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Measurement of the Inclusive Semi-electronic D^0 Branching Fraction

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Abstract

Using the angular correlation between the π^+ emitted in a $D^{*+} \to D^0\pi^+$ decay and the e^+ emitted in the subsequent $D^0 \to Xe^+\nu$ decay, we have measured the branching fraction for the inclusive semi-electronic decay of the D^0 meson to be:

$$\mathcal{B}(D^0 \to Xe^+\nu) = [6.64 \pm 0.18(stat.) \pm 0.29(syst.)]\%.$$

The result is based on 1.7 fb^{-1} of e^+e^- collisions recorded by the CLEO II detector located at the Cornell Electron Storage Ring (CESR). Combining the analysis presented in this paper with previous CLEO results we find,

$$\frac{\mathcal{B}(D^0 \to Xe^+\nu)}{\mathcal{B}(D^0 \to K^-\pi^+)} = 1.684 \pm 0.056(stat.) \pm 0.093(syst.)$$

and

$$\frac{\mathcal{B}(D \to K^-e^+\nu)}{\mathcal{B}(D \to Xe^+\nu)} = 0.581 \pm 0.023(stat.) \pm 0.028(syst.).$$

The difference between the inclusive rate and the sum of the measured exclusive branching fractions (measured at CLEO and other experiments) is $(3.3 \pm 7.2)\%$ of the inclusive rate.

1 Introduction

In this paper, we present a new measurement of the inclusive semi-electronic branching fraction of the D^0 meson. The comparison of the measured inclusive semi-leptonic branching fraction with the sum of the observed exclusive semi-leptonic branching fraction provides a measure of *missing* or unobserved modes. Recent experimental progress on exclusive measurements has yielded precise measurements of the dominant Cabibbo favored modes, observation and measurement of the Cabibbo suppressed branching fractions and stringent upper limits on suppressed Cabibbo favored branching fractions, but has not yielded an improvement in the measured inclusive semi-leptonic branching fraction. The inclusive branching fraction measurement presented here and previous measurements of exclusive branching fractions allows for more accurate comparison than previously performed. For a complete review of experimental and theoretical developments we refer the reader to recent reviews [1, 2].

In addition, we combine the inclusive result presented here with previous CLEO results on $\mathcal{B}(D^0 \to K^-\pi)$ and $\mathcal{B}(D^0 \to K^-e^+\nu)/\mathcal{B}(D^0 \to K^-\pi^+)$ [3, 4] to obtain the ratio, $\mathcal{B}(D^0 \to K^-e^+\nu)/\mathcal{B}(D^0 \to Xe^+\nu)$. As a check of the method the observed inclusive electron momentum spectrum is also extracted from the data and compared with a Monte Carlo simulation.

2 Analysis Technique and Event Selection

The technique to measure the absolute inclusive semi-electronic branching fraction of D^0 mesons is similar to the previous CLEO absolute branching fraction measurement of $D^0 \to K^-\pi^+$ [3]. Common to both analyses is the method used to determine the number of $D^{*+} \to D^0\pi^+$ decays in the data with minimal systematic bias. It is based on the unique two body kinematics of the $D^{*+} \to D^0\pi^+$ decay and the topology of $e^+e^- \to c\bar{c}$ reactions at a center of mass energy of 10.5 GeV. Briefly, the idea is that the thrust axis (defined to be that axis along which the projected momentum is a maximum) for the event approximates the D^{*+} direction in the lab. The limited amount of available phase space in the $D^{*+} \to D^0\pi^+$ decay, results in a small angle between the thrust axis and the charged pion. We denote this angle between the thrust axis and the charged pion as α. Also, the magnitude of the pion momentum is correlated to the parent D^{*+} momentum. Pions with momentum greater than 225 MeV/c are kinematically forbidden to come from the $\Upsilon(4S) \to B\bar{B}$, $\bar{B} \to D^{*+}X$, $D^{*+} \to D^0\pi^+$ decay chain. This selection assures that the D^{*+} is from $e^+e^- \to c\bar{c}$ production and the event has a well defined thrust axis. The top plot in Figure 1 shows the $\sin^2\alpha$ distribution for all pions with momentum between 225 and 425 MeV/c in the data. The peaking at low $\sin^2\alpha$ is evidence for $D^{*+} \to D^0\pi^+$ decays. The total number, $N(D^{*+} \to D^0\pi^+)$, of decays in the sample is $165658 \pm 1149(stat.) \pm 2485(syst.)$. This total is identical to that presented in Ref. [3], as the same data and selection criteria are used in both analyses.

The total number of semi-electronic decays, $N(D^{*+} \to D^0\pi^+,\ D^0 \to Xe^+\nu)$, is determined by identifying an e^+ within a cone around the π^+ direction and plotting the $\sin^2\alpha$ distribution for those π^+ with an associated e^+. This is achieved by studying the sign correlated πe combinations in the data. "Right sign" combinations, π^+e^+, provide the signal distribution and "wrong sign" combinations, π^+e^-, are studied to aid background determinations. Once the number of $D^0 \to Xe^+\nu$ decays has been determined, the branching fraction is then,

$$B(D^0 \to Xe^+\nu) = \frac{N(D^{*+} \to D^0\pi^+, D^0 \to Xe^+\nu)}{N(D^{*+} \to D^0\pi^+) \times \epsilon(D^0 \to Xe^+\nu)} \tag{1}$$

where $\epsilon(D^0 \to Xe^+\nu)$ is the efficiency for detecting the electron.

A detailed description of the CLEO II detector can be found in Ref. [5]. Electrons and positrons [6] are identified principally from the ratio of the energy measured by the CsI calorimeter and the momentum measured by the drift chamber (E/p). Additional information on energy loss in the drift chamber and shower shape in the calorimeter is also used to maximize the identification efficiency and minimize the mis-identification of hadronic tracks. The electrons are required to have momentum greater than 0.7 GeV/c and a polar angle with respect to the beam axis (θ) between 45^o and 135^o, to insure a well determined efficiency and minimal uncertainty due to mis-identified hadronic tracks. Furthermore it is important to reduce the number of electrons from $D^0 \to X\pi^0$; $\pi^0 \to e^+e^-\gamma$, where the $e^+e^-\gamma$ final state is due to either a Dalitz decay of the π^0 or a γ conversion in the detector material. This is accomplished by requiring that the identified electron, when combined with each opposite sign track in the event, does not yield an electron-positron mass below 0.050 GeV/c^2. Every opposite sign track is used to form these pairs, whether or not it is identified as an electron.

In order to correlate the π^+ with an e^+ a fiducial angle cut is applied in the lab frame. We require that $\cos(\Theta_{e-\pi}) > 0.8$, where $\Theta_{e-\pi}$ is the angle between the π emitted in the initial $D^{*+} \to D^0\pi^+$ decay and the electron from the subsequent $D^0 \to Xe^+\nu$ decay. The bottom histogram in Figure 1 shows the $\sin^2\alpha$ distributions for π's after requiring an electron within this angular region; the solid squares are for π^+e^+ combinations (right sign) and the open squares are for π^+e^- combinations(wrong sign).

3 Extraction of yields

As previously stated, the yield of $D^{*+} \rightarrow D^0\pi^+$ decays is identical to that presented in Ref. [3]. In this section we detail the determination of the number of $D^0 \rightarrow Xe^+\nu$ decays associated with the initial $D^{*+} \rightarrow D^0\pi^+$ decay.

The $\sin^2\alpha$ distribution for π^+e^+ (right sign) combinations contains three distinct components: signal and two types of background. One background has a $\sin^2\alpha$ distribution that is identical to the signal as it originates from the decay $D^{*+} \rightarrow D^0\pi^+$, $D^0 \rightarrow Xf_{\pi^+e^+}$, where $f_{\pi^+e^+}$ denotes a e^+ from either a hadronic track mis-identified as a electron or an electron from a $\pi^0 \rightarrow e^+e^-\gamma$ final state. The other background is due to random soft pions (225 to 425 MeV/c in momentum), in coincidence with an electron, and is not as sharply peaked near $\sin^2\alpha = 0$ as the signal distribution.

The $\sin^2\alpha$ distribution for π^+e^- (wrong sign) combinations is devoid of signal but contains the same two sources of background as the right sign distribution [7]. The shapes for these backgrounds in the right sign and wrong sign distributions are identical, although the normalizations differ. This difference in normalization is the result of the hadronic track mixture (π/K ratio) combined with the hadronic track mis-identification rates. For the non-D^{*+} pion background, the normalization is different due to charge conservation in the event.

To use as much information as possible, the right sign and wrong sign distributions are fit simultaneously to the following functional forms:

$$\begin{aligned}\mathcal{G}_{rs}(p_\pi, \sin^2\alpha) &= N_{D^0\rightarrow Xe^+\nu}(p_\pi)g_{D^0\rightarrow Xe^+\nu}(\sin^2\alpha, p_\pi) + \\ &\quad B_{rs}(p_\pi)P_2(\sin^2\alpha) \end{aligned} \tag{2}$$

$$\begin{aligned}\mathcal{G}_{ws}(p_\pi, \sin^2\alpha) &= N_{D^0\rightarrow Xf_e}(p_\pi)g_{D^0\rightarrow Xf_e}(\sin^2\alpha, p_\pi) + \\ &\quad B_{ws}(p_\pi)P_2(\sin^2\alpha). \end{aligned} \tag{3}$$

The expected $\sin^2\alpha$ distributions, $g_{D^0\rightarrow Xe^+\nu}(\sin^2\alpha, p_\pi)$ and $g_{D^0\rightarrow Xf_e}(\sin^2\alpha, p_\pi)$, for the right sign and wrong sign distributions are obtained from a Monte Carlo simulation. The Monte Carlo simulation correctly reproduces the measured D^{*+} production momentum distribution, and simulates $D^0 \rightarrow Xe^+\nu$ decays via the "cocktail" of exclusive modes presented in Appendix A. The second order polynomial, P_2, is constrained to have the same shape for

$p(\pi)$	Yields	
(MeV/c)	Right Sign	Wrong Sign
225 - 250	1232 ± 53	32 ± 31
250 - 275	1071 ± 49	74 ± 29
275 - 300	935 ± 44	45 ± 25
300 - 325	689 ± 38	39 ± 22
325 - 350	414 ± 32	-29 ± 18
350 - 375	259 ± 25	36 ± 17
375 - 400	166 ± 20	-4 ± 12
400 - 425	79 ± 15	0 ± 11
Total	4845 ± 104	193 ± 62

Table 1: The total yield of right sign and wrong sign events as a function of pion momentum. Backgrounds have not yet been subtracted.

both the wrong and right sign distributions. The yield of $D^0 \to Xe^+\nu$ decays, $N_{D^0\to Xe^+\nu}(p_\pi)$, the yield of mis-identified hadrons and electrons from $\pi^0 \to e^+e^-\gamma$ in the wrong sign distribution, $N_{D^0\to Xf_e}(p_\pi)$, the normalizations and shape, $B_{rs}(p_\pi)$, $B_{ws}(p_\pi)$ and P_2, of the background polynomial are determined from the fits to the $\sin^2\alpha$ distribution of the right sign and wrong sign samples in bins of pion momentum, p_π.

The $\sin^2\alpha$ distributions for the data, with the resulting fits overlaid, are shown in Figures 2 and 3 and the right sign and wrong sign yields are presented in Table 1. The right sign yields reported in this table have a contribution due to $D^0 \to Xf_{\pi^+e^+}$ backgrounds.

4 Determination of the background contribution to the signal

In this section the magnitude of the $f_{\pi^+e^+}$ background to the right sign signal yield is determined. The two contributions to this background are the following decay chains: $D^{*+} \to D^0\pi^+$, $D^0 \to Xh^+$, where the h^+ is a hadronic track mis-identified as an electron and $D^{*+} \to D^0\pi^+$, $D^0 \to X\pi^0$,

$\pi^0 \to e^+e^-\gamma$. We denote this sum for the right sign background as

$$N_{rs} = N(X\pi^0) f_{\pi^+e^+}(X\pi^0) + N(Xh^+) f_{\pi^+e^+}(Xh^+), \tag{4}$$

where $N(X\pi^0)$ $[N(Xh^+)]$ is the number of inclusive $D^0 \to X\pi^0$ $[D^0 \to Xh^+]$ decays in the data and $f_{\pi^+e^+}(X\pi^0)$ $[f_{\pi^+e^+}(Xh^+)]$ is the efficiency for detecting this background as signal. We can define the same sum for the wrong sign yield as

$$N_{ws} = N(X\pi^0) f_{\pi^+e^-}(X\pi^0) + N(Xh^-) f_{\pi^+e^-}(Xh^-). \tag{5}$$

The only difference between the wrong sign yield and the right sign background is due to the fact that the positive tracks from D^0 decays are much less likely to be kaons than negative tracks from D^0 decays. Using $f_{\pi^+e^-}(X\pi^0) = f_{\pi^+e^+}(X\pi^0)$, we find

$$N_{rs} = N_{ws} - N(Xh^-) f_{\pi^+e^-}(Xh^-) + N(Xh^+) f_{\pi^+e^+}(Xh^+). \tag{6}$$

If $N(Xh^+) f_{\pi^+e^+}(Xh^+) = N(Xh^-) f_{\pi^+e^-}(Xh^-)$, then the wrong sign yield would be equal to the background contribution to the right sign yield. However, the $\pi^+ : K^+$ ratio of h^+ tracks originating from D^0 mesons is quite different from the $\pi^- : K^-$ ratio. Using world averages [2] of the measured D^0 branching fractions, the π^+:K^+ ratio is 96:4 while the π^-:K^- ratio is 42:58, for pions and kaons from D^0 mesons which pass the same geometry and momentum criteria as for electrons. This difference coupled with different mis-identification rates for π's and K's leads to a small correction to the wrong sign yield.

The probability for a π^+ track to be mis-identified as a e^+ is determined by studying a large sample $K_s^0 \to \pi^+\pi^-$ decays in the data. This sample is large enough to determine the mis-identification probability for charged pions as a function of their momentum. This probability is measured to be $(0.056 \pm 0.015)\%$ for pions with momentum between 0.7 and 0.9 GeV/c. It rises as a function of pion momentum, such that for pions with momentum between 1.9 and 2.5 GeV/c, it is measured to be $(0.250 \pm 0.059)\%$. Convoluting the momentum dependent mis-identification probability with a Monte Carlo simulation of the π^+ momentum distribution from D^0 and $\bar{D}^0$ decays, we find the mis-identification probability integrated over pion momentum to be $(0.102 \pm 0.016)\%$ for the right sign pions and $(0.093 \pm 0.011)\%$ for the wrong sign pions. These numbers differ due to the different momentum spectrum for right sign and wrong sign pions. The error is due to the statistical uncertainty in the mis-identification probability per track as a function of momentum.

For charged K's the data do not provide a statistically rich and clean sample as for pions. The cleanest sample of charged kaons comes from reconstructed $D^0 \to K^-\pi^+(\pi^0)$ decays. With 19742 ± 221 reconstructed D^0's with a K^- that passed the momentum cuts, 4.5 ± 5.5 were consistent with the K^- being identified as an electron. This yields a central value of $(0.02 \pm 0.03)\%$ for the mis-identification probability due to kaons. As no momentum dependence measurement is possible we use $(0.02 \pm 0.03)\%$ as the mis-identification probability for charged kaons over the whole momentum range of interest.

Multiplying these mis-identification probabilities by the $\pi : K$ fractions, we find the total mis-identification probability of $f_{\pi^+e^+}(Xh^+) = (0.099 \pm 0.016)\%$ for the right sign hadronic tracks and $f_{\pi^+e^-}(Xh^-) = (0.051 \pm 0.016)\%$ for the wrong sign hadronic tracks. This is almost a factor of two difference between the two mis-identification rates. These mis-identification probabilities represent the rate per hadronic track from $D^{*+} \to D^0\pi^+$ decays where the π^+ had momentum between 225 and 425 MeV/c. Since the extraction of yields is done in eight 25 MeV/c momentum bins, these mis-identification probabilities are determined for each of the eight bins individually. Small variations arise due to different D^0 momentum spectra and small changes in the $\pi : K$ ratio.

To turn these mis-identification probabilities into the actual yield of mis-identified tracks, the inclusive right sign and wrong sign rate [$N(Xh^+)$ and $N(Xh^-)$] is determined from the data. The number of wrong sign and right sign hadronic tracks associated with $D^{*+} \to D^0\pi^+$ decays is determined by using the same code and technique as for identified electrons, with the requirement that the hadronic track not be identified as an electron. The resulting estimated mis-identified charged track contribution to the right and wrong sign yields is given in Table 2 as well as the final estimated total background to the right sign yield.

5 Efficiency

The efficiency for detecting the e^+ determined by the Monte Carlo simulation depends on the cocktail of exclusive modes used to generate the inclusive

$p(\pi)$ (MeV/c)	N_{ws}	F_{h^-}	F_{h^+}	N_{rs}
225 - 250	32 ± 31	13 ± 3	24 ± 3	43 ± 31
250 - 275	74 ± 29	13 ± 3	22 ± 3	83 ± 29
275 - 300	45 ± 25	9 ± 2	17 ± 2	53 ± 25
300 - 325	39 ± 22	7 ± 2	13 ± 2	45 ± 22
325 - 350	-29 ± 18	5 ± 1	9 ± 1	-25 ± 18
350 - 375	36 ± 17	4 ± 1	6 ± 1	38 ± 17
375 - 400	-4 ± 12	2 ± 1	4 ± 1	-2 ± 12
400 - 425	0 ± 11	1 ± 1	2 ± 1	1 ± 11
Total	193 ± 62	54 ± 5	97 ± 6	236 ± 64

Table 2: Summary of the expected background contribution as a function of pion momentum to the right sign yield, where $N_{rs} = N_{ws} - F_{h^-} + F_{h^+}$, $F_{h^-} = N(Xh^-)f_{\pi^+e^-}(Xh^-)$ and $F_{h^+} = N(Xh^+)f_{\pi^+e^+}(Xh^+)$.

semi-electronic decays. The ratios of exclusive rates presented in Appendix A are used to calculate the ratios $X_m = \mathcal{B}(D^0 \to me^+\nu)/\sum_n \mathcal{B}(D^0 \to ne^+\nu)$, where $m, n = K^-, K^{*-}, K_1^-(1270), K^{*-}(1430), \pi^-$, and ρ^- mesons. The efficiency for each of these modes is obtained from a Monte Carlo simulation of each individual mode. The inclusive efficiency is obtained from

$$\epsilon(Xe^+\nu) \;=\; \sum_m X_m \times \epsilon(D^0 \to me^+\nu). \tag{7}$$

The extraction of yields is done in eight pion momentum bins from 225 to 425 MeV/c, as in the $D^0 \to K^-\pi^+$ analysis. Table 3 contains the efficiency in each of the eight pion momentum bins. The efficiencies for the individual exclusive channels are in Table 13 (Appendix A). The total systematic error due to uncertainties in the cocktail is determined by varying the ratios in Table 12 by one standard deviation, individually and collectively. The largest variation in the overall efficiency is seen when X_K and X_π are both raised or both lowered and the other modes are changed in the opposite direction. This causes a $\pm 2\%$ change in the efficiency and is the estimated systematic error due to the uncertainties in the cocktail of exclusive modes.

In addition to changing the cocktail the effect of the assumed q^2 dependence of the form factors is studied by changing the ISGW slope (κ) [8].

The value used to generate the decays is $\kappa = 0.57 \pm 0.07$, measured in a large sample of $D^0 \to K^- e^+ \nu$ decays by CLEO [4]. Variations of one sigma on κ resulted in a $\pm 0.6\%$ variation in efficiency. The longitudinal and transverse contributions from $D^0 \to K^{*-} e^+ \nu$ decays were varied by one sigma of their measured value and the total efficiency changed by less than $\pm 0.08\%$ [9].

6 Results

6.1 $\mathcal{B}(D^0 \to X e^+ \nu)$

The relevant measurements for determining $\mathcal{B}(D^0 \to X e^+ \nu)$ are given in Table 3. The first column gives the inclusive $D^{*+} \to D^0 \pi^+$ yields from Ref. [3], the second gives the background subtracted yield of $D^0 \to X e^+ \nu$ decays, followed by a column of efficiencies. The last column is the branching fraction for $D^0 \to X e^+ \nu$ for the eight momentum bins. As a check that the eight measurements are self consistent, the χ^2 was calculated under the assumption that all eight branching fraction measurements come from the weighted average. The result is a χ^2 of 9.4 for 7 degrees of freedom.

Sources of systematic effects and their estimated magnitude are listed in Table 4. The dominant systematic uncertainty is the evaluation of the electron identification efficiency. The electron identification algorithm was developed using clean radiative Bhabha events in the data sample. Its performance on continuum events is studied using $\pi^0 \to \gamma e^+ e^-$ where the $e^+ e^-$ pair could originate from either a Dalitz decay of the π^0 or a γ conversion in material. This study resulted in a conservative estimate of the electron identification systematic uncertainty of $\pm 3\%$.

The inclusive semi-electronic branching fraction is measured to be

$$\mathcal{B}(D^0 \to X e^+ \nu) = [6.64 \pm 0.18 \pm 0.29]\% \tag{8}$$

where the first error is statistical and the second error is the estimated systematic effect. Sources of model dependence have been minimized by relying on the experimental measurements of the exclusive rates of the observed modes and experimental measurements of the $d\Gamma/dq^2$ spectrum in $D^0 \to K^- e^+ \nu$ decays. Models have been used only for the $d\Gamma/dq^2$ spectrum of the other exclusive modes. The previous value of $[7.01 \pm 0.62]\%$ agrees with this result [2].

$p(\pi)$ (MeV/c)	$N(D^{*+} \to D^0\pi^+)$	$N(D^{*+} \to D^0\pi^+, D^0 \to Xe^+\nu)$	$\epsilon(Xe^+\nu)$ (%)	$\mathcal{B}(D^0 \to Xe^+\nu)$ (%)
225 - 250	44161 ± 611	1189 ± 61	37.9	7.10 ± 0.38
250 - 275	39114 ± 562	988 ± 57	40.1	6.30 ± 0.38
275 - 300	29482 ± 475	882 ± 51	42.7	7.01 ± 0.42
300 - 325	21120 ± 396	644 ± 44	43.7	6.97 ± 0.49
325 - 350	14973 ± 334	439 ± 37	45.5	6.42 ± 0.56
350 - 375	9165 ± 267	221 ± 30	48.0	5.02 ± 0.70
375 - 400	5492 ± 208	168 ± 23	49.5	6.18 ± 0.88
400 - 425	2151 ± 147	78 ± 19	50.7	7.15 ± 1.8
Total	165658 ± 1149	4609 ± 121		6.64 ± 0.18

Table 3: The yields of inclusive $D^{*+} \to D^0\pi^+$ decays, $D^0 \to Xe^+\nu$ decays, the efficiency for detecting the $Xe^+\nu$ final state and the calculated branching fraction, as a function of the initial D^{*+} pion momentum. The errors are statistical only and include the statistical error on the background subtraction.

Source	Estimated Systematic Error (%)
Electron Identification Efficiency	± 3.0
$Xe^+\nu$ Cocktail	± 2.0
$N(D^{*+})$	± 1.5
Track Reconstruction	± 1.0
Monte Carlo Statistics	± 1.0
Electron fake rate	± 1.0
Form Factor slope κ	± 0.6
Total	± 4.3

Table 4: Estimate of the systematic uncertainty in the measurement of $\mathcal{B}(D^0 \to Xe^+\nu)$

$p(\pi)$ (MeV/c)	$N(D^{*+} \to D^0\pi^+, D^0 \to K^-\pi^+)$	$\epsilon(K\pi)$ (%)	$N(D^{*+} \to D^0\pi^+, D^0 \to Xe^+\nu)$	$\epsilon(Xe^+\nu)$ (%)	$\frac{\mathcal{B}(D^0 \to Xe^+\nu)}{\mathcal{B}(D^0 \to K^-\pi^+)}$
225 - 250	1129 ± 44	64.6	1189 ± 61	37.9	1.80 ± 0.12
250 - 275	945 ± 40	64.3	988 ± 57	40.1	1.68 ± 0.12
275 - 300	741 ± 34	64.4	882 ± 51	42.7	1.80 ± 0.13
300 - 325	528 ± 30	65.1	644 ± 44	43.7	1.82 ± 0.16
325 - 350	393 ± 25	66.0	439 ± 37	45.5	1.62 ± 0.18
350 - 375	262 ± 19	66.4	221 ± 30	48.0	1.17 ± 0.18
375 - 400	153 ± 15	68.8	168 ± 23	49.5	1.53 ± 0.26
400 - 425	57 ± 9	63.1	78 ± 19	50.7	1.70 ± 0.50
Total	4208 ± 83		4609 ± 121		1.684 ± 0.056

Table 5: The yields of $D \to K^-\pi^+$ decays, the efficiency for detecting the $K^-\pi^+$ final state, the yields of $D^0 \to Xe^+\nu$ decays, the efficiency for detecting the $Xe^+\nu$ final state and the calculated ratio of branching fractions, as a function of the initial D^{*+} pion momentum. The errors on the data yields are statistical only. The ratio of branching fractions error is statistical only.

6.2 $\mathcal{B}(D^0 \to Xe^+\nu)/\mathcal{B}(D^0 \to K^-\pi^+)$

In addition to measuring the absolute $D^0 \to Xe^+\nu$ branching fraction, it is straightforward to combine the yields presented here with those in Ref. [3] to obtain a measurement of the ratio $\mathcal{B}(D^0 \to Xe^+\nu)/\mathcal{B}(D^0 \to K^-\pi^+)$. This tabulation is done in Table 5. This ratio is independent of systematics associated with the inclusive $D^{*+} \to D^0\pi^+$ yields. The contributions to the systematic error are given in Table 6. The result is

$$\mathcal{B}(D^0 \to Xe^+\nu)/\mathcal{B}(D^0 \to K^-\pi^+) = 1.684 \pm 0.056 \pm 0.093. \tag{9}$$

Again the first error is statistical and the second error is the estimated systematic effect, where the use of a common dataset allowed cancelation of some systematic effects present in the individual results.

This ratio allows for a check of the ratio

$$X_K = \mathcal{B}(D \to K^-e^+\nu)/\mathcal{B}(D \to Xe^+\nu) \tag{10}$$

$$= \frac{\mathcal{B}(D \to K^-e^+\nu)}{\mathcal{B}(D \to K^-\pi^+)} \times \frac{\mathcal{B}(D \to K^-\pi^+)}{\mathcal{B}(D \to Xe^+\nu)} \tag{11}$$

Source	Estimated Systematic Error (%)
Electron Efficiency	± 3.0
$Xe^+\nu$ Cocktail	± 2.0
Track Reconstruction	± 3.8
Monte Carlo Statistics	± 1.2
Electron fake rate	± 1.0
Form Factor slope κ	± 0.6
$K^-\pi^+$ (mass fit and momentum cut)	± 0.7
Total	± 5.5

Table 6: Estimate of the systematic uncertainty in the measurement of $\mathcal{B}(D^0 \to Xe^+\nu)/\mathcal{B}(D^0 \to K^-\pi^+)$.

which is used in the $D^0 \to Xe^+\nu$ cocktail. To obtain the most precise value possible, we take advantage of the fact that the CLEO results for $\mathcal{B}(D^0 \to K^-e^+\nu)/\mathcal{B}(D^0 \to K^-\pi^+)$ were obtained with the same detector, allowing reduction in the systematic bias due to lepton identification (reduced to $\pm 1.7\%$) and the systematic bias due to tracking reconstruction (reduced to $\pm 2\%$). There is also a large overlap of $D^0 \to K^-\pi^+$ events which were used to calculate the two ratios which appear in Eq. 11 [10]. Using only CLEO results and taking these common systematic effects into account we obtain $X_K = 0.581 \pm 0.023 \pm 0.028$. Using all measurements of $\mathcal{B}(D^0 \to K^-e^+\nu)/\mathcal{B}(D^0 \to K^-\pi^+)$ and taking advantage of the common CLEO systematic errors results in a value of $X_K = 0.552 \pm 0.035$ [12]. These results agree well with the input value of X_K listed in Table 12.

6.3 Comparison of inclusive measurement to the sum of the exclusive rates

The measurement of the inclusive semi-electronic branching fraction is often compared to the sum of the measured exclusive channels [15]. This comparison provides a measure of the consistency of the experimental measurements.

In terms of the branching fraction ratios, $R_m = \mathcal{B}(D^0 \to me^+\nu)/\mathcal{B}(D^0 \to K^-e^+\nu)$, which are used in Appendix A for tabulating the $D^0 \to Xe^+\nu$

cocktail listed in Table 12, the ratio between the difference of the inclusive and the sum of the exclusive rates can be written as:

$$\frac{\mathcal{B}(D^0 \to Xe^+\nu) - \sum_m \mathcal{B}(D^0 \to me^+\nu)}{\mathcal{B}(D^0 \to Xe^+\nu)} = 1 - X_K(1 + R_{K^*} + R_\pi + R_\rho). \tag{12}$$

Performing the comparison using only CLEO data ($X_K = 0.581 \pm 0.036$, and $1 + R_{K^*} + R_\pi = 1.724 \pm 0.078$) results in a value of:

$$\frac{\mathcal{B}(D^0 \to Xe^+\nu) - \sum_m \mathcal{B}(D^0 \to me^+\nu)}{\mathcal{B}(D^0 \to Xe^+\nu)} = (-0.2 \pm 7.7)\%. \tag{13}$$

This CLEO result does not include a contribution from R_ρ as CLEO has not reported a value for this ratio. Inclusion of the small contribution for R_ρ will result in a central value further from zero, while still entirely consistent with zero given the experimental errors. Using the value of $X_K = 0.552 \pm 0.035$ obtained in the previous section and $1 + R_{K^*} + R_\pi + R_\rho = 1.751 \pm 0.067$ (see Table 12) we find,

$$\frac{\mathcal{B}(D^0 \to Xe^+\nu) - \sum_m \mathcal{B}(D^0 \to me^+\nu)}{\mathcal{B}(D^0 \to Xe^+\nu)} = (3.3 \pm 7.2)\%. \tag{14}$$

These results are consistent with the upper limits obtained by direct searches for the unobserved exclusive modes [13].

6.4 The inclusive electron momentum spectrum

The lepton spectrum from semi-leptonic charm decays has not been updated since the DELCO results [14]. Because the measurement presented here is not made in the rest frame of the D^0 we compare the observed lepton spectrum in the lab frame with that of the Monte Carlo simulation. To obtain the momentum spectrum for inclusive $D^0 \to Xe^+\nu$ decays, events were selected if they pass all the selection criteria previously described. An additional cut of $\sin^2\alpha < 0.12$ is applied. This cut retains 90% of the signal and is large enough that systematics associated with modeling the thrust axis are minimized. There is still background in this sample whose shape is provided by the wrong sign $\sin^2\alpha$ distribution. The normalization of this background is obtained by normalizing the wrong sign $\sin^2\alpha$ distribution to the right sign $\sin^2\alpha$ distribution for values of $\sin^2\alpha > 0.2$. As this result is focused on the distribution of the electron momentum not the normalization, a $\pm 1\%$ uncertainty in the level of background spread over the momentum range is negligible. In Figure 4 the background subtracted momentum spectrum for the electrons is shown along with the momentum spectrum obtained from the Monte Carlo simulation. The two distributions are normalized to the same number of events, resulting in a 75% confidence level. The comparison shows that the simulation is correctly producing D^{*+}, D^0 mesons and the inclusive $D^0 \to Xe^+\nu$ decays. Any deviations would indicate a problem in the simulation, either in the production or decay dynamics. We conclude that the Monte Carlo provides a good simulation of the data.

7 Conclusions

We have presented a new measurement of the inclusive branching fraction for $D^0 \to Xe^+\nu$ decays. The final result is,

$$\mathcal{B}(D^0 \to Xe^+\nu) = [6.64 \pm 0.18(stat.) \pm 0.29(syst.)]\%. \tag{15}$$

We find that the difference between this inclusive rate and the sum of the observed exclusive channels is $(3.3 \pm 7.2)\%$ of the inclusive rate. This corresponds to an upper limit on the unobserved modes of 14% of the inclusive rate (at the 90% C.L.). The experimental upper limits obtained using direct searches for specific unobserved exclusive semi-electronic modes are lower than the limit quoted here. However, the upper limit obtain in this paper is less sensitive to the assumption of what exclusive channels are unobserved. The two methods, direct searches and inclusive-exclusive rate comparison, both suggest that the remaining unobserved exclusive semi-leptonic modes occur at small rates. In addition the observed electron momentum spectrum from inclusive $D^0 \to Xe^+\nu$ decays is seen to be well described by the exclusive semi-electronic cocktail.

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A Determination of the $D^0 \to Xe^+\nu$ cocktail

In this appendix, the exclusive semi-leptonic branching fractions, and a list of their averages are presented. This list, which is referred to as the $D^0 \to Xe^+\nu$ cocktail, is used to calculate the electron detection efficiency. The $D^0 \to Xe^+\nu$ cocktail is determined using world averages to obtain the following ratios:

$$R_{K^*} = \mathcal{B}(D^0 \to K^{*-}e^+\nu)/\mathcal{B}(D^0 \to K^-e^+\nu) \tag{16}$$
$$R_\pi = \mathcal{B}(D^0 \to \pi^-e^+\nu)/\mathcal{B}(D^0 \to K^-e^+\nu) \tag{17}$$
$$R_\rho = \mathcal{B}(D^0 \to \rho^-e^+\nu)/\mathcal{B}(D^0 \to K^-e^+\nu). \tag{18}$$

Experimental upper limits are used to obtain estimates for the unobserved modes:

$$R_{K(1270)} = \mathcal{B}(D^0 \to K_1^-(1270)e^+\nu)/\mathcal{B}(D^0 \to K^-e^+\nu) \tag{19}$$
$$R_{K^*(1430)} = \mathcal{B}(D^0 \to K^{*-}(1430)e^+\nu)/\mathcal{B}(D^0 \to K^-e^+\nu). \tag{20}$$

The central value used for these unobserved modes is set to half the 90% confidence level upper limit and with an error equal to $\pm 100\%$ of the central value.

The ratio of an exclusive channel to the inclusive rate is then obtained from the following formulas:

$$S = 1 + R_{K^*} + R_\pi + R_\rho + R_{K(1270)} + R_{K^*(1430)} \tag{21}$$
$$X_K = 1/S \tag{22}$$
$$X_{K^*} = R_{K^*}/S \tag{23}$$
$$X_\pi = R_\pi/S \tag{24}$$
$$X_\rho = R_\rho/S \tag{25}$$

$$
\begin{aligned}
X_{K(1270)} &= R_{K(1270)}/S & (26)\\
X_{K^*(1430)} &= R_{K^*(1430)}/S. & (27)
\end{aligned}
$$

Throughout this appendix the results are written in terms of the D^0 branching fractions. Results from the D^+ sector are converted into D^0 equivalent branching fractions using isospin and the measured D^0 and D^+ lifetimes. Also semi-muonic measurements are converted into semi-electronic results by correcting for the phase space difference between the muonic and electronic modes [2]. In several of the tables, two averages are presented, one which includes all the data presented in the table, and another with CLEO results excluded. This is done to avoid double weighting in the CLEO data when performing calculations.

A.1 $R_{K^*} = \mathcal{B}(D^0 \to K^{*-}e^+\nu)/\ \mathcal{B}(D^0 \to K^-e^+\nu)$

There are two methods to measure this ratio: direct and indirect. The direct measurements, given in Table 7, can only be performed when both the K and K^* modes are reconstructed through the same parent species within the same experiment. The indirect measurement compares the $K^*e^+\nu$ width measured in D^+ decays to the $K^-e^+\nu$ width measured in D^0 decays, via

$$
\begin{aligned}
R_{K^*}^{indirect} &= \frac{\mathcal{B}(D^+ \to \bar{K}^{*0}e^+\nu)}{\mathcal{B}(D^+ \to K^-\pi^+\pi^+)} \times \\
&\quad \frac{\mathcal{B}(D^0 \to K^-\pi^+)}{\mathcal{B}(D^0 \to K^-e^+\nu)} \times \\
&\quad \frac{\mathcal{B}(D^+ \to K^-\pi^+\pi^+)}{\mathcal{B}(D^0 \to K^-\pi^+)} \times \frac{\tau_{D^0}}{\tau_{D^+}}. \qquad (28)
\end{aligned}
$$

Table 8 contains the world average for $\mathcal{B}(D^0 \to K^-e^+\nu)/\mathcal{B}(D^0 \to K^-\pi^+)$ and Table 9 contains the world average for $\mathcal{B}(D^+ \to \bar{K}^{*0}e^+\nu)/\mathcal{B}(D^+ \to K^-\pi^+\pi^+)$ where the CLEO measurements have been specifically excluded as these measurements are used in the direct determination of R_{K^*}. To determine $R_{K^*}^{indirect}$, the ratio of normalizing modes $K\pi\pi/K\pi$ presented in Table 10 is used. Using the world average for this ratio of branching fractions and the D^+/D^0 lifetime ratio [2] the value for $R_{K^*}^{indirect}$ is measured to be 0.559 ± 0.068. Averaging $R_{K^*}^{direct}$ and $R_{K^*}^{indirect}$ yields

$$
R_{K^*} = 0.579 \pm 0.049. \qquad (29)
$$

Experiment	Reference	$\bar{K}^*e^+\nu/\bar{K}e^+\nu$
CLEO93	[4]	0.62 ± 0.08
CLEO91	[16]	0.51 ± 0.19
Average		0.60 ± 0.07

Table 7: Direct measurements of the $\mathcal{B}(D \to K^*e^+\nu)/\mathcal{B}(D \to Ke^+\nu)$ ratio and their weighted average.

Experiment	Reference	$K^-e^+\nu/K^-\pi^+$
CLEO93	[4]	0.978 ± 0.052
E687 (94)	[17]	0.865 ± 0.051
CLEO91	[16]	0.86 ± 0.07
E691	[18]	0.91 ± 0.13
E687 (90)	[19]	0.84 ± 0.19
Average without CLEO		0.869 ± 0.046
Average		0.906 ± 0.031

Table 8: Measurements of the $\mathcal{B}(D^0 \to K^-e^+\nu)/\mathcal{B}(D^0 \to K^-\pi^+)$ ratio and their weighted average. The average without CLEO measurements is also calculated separately to avoid multiple use of the CLEO results in determining R_{K^*}.

Experiment	Reference	$\bar{K}^{*0}e^+\nu/K^-\pi^+\pi^+$
E691	[20]	0.49 ± 0.06
E687	[21]	0.59 ± 0.07
CLEO	[4]	0.67 ± 0.11
E653	[22]	0.48 ± 0.11
Argus	[23]	0.55 ± 0.13
WA82	[24]	0.62 ± 0.17
average	without CLEO	0.527 ± 0.041
average		0.547 ± 0.038

Table 9: Measurements of the $\mathcal{B}(D^+ \to \bar{K}^{*0}e^+\nu)/\mathcal{B}(D^+ \to K^-\pi^+\pi^+)$ ratio and their weighted average.

Experiment	Reference	$\mathcal{B}(D^0 \to K^-\pi^+)$ (%)	$\mathcal{B}(D^+ \to K^-\pi^+\pi^+)$ (%)	$\frac{\mathcal{B}(D^+\to K^-\pi^+\pi^+)}{\mathcal{B}(D^0\to K^-\pi^+)}$
CLEO	[3]	3.91 ± 0.19	9.3 ± 1.0	2.35 ± 0.23
ARGUS	[25]	3.41 ± 0.30		
ALEPH	[26]	3.89 ± 0.33		
Mark III	[27]	4.2 ± 0.6	9.1 ± 1.4	
Mark II	[28]	4.1 ± 0.6	9.1 ± 1.9	
ARGUS	[29]	4.5 ± 0.7		
HRS	[30]	4.50 ± 0.94		
Mark I	[31]	4.3 ± 1.0	8.6 ± 2.0	
average	without CLEO	3.84 ± 0.18	8.98 ± 0.98	2.34 ± 0.28
average		3.87 ± 0.13	9.1 ± 0.7	2.35 ± 0.18

Table 10: Measurements of the hadronic normalizing modes, $D^0 \to K^-\pi^+$, $D^+ \to K^-\pi^+\pi^+$ and their ratio. The CLEO result on $\mathcal{B}(D^+ \to K^-\pi^+\pi^+)/\mathcal{B}(D^0 \to K^-\pi^+)$ is a direct measurement of this ratio, and is not obtained by dividing the individual CLEO results.

Experiment	Reference	Mode	$\pi^- e^+\nu/K^- e^+\nu$
CLEO	[33]	$\pi^- e^+\nu$	0.103 ± 0.041
Mark III	[34]	$\pi^- e^+\nu$	0.115 ± 0.051
CLEO	[35]	$\pi^0 e^+\nu$	0.17 ± 0.06
average			0.121 ± 0.028

Table 11: Measurements of the $\mathcal{B}(D^0 \to \pi^- e^+\nu)/\mathcal{B}(D^0 \to K^- e^+\nu)$ ratio and their weighted average.

A.2 $R_\pi = \mathcal{B}(D^0 \to \pi^- e^+\nu)/(D^0 \to K^- e^+\nu)$

The Cabibbo suppressed decay $D^0 \to \pi^- e^+\nu$ has been observed at Mark III. CLEO has made measurements of both the $D^0 \to \pi^- e^+\nu$ and the $D^+ \to \pi^0 e^+\nu$ decay chains. There is factor of two due to isospin that is needed to convert the $D^+ \to \pi^0 e^+\nu$ measurement to a $D^0 \to \pi^- e^+\nu$ branching fraction. The results are presented in Table 11.

A.3 $R_\rho = \mathcal{B}(D^0 \to \rho^- e^+\nu)/\mathcal{B}(D^0 \to K^- e^+\nu)$

Fermilab experiment E653 has published an observation of four $D^+ \to \rho^0\mu^+\nu$ events based on a kinematic separation of the Cabibbo suppressed $\rho^0\mu^+\nu$ signal from the more copious $\bar{K}^{*0}\mu^+\nu$ mode [36]. They measure $\mathcal{B}(D^+ \to \rho^0\mu^+\nu)/\mathcal{B}(D^+ \to \bar{K}^{*0}\mu^+\nu) = 0.044^{+0.031}_{-0.025} \pm 0.014$. To obtain R_ρ this measurement needs be corrected by the isospin factor and multiplied by R_{K^*} which gives; $R_\rho = \mathcal{B}(D^+ \to \rho^0\mu^+\nu)/\mathcal{B}(D^+ \to \bar{K}^{*0}\mu^+\nu) \times R_{K^*} \times I_\rho = (0.044^{+0.031}_{-0.025} \pm 0.014) \times (0.579 \pm 0.049) \times 2 = 0.051 \pm 0.037$. For Monte Carlo generation it is assumed that the form factor ratios for $D^0 \to \rho^- e^+\nu$ decay are identical to that of the well measured $D^0 \to K^{*-} e^+\nu$ decay.

A.4 $\mathcal{B}(D^0 \to (\bar{K}^*\pi)^- e^+\nu)$ upper limits

Searches for higher $K^{(*)}$ resonances and possible non-resonant contributions to D semi-leptonic decay have been performed by the fixed target experiments [13]. Although no evidence for these decays has been demonstrated we include $D^0 \to K^-(1270)e^+\nu$ and $D^0 \to K^{*-}(1430)e^+\nu$ in the Monte

Mode	R_m	X_m
$D^0 \to K^- e^+ \nu$	1.0	0.555 ± 0.024
$D^0 \to K^{*-} e^+ \nu$	0.579 ± 0.049	0.321 ± 0.021
$D^0 \to \pi^- e^+ \nu$	0.121 ± 0.028	0.067 ± 0.015
$D^0 \to \rho^- e^+ \nu$	0.051 ± 0.037	0.028 ± 0.020
$D^0 \to K_1^-(1270) e^+ \nu$	0.03 ± 0.03	0.017 ± 0.016
$D^0 \to K^{*-}(1430) e^+ \nu$	0.02 ± 0.02	0.011 ± 0.011
Sum	1.801 ± 0.077	

Table 12: The world average or estimate of the ratio of exclusive channels relative to the $D^0 \to K^- e^+ \nu$ decay mode, $R_m = \mathcal{B}(D^0 \to m e^+ \nu)/\mathcal{B}(D^0 \to K^- e^+ \nu)$. The third column, is the ratio of the exclusive rate to the sum of the exclusive rates, $X_m = \mathcal{B}(D^0 \to m e^+ \nu)/\mathrm{Sum}$.

Carlo simulation. The decays are generated unpolarized and with the following strengths and errors, $R_{K(1270)} = \mathcal{B}(D^0 \to K_1^-(1270) e^+ \nu)/\mathcal{B}(D^0 \to K^- e^+ \nu) = 0.03 \pm 0.03$ and $R_{K^*(1430)} = \mathcal{B}(D^0 \to K^{*-}(1430) e^+ \nu)/\mathcal{B}(D^0 \to K^- e^+ \nu) = 0.02 \pm 0.02$. It is assumed that any non-resonant contribution to the inclusive rate will have a similar electron momentum spectrum distribution as these higher order modes.

A.5 Calculation of the $D^0 \to X e^+ \nu$ cocktail

Table 12 summarizes the relative rates, R_m (relative to $D^0 \to K^- e^+ \nu$) obtained in the previous sections. The sum of these rates is then used to determine the ratio of each exclusive rate to the sum of all the exclusive rates as per Eqs. 21- 27. Table 13 contains the efficiencies for these exclusive modes to pass the selection criteria.

A.6 Comparison of the inclusive rate to the sum of the exclusive measurements.

One of the most frequent comparisons in the literature [1, 2, 15] is the sum of the observed exclusive channels to the measured inclusive rate. The method

$p(\pi)$ MeV/c	$\epsilon(K^- e^+\nu)$ (%)	$\epsilon(K^{*-} e^+\nu)$ (%)	$\epsilon(\pi^- e^+\nu)$ (%)	$\epsilon(\rho^- e^+\nu)$ (%)	$\epsilon(K_1^-(1270) e^+\nu)$ (%)	$\epsilon(K^{*-}(1430) e^+\nu)$ (%)
225-250	40.4	34.4	42.4	38.1	20.4	10.9
250-275	42.8	36.3	45.4	39.5	22.9	11.7
275-300	45.6	38.6	47.6	42.6	20.6	12.0
300-325	46.2	40.4	49.2	43.8	23.4	13.0
325-350	48.6	41.0	51.0	46.6	27.2	12.0
350-375	50.8	44.1	54.6	43.7	30.7	14.6
375-400	51.9	46.1	56.6	48.2	29.4	19.5
400-425	53.9	45.2	57.7	57.4	21.4	34.3

Table 13: Efficiencies for the exclusive decay channels used in the $Xe^+\nu$ cocktail.

of comparing the inclusive measurement to the sum of the ratio of exclusive measurements is presented here.

The following set of equations are used to calculate the branching fraction for the observed exclusive decays:

$$\mathcal{B}(D^0 \to K^- e^+\nu) = r_{K\pi}^{Ke^+\nu} \times \mathcal{B}(D^0 \to K^-\pi^+) \tag{30}$$

$$\mathcal{B}(D^0 \to K^{*-} e^+\nu) = r_{K\pi}^{Ke^+\nu} \times \mathcal{B}(D^0 \to K^-\pi^+) \times R_{K^*} \tag{31}$$

$$\mathcal{B}(D^0 \to \pi^- e^+\nu) = r_{K\pi}^{Ke^+\nu} \times \mathcal{B}(D^0 \to K^-\pi^+) \times R_{\pi} \tag{32}$$

$$\mathcal{B}(D^0 \to \rho^- e^+\nu) = r_{K\pi}^{Ke^+\nu} \times \mathcal{B}(D^0 \to K^-\pi^+) \times R_{\rho} \tag{33}$$

The sum of the observed exclusive rates is then:

$$\sum_m \mathcal{B}(D^0 \to m e^+\nu) = r_{K\pi}^{Ke^+\nu} \times \mathcal{B}(D^0 \to K^-\pi^+) \times (1 + R_{K^*} + R_\pi + R_\rho) \tag{34}$$

The quantities $r_{K\pi}^{Ke^+\nu} = \mathcal{B}(D^0 \to K^- e^+\nu)/\mathcal{B}(D^0 \to K^-\pi^+)$ and $\mathcal{B}(D^0 \to K^-\pi^+)$ are common to all derived exclusive branching fractions, and thereby effect the entire scale.

References

[1] Jeffery D. Richman and Patricia R. Burchat, UCSB-HEP-95-08 or Stanford-HEP-95-01, to be published in *Reviews of Modern Physics*. In this review of charm and beauty semi-leptonic decays, they obtain a value of $[5.73 \pm 0.25]\%$ for the sum of the exclusive channels and $[7.10 \pm 0.29]\%$ for the inclusive D semi-leptonic decay branching fraction. This results in a difference between the inclusive rate and the exclusive rate of $(19.3 \pm 4.8)\%$ of the inclusive rate.

[2] L. Montanet *et al.* , Review of Particle Properties Phys. Rev. **D50**, 1994. Within the *Note on Semi-Leptonic Decays of D and B Mesons, Part I* by R.J. Morrison and J. D. Richman, they obtain $[5.73 \pm 0.25]\%$ for the sum of exclusive semi-leptonic D decay branching fractions and $[7.01 \pm 0.62]\%$ for the inclusive D semi-leptonic branching fraction. This results in a difference between the inclusive rate and the exclusive rate of $(18.3 \pm 8.1)\%$ of the inclusive rate.

[3] D.S. Akerib *et al.* , (CLEO Collaboration), Phys. Rev. Lett. **71**, 3070 (1993).

[4] A. Bean *et al.* , (CLEO Collaboration), Phys. Lett. **B317**, 647 (1993).

[5] Y. Kubota *et al.* , (CLEO Collaboration), Nucl. Instr. Meth. **A320**, 66 (1992).

[6] Charge conjugation is implied throughout this paper, the word electron will be used to refer to both e^-'s and e^+'s.

[7] Other sources of right and wrong sign electrons are $D^0 - \bar{D}^0$ mixing and flavor changing neutral currents, $D^0 \to e^+e^-$. Experimental limits and theoretical expectations on these rare processes make their contribution to the right and wrong sign yields negligible [2].

[8] N. Isgur, D. Scora, B. Grinstein and M. B. Wise, Phys. Rev. **D39**, 799 (1989).

[9] The $D \to \bar{K}^* e^+ \nu$ decay has three helicity states in the limit of negligible lepton mass. In Monte Carlo generation the world average [20, 21, 22]

of the three form factors involved in the decay is used to determined the ratio of longitudinal to transverse alignments ($\Gamma_L/\Gamma_T = 1.23 \pm 0.13$).

[10] When combining the CLEO result for $\mathcal{B}(D^0 \to K^- e^+ \nu)/\mathcal{B}(D^0 \to K^- \pi^+)$ and the CLEO result for $\mathcal{B}(D^0 \to X e^+ \nu)/\mathcal{B}(D^0 \to K^- \pi^+)$ we use the following values which have reduced systematic errors:

$$\mathcal{B}(D^0 \to K^- e^+ \nu)/\mathcal{B}(D^0 \to K^- \pi^+) = 0.978 \pm 0.025 \pm 0.032 \quad (35)$$
$$\mathcal{B}(D^0 \to X e^+ \nu)/\mathcal{B}(D^0 \to K^- \pi^+) = 1.684 \pm 0.049 \pm 0.061. \quad (36)$$

[11] When combining the CLEO result for $\mathcal{B}(D^0 \to K^- e^+ \nu)/\mathcal{B}(D^0 \to K^- \pi^+)$ and the CLEO result for $\mathcal{B}(D^0 \to X e^+ \nu)/\mathcal{B}(D^0 \to K^- \pi^+)$ we use the following values with reduced systematic errors:

$$\mathcal{B}(D^0 \to K^- e^+ \nu)/\mathcal{B}(D^0 \to K^- \pi^+) = 0.978 \pm 0.025 \pm 0.032 \quad (37)$$
$$\mathcal{B}(D^0 \to X e^+ \nu)/\mathcal{B}(D^0 \to K^- \pi^+) = 1.684 \pm 0.049 \pm 0.061. \quad (38)$$

[12] This value for X_K includes the CLEO and non-CLEO results for $R = \mathcal{B}(D^0 \to K^- e^+ \nu)/\mathcal{B}(D^0 \to K^- \pi^+)$. The weighted average of $R(CLEO) = 0.978 \pm 0.041$ (from [10]) and $R(non - CLEO) = 0.869 \pm 0.046$ is $R(wtd) = 0.930 \pm 0.031$. To calculate X_K one divides by the branching fraction ratio $R' = \mathcal{B}(D^0 \to X e^+ \nu)/(D^0 \to K^- \pi^+)$. Using only CLEO data to calculate X_K, the statistical and systematic errors of R and R' are reduced because of the cancelation of the contributions from $\mathcal{B}(D^0 \to K^- \pi^+)$, and one calculates $X_K = R/R' = (0.978 \pm 0.041)/(1.684 \pm 0.078) = 0.581 \pm 0.036$, where R and R' are taken from [10]. Using $R(non - CLEO)$ to calculate X_K, one obtains $X_K = R/R' = (0.869 \pm 0.046)/(1.684 \pm 0.109) = 0.516 \pm 0.043$. In this case the full errors for R' are used (see equation 9), because the non-CLEO measurement of R has no experimental correlation with the CLEO measurement of R'. To combine the CLEO and non-CLEO results for R to calculate X_K, one needs a weighted value of R' from the two values given above. $R'(wtd) = 1.684 \pm [0.078 f(correlated) + 0.109 f(uncorrelated)]$, where $f(uncorrelated) = 1 - f(correlated) = (1/0.046)^2/[(1/0.046)^2 + (1/0.041)^2] = 0.438$. This results in $R'(wtd) = 1.684 \pm 0.092$. Taking the ratio $R(wtd)/R'(wtd) = (0.930 \pm 0.030)/(1.684 \pm 0.092)$ one calculates $X_K = 0.552 \pm 0.035$.

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