CLNS 95/1371

Charmed Strange Pentaquarks in the Large N_c Limit

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(November 7, 1995)

Abstract

The properties of pentaquarks containing a heavy anti-quark and strange quarks are studied in the bound state picture. In the flavor SU(3) limit, there are many pentaquark states with the same binding energy. When the SU(3) symmetry breaking effects are included, however, three states become particularly stable due to a "Gell-Mann–Okubo mechanism". They are the $\bar{Q}suud$ and $\bar{Q}sudd$ states discussed by Lipkin, and a previously unstudied $\bar{Q}ssud$ state. These states will have $J^P = \frac{1}{2}^+$ and their masses are estimated. These states, if exist, may be seen in experiments in the near future.

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Quark confinement mandates that hadrons must be singlets under SU(3) color. Ordinary hadrons, i.e., mesons as $q\bar{q}$ bound states and baryons as qqq bound states, do conform to this rule. On the other hand, it is also possible to construct other SU(3) singlet quark states. The most famous of these multiquark exotic states are the Jaffe tetraquark $qq\bar{q}\bar{q}$ [1,2] and hexaquark $qqqqqq$ [3]. Five quark bound states $\bar{q}qqqq$, known as pentaquarks, are first suggested in Ref. [4] and are subsequently studied in Ref. [5–10]. In particular, Lipkin [5,9] suggested that the states $\bar{Q}suud$ and $\bar{Q}sudd$, where Q is a heavy quark (c or b), are especially stable under the "flavor antisymmetry principle". These states, if exist, may be discovered soon in the Fermilab E791 experiment.

In a previous article [10], the author has discussed the possibility of constructing heavy pentaquarks $\bar{Q}qqqq$ in the large N_c limit. These pentaquarks appear as bound states of heavy mesons to chiral *antisolitons*. However, that investigation was made under $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ chiral symmetry, in which all the states have zero strangeness. In order to take the strange quark into account, we have to incorporate SU(3) flavor symmetry into our model, which is the objective of this article. We will see that, when SU(3) flavor symmetry is unbroken, there will be a large number of degenerate pentaquark states, all of them weakly bounded but may be destabilized by higher order $1/M_Q$ and/or $1/N_c$ corrections. When, however, the SU(3) flavor symmetry breaking effect is taken into account, the states studied by Lipkin will be even more tightly bounded by a "Gell-Mann–Okubo mechanism" analogous to which cause the $\Sigma - \Lambda$ splitting. As a result, these states will have the best chances to survive the higher order corrections and remain bounded in the real world.

It will help to review the relevant results in Ref. [10]. In the large N_c limit, the nucleon N and the Delta Δ appear as topological solitons of the SU(2) pion fields, which are the Goldstone field of the spontaneous symmetry breaking $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \to \mathrm{SU}(2)_V$ [11]. Then the interaction of such chiral solitons with heavy mesons can be studied under the chiral lagrangian [12–14]. It turns out that only states with $K = I + s_\ell = 0$ are bound [15–17]. The binding energy V is

$$V_0 = V(K=0) = -\tfrac{3}{2} g F'(0), \tag{1}$$

where g is the axial current coupling constant in the chiral lagrangian and $F'(0)$ is the derivative of the soliton profile function at the origin. Since $F'(0) > 0$, the negative sign in Eq. (1) implies that the binding energy is negative and the resultant state is really bounded. These states can be identified as Λ_Q with $I = s_\ell = 0$ and Σ_Q with $I = s_\ell = 1$. On the other hand, the $K = 1$ states are unbounded, with binding energy

$$V(K=1) = -\tfrac{1}{3} V_0. \tag{2}$$

On the other hand, pentaquarks merge naturally as heavy meson–chiral antisoliton bound states in the same picture. If only terms with at most one derivative are retained in the chiral lagrangian (the truncated lagrangian in Ref. [10]), the binding energy, denoted by $\tilde{V}$ in this case, is just V with the opposite sign.

$$\tilde{V}(K) = -V(K). \tag{3}$$

Hence, the pentaquark bound states are those with $K = 1$, i.e., $(I, s_\ell) = (1,0)$, $(1,1)$ or $(0,1)$, with binding energy a third that of a normal heavy baryon.

$$\tilde{V}(K=1) = \tfrac{1}{3} V(K=0) = \tfrac{1}{3} V_0. \tag{4}$$

Under flavor SU(3), the nucleon isodoublet becomes a part of the ground state baryon octet. We will follow the notation of section 7 of Ref. [15] and denote an irreducible SU(3) representation as (m, n), which is a traceless tensor completely symmetric in m upper and n lower indices, and have dimension

$$\dim(m,n) = \tfrac{1}{2}(m+1)(n+1)(m+n+2). \tag{5}$$

For example, the fundamental triplet **3** is $(1,0)$, sextet **6** is $(2,0)$, octet **8** (1,1), decuplet **10** (3,0), *etc.* The conjugate representations are obtained by interchanging m and n, i.e., antitriplet $\bar{3}$ is $(0,1)$, *etc.* Note that there are four irreducible representations of dimension 15: $(4,0)$, $(2,1)$ and their conjugates.

Now we are ready to generalize the SU(2) results above to SU(3) [15]. First we will generalize the results for the normal heavy baryons. The baryon octet–heavy meson antitriplet bound states are,

$$(1,1) \otimes (0,1) = (0,1) \oplus (2,0) \oplus (1,2)$$
$$\mathbf{8} \otimes \bar{\mathbf{3}} = \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \mathbf{1\bar{5}}, \tag{6a}$$

while the baryon decuplet–heavy meson antitriplet bound states are,

$$(3,0) \otimes (0,1) = (2,0) \oplus (3,1)$$
$$\mathbf{10} \otimes \bar{\mathbf{3}} = \mathbf{6} \oplus \mathbf{24}. \tag{6b}$$

The stable states are those connected to the states stable in SU(2) by an SU(3) rotation. For example, the $I = s_\ell = 0$ Λ_Q state is an element of the $(0,1)$ in Eq. (6a) and hence the whole antitriplet, with $s_\ell = 0$, will have binding energy V_0. Physically it represents the (Λ_Q, Ξ_Q) antitriplet. Similarly, $\Sigma_Q^{(*)}$, with $I = s_\ell = 1$, is an element of a particular combination of the $(2,0)$ in Eq. (6a) (denoted as $(2,0)_8$) and that in Eq. (6b) (denoted as $(2,0)_{10}$).

$$\Sigma_Q \in (2,0)_\Sigma = \sqrt{\frac{1}{3}}(2,0)_8 + \sqrt{\frac{2}{3}}(2,0)_{10}. \tag{7}$$

Thus the whole $(2,0)_\Sigma$ sextet, with $s_\ell = 1$ will have binding energy V_0. It corresponds to the $(\Sigma_Q^{(*)}, \Xi_Q^{\prime(*)}, \Omega_Q^{(*)})$ sextet in the real world.

A similar analysis can be done in the pentaquarks sector. Since the heavy anti-mesons form a triplet under flavor SU(3), the counterparts of Eqs. (6) are,

$$(1,1) \otimes (1,0) = (1,0) \oplus (0,2) \oplus (2,1)$$
$$\mathbf{8} \otimes \mathbf{3} = \mathbf{3} \oplus \bar{\mathbf{6}} \oplus \mathbf{15}, \tag{8a}$$

$$(3,0) \otimes (1,0) = (2,1) \oplus (4,0)$$
$$\mathbf{10} \otimes \mathbf{3} = \mathbf{15} \oplus \mathbf{15'}. \tag{8b}$$

Note that the two **15**'s in Eq. (8b) are neither equivalent nor conjugate to each other. Recall that, in the SU(2) case, we have identified three stable bound states with $(I, s_\ell) = (0, 1)$, $(1, 1)$ and $(1, 0)$. The $(I, s_\ell) = (0, 1)$ state now is a part of the $(0, 2)$ representation in Eq. (8a), while $(I, s_\ell) = (1, 0)$ state is inside the $(2, 1)$ in Eq. (8a). The $(I, s_\ell) = (1, 1)$ state falls in the linear combination of the $(2, 1)$'s in Eq. (8a) and that in Eq. (8b) shown below.

$$(2, 1)_\Sigma = \sqrt{\frac{2}{3}}(2, 1)_8 - \sqrt{\frac{1}{3}}(2, 1)_{10}. \tag{9}$$

In the large N_c limit, all these states will be degenerate. In the real world, the baryon decuplet is heavier than the octet by $M_\Delta - M_N \sim 300$ MeV. This would cause the $(2, 1)_\Sigma$ state to be heavier than the other two states by $\frac{1}{3}(M_\Delta - M_N) \sim 100$ MeV. (This is the same mechanism which breaks the $\Sigma_Q^{(*)} - \Lambda_Q$ degeneracy, $M_{\Sigma_Q^{(*)}} - M_{\Lambda_Q} = \frac{2}{3}(M_\Delta - M_N)$.) Since pentaquarks are only weakly bounded (if bounded at all) in all existing models, it is probable that this 100 MeV mass difference will destabilize the $(2, 1)_\Sigma$ pentaquarks. Hence we will ignore this state for the rest of our discussion. This leaves us with the $s_\ell = 1$ $\bar{\mathbf{6}}$ and the $s_\ell = 0$ **15**, both just involving the baryon octet and hence degenerate (up to order N_c^0) even at finite N_c. All of them will have binding energy $-\frac{1}{3}V_0$. We will see that the flavor SU(3) symmetry breaking is going to pick out the Lipkin states (and one other state) as the most tightly bounded ones.

To investigate the effect of SU(3) symmetry breaking effects, it helps to review the SU(3) symmetry breaking in the baryon octet, in which the baryon masses are governed by the Gell-Mann–Okubo formula [18,19],

$$M = M_0 + aY + b(I(I+1) - \tfrac{1}{4}Y^2). \tag{10}$$

The last term is responsible for the physical $\Sigma - \Lambda$ splitting, without which all four $S = 1$ baryons will be degenerate and be exactly half way between N and Ξ on the mass spectrum. In the real world, b is positive and hence Σ is heavier than Λ by about 78 MeV. Since the Gell-Mann–Okubo formula is the consequence of just SU(3) group theory, a similar formula with different coefficients also governs the pentaquark masses in each of the representations.

Then this "Gell-Mann–Okubo mechanism" may also break the degeneracy between states with the same strangeness in the $(2,1)$ representation. In particular, for $S = 1$, there exists both an isodoublet and an isoquartet in the $(2,1)$ representation. The octet example leads us to expect the isodoublet to be lowered and the isoquartet raised. If this is indeed the case, the most stable bound states will be the $\bar{Q}suud$ and $\bar{Q}sudd$ isodoublet, i.e., exactly the states Lipkin predicted.

To verify this conjecture, one must project the Lipkin states back into the heavy meson–octet baryon product space. In general,

$$|\bar{Q}suud\rangle_L = w|\bar{Q}s\rangle|p\rangle + x|\bar{Q}d\rangle|\Sigma^+\rangle + y|\bar{Q}u\rangle|\Sigma^0\rangle + z|\bar{Q}u\rangle|\Lambda\rangle, \tag{11}$$

with the normalization condition $|w|^2 + |x|^2 + |y|^2 + |z|^2 = 1$, and the subscript L stands for Lipkin. The algebra, consists of considerations of the U-spin, V-spin and the isospin as well as the orthogonality of states, is straightforward but cumbersome and will not be repeated here. The result is,

$$(|w|^2, |x|^2, |y|^2, |z|^2) = (\tfrac{3}{8}, \tfrac{1}{24}, \tfrac{1}{48}, \tfrac{9}{16}). \tag{12}$$

As expected, the Lipkin states is mainly a Λ bound state while the Σ contributions are small $(|x|^2, |y|^2 \ll |z|^2)$. By isospin symmetry, the same conclusion holds for $|\bar{Q}sudd\rangle$. We can also do the same decomposition for the $S = 2$ $I = 0$ state $|\bar{Q}ssud\rangle_L$.

$$|\bar{Q}ssud\rangle_L = w_s|\bar{Q}u\rangle|\Xi^-\rangle + x_s|\bar{Q}d\rangle|\Xi^0\rangle + y_s|\bar{Q}s\rangle|\Sigma^0\rangle + z_s|\bar{Q}s\rangle|\Lambda\rangle. \tag{13}$$

The results are

$$(|w_s|^2, |x_s|^2, |y_s|^2, |z_s|^2) = (\tfrac{1}{8}, \tfrac{1}{8}, 0, \tfrac{3}{4}). \tag{14}$$

This state is again predominantly a Λ bound state. As a result, these three Lipkin states are expected to be stabilized by the "Gell-Mann–Okubo mechanism", which makes them lighter than other pentaquark states with the same strangeness by (a fraction of) $M_\Sigma - M_\Lambda =$ 78 MeV. For a weakly bounded system, this can be a huge increase in stability. Thus these Lipkin states are the ones most likely to survive the destabilizing perturbations, which appear in higher orders in the bound state picture.

Many properties of the Lipkin states can be predicted in the bound state picture. They consist of a $S=1$ isodoublet and a $S=2$ isosinglet, which look like an SU(3) triplet (or, if the $S=0$ pentaquarks also exist, an SU(3) sextet) but is in fact part of a **15**. With $s_\ell = 0$ and same parity as the normal heavy baryon bound state, they have $J^P = \frac{1}{2}^+$. The binding energy $\frac{1}{3}V_0$ can be extracted from the heavy baryon sector. In the charm sector, we have this estimate of V_0.

$$\tfrac{1}{4}(M_D + 3M_D^*) + M_N - M_{\Lambda_c} = (1973 + 938 - 2285)\ \text{MeV} = 626\ \text{MeV}, \tag{15}$$

which gives $\frac{1}{3}V_0 = -209$ MeV.

The masses of the pentaquark states are given by the hamiltonian,

$$H_L = H_{heavy} + H_{baryon} + \tfrac{1}{3}V_0, \tag{16}$$

where the heavy meson mass term H_{heavy} should be taken as the spin-averaged mass of the ground state pseudoscalar and vector mesons. Note that, in our notation, V_0 is negative and hence the last term is stabilizing. Immediately this provides an estimation of the mass of the $S=0$ charmed pentaquarks $|\bar{c}qqqq\rangle$.

$$|\bar{c}qqqq\rangle \sim 2702\ \text{MeV}. \tag{17}$$

These states may have (I, s_ℓ) either $(1,0)$ or $(0,1)$. The masses of the Lipkin states $|\bar{c}suud\rangle_L$, $|\bar{c}sudd\rangle_L$ and $|\bar{c}ssud\rangle_L$ are also predicted,

$$|\bar{c}suud\rangle_L = |\bar{c}sudd\rangle_L \sim 2857\ \text{MeV}, \tag{18a}$$

$$|\bar{c}ssud\rangle_L \sim 3009\ \text{MeV}. \tag{18b}$$

For comparison the masses of the $|\bar{c}suud\rangle$ state in the **3**, $\bar{\mathbf{6}}$ and the $I=\frac{3}{2}$ state in the **15** have also been calculated.

$$|\bar{c}suud\rangle_3 \sim 2896 \text{ MeV}, \quad (|w|^2, |x|^2, |y|^2, |z|^2) = (\tfrac{3}{8}, \tfrac{3}{8}, \tfrac{3}{16}, \tfrac{1}{16}), \tag{19a}$$

$$|\bar{c}suud\rangle_{\bar{6}} \sim 2890 \text{ MeV}, \quad (|w|^2, |x|^2, |y|^2, |z|^2) = (\tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{8}, \tfrac{3}{8}), \tag{19b}$$

$$|\bar{c}suud\rangle_{15\ I=\frac{3}{2}} \sim 2957 \text{ MeV}, \quad (|w|^2, |x|^2, |y|^2, |z|^2) = (0, \tfrac{1}{3}, \tfrac{2}{3}, 0). \tag{19c}$$

Indeed the "Gell-Mann–Okubo mechanism" is at work: the Lipkin states are pushed down below the **3** and the $\bar{\mathbf{6}}$ while the $I = \frac{3}{2}$ state in the **15** is pushed up. The analogous comparisons for the $|\bar{c}ssud\rangle$ states are,

$$|\bar{c}ssud\rangle_3 \sim 3060 \text{ MeV}, \quad (|w_s|^2, |x_s|^2, |y_s|^2, |z_s|^2) = (\tfrac{3}{8}, \tfrac{3}{8}, 0, \tfrac{1}{4}), \tag{20a}$$

$$|\bar{c}ssud\rangle_{\bar{6}} \sim 3079 \text{ MeV}, \quad (|w_s|^2, |x_s|^2, |y_s|^2, |z_s|^2) = (\tfrac{3}{8}, \tfrac{3}{8}, \tfrac{1}{4}, 0), \tag{20b}$$

$$|\bar{c}ssud\rangle_{15\ I=1} \sim 3066 \text{ MeV}, \quad (|w_s|^2, |x_s|^2, |y_s|^2, |z_s|^2) = (\tfrac{1}{8}, \tfrac{1}{8}, 0, \tfrac{1}{4}), \tag{20c}$$

Again the Lipkin state merges as the lightest one.

The analysis in the bottom sector is similar. The estimate of V_0 from the Λ_b mass

$$\tfrac{1}{4}(M_B + 3M_{B^*}) + M_N - M_{\Lambda_b} = (5314 + 938 - 5641) \text{ MeV} = 611 \text{ MeV}, \tag{21}$$

agrees nicely with that in the charmed sector. With $\frac{1}{3}V_0 = -204$ MeV we obtained these predictions,

$$|\bar{b}suud\rangle_L = |\bar{b}sudd\rangle_L \sim 6203 \text{ MeV}, \tag{22a}$$

$$|\bar{b}ssud\rangle_L \sim 6355 \text{ MeV}. \tag{22b}$$

Eqs. (18) and (22) are the main results of this paper. The bound state picture agrees with Lipkin's model that heavy pentaquarks probably exist. It should be noted that Lipkin's model is dictated by the principle of "maximal flavor antisymmetry", in which the most stable states will have the wave function in the flavor space maximally antisymmetrized. In the baryon octet, the only state satisfying this criterion is the Λ. In this light, it is not surprising that our results that the stable states as heavy meson bounded to predominantly Λ baryons agree so well with his.

It must be stressed that, even within our model, we have *not* proven the non-existence the non-Lipkin pentaquark states. For example, the states in the $\bar{\mathbf{6}}$ may as well have negative binding energies. These states, however, are expected to be more massive than the Lipkin states and hence will decay to the Lipkin states electromagnetically. Hence they are expected to be short-lived (when compared to $\tau_{\text{Lipkin}} \sim 10^{-12}$ sec) and much wider than the Lipkin states.

On the other hand, there are important corrections to our predictions which have not been incorporated in our model. For example, the hyperfine splittings $M_{D^*} - M_D = 146$ MeV, which is an $1/m_c$ correction, is significant for the stability of a pentaquark, the binding energy of which is typically of the order of 100 MeV. The effect of hyperfine splitting has been taken into account in our calculations, but the effects of other $1/m_c$ corrections may also be important. Another potentially dangerous source of correction comes from the terms in the chiral lagrangian with more than one derivatives. It is not clear to the author how these effect can be estimated.

In conclusion, in the bound state picture we have calculated the spin, isospin and parities of the charmed pentaquarks with $S = 0$, 1 and 2 and also estimated their masses. All these predictions await experimental verifications, possibly by E791 at Fermilab. Since these pentaquarks are stable with respect to QCD, they must decay weakly. If we assume the Λ is just a spectator in the decay, a reasonable assumption for a weakly bounded system, then the pentaquark life time will be related to that of the constituent heavy meson.

$$\tau_{\bar{Q}suud} \sim \tau_{\bar{Q}u}, \tag{23a}$$

$$\tau_{\bar{Q}sudd} \sim \tau_{\bar{Q}d}, \tag{23b}$$

$$\tau_{\bar{Q}ssud} \sim \tau_{\bar{Q}s}. \tag{23c}$$

In particular, the relation $\tau_{D^0} < \tau_{D^s} < \tau_{D_+}$ is expected to translate into $\tau_{\bar{c}suud} < \tau_{\bar{c}ssud} < \tau_{\bar{c}sudd}$ in the pentaquark sector, as the valence s and u quarks can lead to $\bar{c}s$ annihilation diagrams and W exchange diagrams, which interferes constructively with the spectator diagrams. It is important, howevere, to note that[1] that the W exhange diagrams are helicity suppressed in D decays while unsuppressed in the pentaquark case. It is not clear if this difference will upset the lifetime hierarchy mentioned above. In any case, decay channels like $(\Lambda D) \rightarrow \Lambda K\pi$ can be measured by identifying the decay products, and the charmed pentaquarks may appear as a peak in the $\Lambda K\pi$ invariant mass plot.

ACKNOWLEDGMENTS

I must thank S. Kwan for urging me to generalize the results in Ref. [10] to include pentaquarks with non-zero strangeness, and T.M. Yan for many helpful discussions.

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[1]This point is raised by J. Appel and brought to my attention by S. Kwan.

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