is of the order m_T^2, the first can be performed by first integrating from 0 to ∞ and then subtracting the result obtained by integrating from 0 to $\sqrt{y^2 + m_T^2}$. The first can be done explicitly, and gives a constant ζ_2 which can be viewed as a shift in μ, the second integral can be approximated to leading order in $\sqrt{y^2 + m_T^2}$, and to this order exactly cancels the leading y dependence of integration of I from 0 to y. A good computational check is that the final result must be independent on the cut-off y, since the cut-off is introduced solely for computational convenience. Thus, to leading order in m_T, the result for I is

$$I = \zeta_2 - \frac{m_T^2}{2}(1 - \frac{m_T}{3}). \tag{40}$$

The first term in I should have no physical effect since in the zero mass limit $\dot{\theta}$ is pure gauge and must have no physical effect whatsoever. It can be viewed as a shift in the chemical potential μ:

$$\bar{\mu} = \mu \mp g_\theta \dot{\theta}, \tag{41}$$

where $\bar{\mu}$ denotes the physical chemical potential with a correct physical interpretation: $\bar{\mu}$ is proportional to the particle density perturbation. With the substitution of Eq. (41), and inserting the above results for I and I_μ, Eq. (36) yields the result for $\bar{\mu}$ given in Eq. (13). (Note that in the main text we have denoted the physical (shifted) chemical potential by μ.)

7 Appendix B: The irrelevance of dissipative effects

In this Appendix we will show that in the case of a second order phase transition dissipation does not effect the evolution of the ϕ field and it is irrelevant for baryogenesis considerations. This is in contrast to the case of a first order phase transition where it is known [32] that dissipation is crucial for the dynamics of the transition and baryogenesis. We will study the case of the ϕ field, but an analogous analysis applies to the relative Higgs phase θ.

The equation of motion for the Higgs condensate is

$$\ddot{\phi} + 3H\dot{\phi} + \gamma_\phi \dot{\phi} + \frac{\partial V(\phi, T)}{\partial \phi} = 0. \tag{42}$$

The form of the dissipative term $\gamma_\phi \dot{\phi}$ can be obtained in the WKB approximation using the method developed in [32]. Assuming that the main source of dissipation comes from the tree-level process in which the Higgs particle scatters off the top and decays, we obtain for the friction

$$\gamma_\phi \simeq \frac{3}{2\pi^4} \lambda \phi^2 \ln \frac{T^2}{\lambda \phi^2} \frac{1}{\Gamma_\phi}, \tag{43}$$

where $\Gamma_\phi \simeq 0.3 \alpha_w y_t^2 T$ is the Higgs decay rate. The lessons to learn from this simple calculation are the following. Firstly, the friction term is local in time. This is to be expected since the time scale on which the Universe evolves $\sim 1/H$ is huge in comparison to the time scale on which perturbations are damped $\tau_\phi \sim 1/\Gamma_\phi$. This is in contrast to what happens at a first order phase transition where the system is perturbed out of equilibrium in a non-local manner in the sense that perturbations are sourced and then propagate across the phase boundary. Secondly, in the case of one fluid, friction is related to viscosity, while in our case of many fluids interacting, the main source of friction are couplings between different species. Thirdly, the friction for the ϕ field is *immense* in comparison to the expansion rate of the Universe.

Since the time scale on which the Universe evolves is given by $\sim 1/H$, we can neglect the first two terms in Eq. (42). We now write Eq. (42) in the following form:

$$\phi = \phi_0 + \varphi \,, \qquad \gamma_\phi \dot{\phi} + 8D(T_0^2 - T^2)\varphi + 3[2D\lambda(T_0^2 - T^2)]^{1/2}\varphi^2 + \lambda \varphi^3 = 0 \tag{44}$$

The solution is

$$\varphi = -\phi_0 \frac{\gamma_\phi H}{T^2} \frac{1}{8D} \frac{1}{(T_0^2/T^2 - 1)^2} \tag{45}$$

which means that the instantaneous value of ϕ is very well approximated by the equilibrium value ϕ_0 in the sense that the relative correction φ/ϕ is suppressed by the expansion rate of the Universe. One can check that indeed all of the following $\dot{\varphi}$, φ^2 and φ^3 are suppressed by at least H in comparison to $\dot{\phi}_0$, which justifies the above approximate solution.

We have thus shown that friction does not alter in any significant manner the evolution of the Higgs condensate and the use of ϕ_0 in the main text was justified. An analogous consideration applies to the θ field.

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