CLNS 95/1332
CLEO 95/6
June 15, 1995

Measurement of α_s from τ Decays

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Abstract

We present measurements of spectral moments extracted from the invariant mass distributions of the final states of hadronic τ decay products recorded in the CLEO detector. From a fit of theoretical predictions to the measurements of spectral moments and the total hadronic decay width of the τ, we determine the strong coupling constant and a set of non-perturbative QCD parameters. The strong coupling constant is measured to be $\alpha_s(m_\tau)$=0.306$\pm$0.024, which when extrapolated to the Z mass, yields $\alpha_s(M_Z)$ = 0.114$\pm$0.003.

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I. INTRODUCTION

The tau lepton, τ, which is the only known lepton heavy enough to decay into hadrons, presents a clean environment to study the hadronic weak currents. The total τ hadronic width has been calculated [1,2] using analyticity and the Operator Product Expansion (OPE). This is one of the very few quantities calculated to $\mathcal{O}(\alpha_s^3)$, and the non-perturbative corrections have been shown to be largely suppressed. Thus inclusive hadronic decays of the tau are expected to provide a unique opportunity to make a precision measurement of α_s.

The invariant mass distributions of the final states in the hadronic decay channels of the τ can provide further insight, but they have not yet been calculated within the existing framework of QCD. However, the spectral moments, built from the weighted integrals of these distributions, have been calculated [2,3] using OPE. In this case, the non-perturbative QCD contributions to the spectral moments may not necessarily be suppressed and this may increase the uncertainty of the theoretical predictions. Indeed, choosing the appropriate weight functions, one can make non-perturbative effects dominate the final results. However, this feature of spectral moments only makes them more interesting, as it can be used to measure the parameters characterizing the non-perturbative dynamics and hence improve our understanding of QCD at long distances [3].

Many methods have been used to determine α_s. For most of these methods [4], theoretical uncertainties dominate over experimental errors, mainly due to the missing higher order corrections in the theory. The ALEPH collaboration [5] used the τ spectral moments to determine the value of α_s from hadronic decays and found that in their case the experimental errors were larger than the theoretical uncertainties. Thus, it becomes of great interest to use the large data sample and the excellent π^0 reconstruction capabilities of CLEO in order to improve the experimental precision on this measurement.

In this article, we report results on the determination of $\alpha_s(m_\tau)$ from: a) the total hadronic decay width, b) the spectral moments, and c) a combined fit to the hadronic width and spectral moments. In the combined fit, we also determine the values for some of the non-perturbative QCD parameters. The final results are obtained from the combined fit. The other fits mentioned above provide a consistency check of the procedure.

II. DEFINITION OF SPECTRAL MOMENTS

In an analogy to the ratio R of the hadronic to leptonic production rate in e^+e^- collisions, one may define the ratio of hadronic to leptonic τ decay widths as:

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau hadrons)}{\Gamma(\tau^- \to \nu_\tau l^- \bar{\nu}_e)} = \frac{1 - B_e - B_\mu}{B_l} = \frac{1}{B_l} - f_e - f_\mu \tag{1}$$

where B_e and B_μ are the branching fractions of τ decays to e and μ respectively, and f_e=1 and f_μ=0.9726 are phase space factors. Theoretically, R_τ may be written as

$$R_\tau = \frac{1}{\Gamma_l}\int_0^{m_\tau^2} ds \frac{d\Gamma_h}{ds} \tag{2}$$

where s is the invariant mass-squared of the hadronic final state. Within existing theoretical skills, it is not possible to calculate $d\Gamma_h/ds$ directly. However, it is possible to calculate R_τ^{kl}, the weighted integrals of $d\Gamma_h/ds$, written as

$$R_\tau^{kl} = \frac{1}{\Gamma_l}\int_0^{m_\tau^2} ds \left(1-\frac{s}{m_\tau^2}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{d\Gamma_h}{ds}. \tag{3}$$

One should note that R_τ is the special case of R_τ^{kl} when k=0 and l=0. The spectral moments, D_τ^{kl}, are defined as:

$$D_\tau^{kl} = \frac{R_\tau^{kl}}{R_\tau^{00}}. \tag{4}$$

The expressions for the R_τ^{kl} have been derived using OPE, and can be written in the form

$$R_\tau^{kl}(u,d) = |V_{ud}|^2 S_{EW} r_{kl}\left[1+\Delta_{m_q}+\Delta_p+\Delta_{np}\right] \tag{5}$$

where V_{ud}=0.9753±0.0004 [6] is a CKM matrix element, $S_{EW} = 1.0194$ is an electroweak correction factor [8], r_{kl} are the terms predicted at parton level, Δ_{m_q} are the quark mass corrections, Δ_p are the perturbative terms in powers of α_s, and Δ_{np} are the non-perturbative corrections. A more detailed discussion of this expression is provided in the Appendix.

The measured values of R_τ and the spectral moments, D_τ^{kl}, are used to perform a global fit in order to determine the value of $\alpha_s(m_\tau)$ and three non-perturbative parameters labelled as $< \frac{\alpha_s}{\pi} GG >$, $\mathcal{O}(6)$ and $\mathcal{O}(8)$ following the notation of Ref. [3] (see Appendix).

III. MEASUREMENT OF SPECTRAL MOMENTS

In order to evaluate the spectral moments, we have measured the invariant mass squared distributions of the final state charged hadrons in all of the significant hadronic decay channels of the τ. The final state hadrons observed in the detector were considered to be pions, and no attempt was made to separate kaons from pions. However, the kaon contribution was considered to be a background. After subtracting the background from this and other sources, the mass distribution for each decay channel was corrected for detector resolution effects and for the mass dependence of the selection efficiency. All the individual mass distributions thus corrected, weighted by the branching ratios compiled by the Particle Data Group [6], were combined into a single histogram. It is this histogram from which the spectral moments were computed.

To perform this analysis we used data from the reaction $e^+e^- \to \tau^+\tau^-$, collected with the CLEO-II detector [7] at CESR at about 10 GeV center of mass energy. The data sample corresponded to an integrated luminosity of about 3 fb^{-1}. We identified τ pair events in which one of the taus decayed leptonically *i.e.* $\tau^- \to l^-\bar{\nu}_\mu\nu_\tau$ (where l is e or μ) to study the hadronic decay of the other τ. (The inclusion of charge conjugate modes is implied throughout the paper). Hadronic τ decays were required to have either one or three charged tracks corresponding to a 1-1 or 1-3 charged track topology for the event.

The opening angle between the two charged tracks in 1-1 topology and the minimum angle between the lepton and any of the other three charged tracks in 1-3 topology were required to exceed 90^0. Only events with zero total charge and 2 (for 1-1 topology) or 4 (for 1-3 topology) "good" charged tracks were considered. A good charged track was defined as having a radial impact parameter less than 0.01 m, momentum less than 85% of the beam momentum, and an angle made with the beam direction greater than 37^0.

The total visible energy of the event was required to exceed 0.15 E_{cm} to suppress two-photon background. To reject the background from electromagnetic processes, it was required that the net transverse momentum of the tracks be greater than 200 MeV and the ratio of the vector to scalar sum of charged track momenta be greater than 0.05. Events with energy greater than 200 MeV deposited in the endcap were rejected.

Electromagnetic clusters which had energy $\geq$ 60 MeV, were more than 20^0 away from the lepton track, were not associated with any charged tracks, and were contained in the barrel calorimeter were used to reconstruct π^0's. In the reconstruction of a π^0, it was required that the angle between two clusters should not exceed 135^0 and that at least one cluster should have a minimum energy of 80 MeV. The reconstructed π^0 was required to lie within 90^0 of the π direction (for the 1-1 topology) and the average direction of 3 pions (for the 1-3 topology). For each $n\pi^0$ case, all possible combinations of clusters forming a π^0 were considered and the one with minimum overall $\chi^2 = \frac{1}{n}\sum_{i=1}^{n}(m_i - m_\pi)^2/\sigma^2_{m_i}$ was selected, where m_i is the invariant mass of the two clusters forming the i'th π^0 candidate, and σ_{m_i} is the uncertainty on m_i calculated from the measured angular and energy resolution of the calorimeter.

Out of a total number of 2.7×10^6 tau pairs produced, the number of events which passed our selection criteria are listed in Table 1 for each considered channel, along with the background fraction f_B. The invariant mass distributions of the hadronic final states were computed from the selected events, separately for electron and muon tagged events. The contributions from the higher multiplicity decay channels were considered negligible, within the experimental accuracy of this analysis, and were ignored. From these distributions, the background estimated from the following sources was subtracted: cross channel contamination from other τ decay channels; contamination from τ decay channels containing kaons; fake π^0's constructed from unrelated photons; and contamination from hadronic events.

Mode	Events	Background(%) f_B
$\pi\pi^0\nu_\tau$	90864	10.0
$\pi 2\pi^0\nu_\tau$	18202	9.2
$\pi 3\pi^0\nu_\tau$	1384	22.4
$\pi 4\pi^0\nu_\tau$	104	22.9
$3\pi\nu_\tau$	53488	16.8
$3\pi\pi^0\nu_\tau$	11179	18.5
$3\pi 2\pi^0\nu_\tau$	685	22.2

Table 1: Number of selected events and the background fractions for various decay channels.

After subtracting the background, the distributions were corrected for detector effects as described below:

1. A Monte Carlo (MC) sample using KORALB [10] was generated and processed through the detector simulation program subject to to the same reconstruction procedure and event selection criteria as the experimental data. From this MC sample, we constructed the probability matrix $P^i(j \rightarrow k)$ for each decay channel

i. The matrix element P^i_{jk} represent the probability that an event generated in decay channel i in bin j of the mass-squared distribution will be observed in bin k in the same decay channel i. After background subtraction, this matrix is used to correct the measured mass-squared distribution for bin migration within each decay channel.

2. The mass-squared distribution corrected for bin migration as above and normalized to unity is then corrected for selection efficiency. This is done for each decay channel separately.

For each of the the muon and electron tagged data samples, the corrected distributions, normalized to unit area, weighted by the branching ratios taken from Particle Data Group [6], were combined into a single distribution presented in Fig. 1. Figure 1 demonstrates excellent agreement between the combined mass histograms from electron tagged and muon tagged data samples. The π^- mass from $\pi^-\nu_\tau$ decay channel enters these distributions as a delta function (not shown in the figure). From these combined distributions, the spectral moments were computed using Equation 3. Moments with $k > 1$ were not used to avoid the low mass region where the validity of the method may be questionable. The values of the spectral moments computed separately from muon and electron tagged data samples were found to be consistent within statistical errors. The two distributions were combined to compute the final values of the spectral moments presented in Table 2, where the experimental errors were obtained by adding in quadrature the statistical errors and the systematic errors. The last column in Table 2 contains the values of the spectral moments computed from the raw data before making any background subtractions or detector corrections. A comparison of the first and the last column in Table 2 demonstrates that the corrections to the data do not have a significant effect on these measurements.

Moment	Value	$\sigma(stat)$	$\sigma(sys)$	$\sigma(exp)$	Value (raw)
$D_\tau^{1,0}$	0.7283	0.0014	0.0034	0.0037	0.7277±0.0014
$D_\tau^{1,1}$	0.1553	0.0004	0.0012	0.0013	0.1544±0.0004
$D_\tau^{1,2}$	0.0559	0.0002	0.0007	0.0007	0.0555±0.0002
$D_\tau^{1,3}$	0.0249	0.0001	0.0004	0.0004	0.0247±0.0001

Table 2: The measured spectral moments from the corrected mass distributions.

IV. DETERMINATION OF α_s

In the determination of α_s we also use R_τ, in addition to the spectral moments. The value of R_τ is determined using equation 1 where B_l is determined by the following three methods: 1) $B_l = B_e/f_e = 0.1801 \pm 0.0018$, 2) $B_l = (\frac{\tau_\tau}{\tau_\mu})(\frac{m_\tau}{m_\mu})^5 = 0.1811 \pm 0.0019$, 3)

$B_l = \frac{B_\mu}{f_\mu} = 0.1816 \pm 0.0024$; where the values for leptonic branching ratios, τ lifetime, and τ mass have all been taken from Ref. [6]. These three measurements yield the weighted average of $B_l = 0.1810 \pm 0.0012$ which yields $R_\tau = 3.559 \pm 0.035$.

In Table 2, only the experimental errors on spectral moments are listed. However, in the fits we also use the theoretical errors which mainly arise from the uncertainties due to the missing higher order corrections, the renormalization scale, and non-perturbative contributions (see the Appendix for detail). The spectral moments are correlated with one another. The correlation coefficients, listed in Table 7, are also used in the fits.

The fits of the theoretical predictions to the experimental measurements were made for various combinations of the spectral moments and the results are listed in Table 3.

Observables used in fit	$\alpha_s(m_\tau)$	$< \frac{\alpha_s}{\pi} GG >$	O(6)	O(8)	χ^2
$R_\tau, D_\tau^{1,0}, D_\tau^{1,1}, D_\tau^{1,2}, D_\tau^{1,3}$	0.306±0.024	0.039±0.012	-0.0053±0.0013	0.0053±0.0013	1.00/1
R_τ	0.327±0.025	0.025	0.0022	0	—
$D_\tau^{1,0}, D_\tau^{1,1}, D_\tau^{1,2}, D_\tau^{1,3}$	0.375±0.051	0.025	0.0022	0	3.25/3
$D_\tau^{1,0}, D_\tau^{1,1}, D_\tau^{1,2}, D_\tau^{1,3}$	0.305±0.029	0.039	-0.0053	0.0053	0.99/3

Table 3: Values of $\alpha_s(m_\tau)$ determined from the fits of the theory to the measured spectral moments. For the non-perturbative parameters, no error indicates that the parameter was kept fixed in the fit.

The simultaneous global fit to the measured values of R_τ and $D_\tau^{1,l}$ is a four-parameter fit where the values of α_s and non-perturbative parameters (NPP) $< \frac{\alpha_s}{\pi} GG >$, $\mathcal{O}(6)$ and $\mathcal{O}(8)$ are determined together. This yields

$$\alpha_s(m_\tau) = 0.306 \pm 0.017 \pm 0.017, \tag{6}$$

where the first error is experimental and the second theoretical. These results are listed in the first row of Table 3. The other rows of Table 3 contain the values of $\alpha_s(m_\tau)$ determined when the NPP are kept fixed at various sets of values. Row 2 is obtained when only the measured value of R_τ is compared with the theory for fixed values of NPP taken from Ref. [2]. Row 3 uses the measured values of $D_\tau^{1,l}$ from Table 2 and the NPP values from Ref. [2]. Row 4 uses the same set of values for $D_\tau^{1,l}$ but the NPP values from the global fit in row 1. The results for $\alpha_s(m_\tau)$ from R_τ alone and $D_\tau^{1,l}$ alone are consistent with each other. This checks the consistency of the method.

In the global fit, as the NPP are determined from the fit, the uncertainties on their values do not contribute to the theoretical errors on the spectral moments. In the fits to $D_\tau^{1,l}$ alone, the uncertainties on the values of the NPP become the dominant source of theoretical errors (Table 6 in the Appendix). This results in the larger errors on the values of $\alpha_s(m_\tau)$ listed in row 3. For row 4, we recompute the theoretical errors on the moments going into the fit according to the errors on the non-perturbative parameters determined from the global fit.

The correlation coefficients for $\alpha_s(m_\tau)$ and the NPP obtained from the global fit are presented in Table 4. The large correlations among the NPP reflect the high correlations among spectral moments.

Observable	$\alpha_s(m_\tau)$	$< \frac{\alpha_s}{\pi} GG >$	O(6)	O(8)
$\alpha_s(m_\tau)$	1	-0.58	0.56	-0.47
$< \frac{\alpha_s}{\pi} GG >$	-0.58	1	-0.86	0.80
O(6)	0.56	-0.86	1	-0.96
O(8)	-0.47	0.80	-0.96	1

Table 4: Correlation coefficients for $\alpha_s(m_\tau)$, $< \frac{\alpha_s}{\pi} GG >$, O(6) and O(8) obtained from the global fit.

V. SYSTEMATIC ERRORS

In this analysis, systematic experimental errors may arise from the following sources:

1. Uncertainties in the branching ratios of the hadronic decay modes of the τ included in the analysis.
2. Uncertainty in the mass scale.
3. Uncertainty on the background evaluation.
4. Uncertainty on π^0 reconstruction.
5. The limited MC statistics to correct the data.
6. Uncertainty in the unfolding procedure.

The effect of the mass scale uncertainty of $\pm 0.25\%$ was estimated by multiplying the mass-squared scale (X-axis) of Fig. 1 by a factor of 1.000 ± 0.005, and taking the resulting shifts in the moments as the corresponding errors. The errors due to the uncertainties in the branching ratios were estimated in a conservative way: spectral moments were recalculated by using the central values of all the branching ratios except one, which was first increased and then decreased by its uncertainty. The larger of the resulting shifts in the central value of a given moment was taken as a symmetric error on the moment due to the uncertainties in that branching fraction. This process was repeated for all branching ratios. The resulting errors for a given moment were added in quadrature to yield the total systematic error due to the uncertainties in the branching fractions. The errors due to the limited MC statistics used to correct the data are also included in the systematic errors. For example, the systematic error of ± 0.0034 on $D_\tau^{1,0}$ contains contributions of ± 0.0009 from mass scale uncertainty, ± 0.0012 from MC statistics, and ± 0.0031 from branching ratio uncertainties. The errors due to other sources listed above are largely included in the errors on the branching fractions, to the extent that these sources do not affect the mass shape. However, we made the following checks on the effects of those uncertainties on our final results:

1. The analysis was repeated without making any corrections to or background subtractions from the data. The final results agreed within the statistical errors.
2. The analysis was repeated without making any bin migration corrections to the data. The final results agreed within the statistical errors.

3. We repeated our analysis with no kaon subtraction from the data, and the final result on $\alpha_s(m_\tau)$ shifted by 0.4%, ie 7% of the overall experimental error.
4. The cut on the χ^2 per degree of freedom of π^0 reconstruction was varied between 4 and 14 for the $\pi 2\pi^0$ channel. The variations in the moments were less than 0.04 % and there was a negligible effect on $\alpha_s(m_\tau)$.
5. The cut on the χ^2 of π^0 reconstruction in the $\pi\pi^0$ channel was varied between 9 and 16. The variations in the moments were less than 0.06 % and there was a negligible effect on $\alpha_s(m_\tau)$.
6. The effect of finite bin widths in evaluating the spectral moments was investigated and found to be negligible.

VI. SUMMARY AND DISCUSSION

In summary, we have determined $\alpha_s(m_\tau)$ from hadronic decays of the τ recorded by the CLEO II detector. By comparing the calculations performed within the OPE framework to the data, we have determined $\alpha_s(m_\tau)$=0.306$\pm$0.017$\pm$ 0.017, where the first error is experimental and the second is due to theoretical uncertainties. Our result agrees with the ALEPH measurement [5] of $\alpha_s(m_\tau) = 0.330\pm$ 0.046.

The agreement between the values of α_s obtained from R_τ alone and from spectral moments D_τ^{1l} points towards the applicability of perturbative QCD calculations in this low energy regime [3]. The values of the non-perturbative parameters obtained from the fits of the theory to the data do not agree very well with those previously obtained from the fits of largely non τ data [2].

Using the formulae of Ref. [13], the value of $\alpha_s(m_\tau)$=0.306$\pm$0.017$\pm$0.017, extrapolated to 10.53 GeV, yields:

$$\alpha_s(10.53GeV) = 0.166 \pm 0.004 \pm 0.004 \tag{7}$$

which is in good agreement with $\alpha_s(10.53) = 0.164\pm0.015$ determined from jet production rates at CLEO [14]. Extrapolated to M_Z, our result yields:

$$\alpha_s(M_Z) = 0.114 \pm 0.002 \pm 0.002 \tag{8}$$

Adding an error of $\pm$0.001 from the uncertainties in the extrapolation procedure arising from the crossings of c and b production thresholds, this result becomes $\alpha_s(M_Z) = 0.114 \pm 0.003$. It is in excellent agreement with the values of $\alpha_s(M_Z)$ determined from other low energy processes [6] such as 0.112$\pm$0.004 from deep-inelastic scattering, and 0.110$\pm$0.006 from bound quark states using lattice QCD. Our value is consistent with the world average [6] of $\alpha_s(M_Z) = 0.117 \pm 0.005$.

The analysis presented here is sensitive to the τ-decay branching ratios. The reported results are obtained by using the branching ratios taken from Particle Data Table [6]. The analysis was repeated by replacing the hadronic branching ratios with those from a more recent compilation [11]. The central value of $\alpha_s(m_\tau)$ shifted from 0.306 to 0.304 which is within the accuracy of the measurement. However if we replace both the hadronic and leptonic branching ratios of the tau with those from this recent compilation [11], the central value of $\alpha_s(m_\tau)$ changes from 0.306 to 0.339. This is mainly due to the change in B_l which changes R_τ in inverse proportion (see Equation 1).

Another word of caution is necessary. The theoretical error of 0.002 in equation (10) above has been determined following the procedure suggested in Ref. [2,3]. However, this procedure may underestimate the theoretical errors due to the absence of non perturbative terms in the leading order of tau mass squared. An additional theoretical error of ± 0.007 has been suggested [15].

VII. ACKNOWLEDGEMENTS

We gratefully acknowledge the effort of the CESR staff in providing us with excellent luminosity and running conditions. J.P.A., J.R.P., and I.P.J.S. thank the NYI program of the NSF, G.E. thanks the Heisenberg Foundation, I.P.J.S. and T.S. thank the TNRLC, K.K.G., M.S., H.N.N., T.S., and H.Y. thank the OJI program of DOE, J.R.P thanks the A.P. Sloan Foundation, and A.W. thanks the Alexander von Humboldt Stiftung for support. This work was supported by the National Science Foundation, the U.S. Department of Energy and the Natural Sciences and Engineering Research Council of Canada. We especially thank F. Le Diberder of Orsay for very helpful communications and for providing the computer program used to compute the theoretical predictions.

Appendix: Theoretical Predictions and Uncertainties

In this section, we present some theoretical formulae, taken from Ref. [3], and a discussion of theoretical uncertainties in this analysis. The R_τ^{kl} have been predicted using OPE in which they can be written as

$$R_\tau^{kl}(u,d) = |V_{ud}|^2 \, S_{EW} r_{kl} \left[1 + \Delta_{m_q}(k,l) + \Delta_p(k,l) + \sum_{D=2,4,\ldots} \Delta^{kl}(m_\tau^2, D) \right]; \tag{9}$$

where V_{ud}=0.9753±0.0004 [6] is a CKM matrix element, $S_{EW} = 1.0194$ an electroweak correction [8], r_{kl} the terms predicted at parton level, and Δ_{m_q} are the quark mass corrections. The zero'th dimension (ie D=0) term, $\Delta_p(k,l)$, is the perturbative contribution and is known to third power in α_s. The higher dimension terms, $\Delta^{kl}(m_\tau^2, D)$, are the non-perturbative contributions which may be expressed as:

$$\Delta^{kl}(m_\tau^2, D) = \frac{c_D(k,l)}{m_\tau^D} \mathcal{O}(D). \tag{10}$$

The terms $\frac{c_D(k,l)}{m_\tau^D}$, computed from Ref [3], are listed in Table 5. The terms $\mathcal{O}$(D) are known as condensates and can, in principle, be obtained from the matrix element in the vacuum of QCD operators [12]. The term $\mathcal{O}(2)$ is considered absent because one cannot build an operator in this dimension from the QCD Lagrangian. The term $\mathcal{O}(4)$ is expressed in terms of quark masses and gluon condensate, $< \frac{\alpha_s}{\pi} GG >$.

D	$\frac{c_D(k,l)}{m_\tau^D}$				
	R_τ	R_τ^{10}	R_τ^{11}	R_τ^{12}	R_τ^{13}
4	0	5.65	-23.67	0	0
6	-3.75	-5.36	-7.49	20.16	0
8	-0.79	-2.83	7.11	6.38	-13.64

Table 5: Numerical values of $\frac{c_D(k,l)}{m_\tau^D}$ computed from Reference [3].

The theoretical predictions were computed using a FORTRAN program supplied by one of the authors of Ref. [3], F. Le Diberder. The spectral moments are highly correlated quantities. The correlations were taken into account and the χ^2 of the fit was defined as:

$$\chi^2 = \sum_{ij} \left[D_\tau^i(exp.) - D_\tau^i(th.) \right] V_{ij}^{-1} \left[D_\tau^j(exp.) - D_\tau^j(th.) \right]; \tag{11}$$

where D_τ^i are R_τ and D_τ^{kl}; and V_{ij}^{-1} are the elements of the inverse of the covariance matrix. The following sources of theoretical uncertainties were considered:

1. The uncertainties on the constant quantities, non-perturbative electroweak and mass corrections, used in the equation 9.
2. The uncertainties due to the missing higher order corrections in the perturbative part of the moments. This uncertainty is evaluated by taking the unknown fourth order coefficients in terms of known second and third order coefficients, namely $K_4 = 2\,|K_3(K_3/K_2)|$ and $\beta_4 = 2\,|\beta_3(\beta_3/\beta_2)|$, and computing the variation in the predicted values of the moments.

3. The contribution of renormalization uncertainty to the moments was evaluated by varying the renormalization scale μ from 1.0 to 2.5 GeV and also by varying the scheme dependent coefficient β_3 from zero to twice its value in $\overline{MS}$ scheme used in these calculations.

The theoretical errors on the moments and the correlation coefficients thus computed using Ref. [2] are listed in Table 6 and Table 7. These errors are used in the fits and enter into the results quoted in Table 3.

Error type	Error				
	R_τ	D_τ^{10}	D_τ^{11}	D_τ^{12}	D_τ^{13}
Σ_{th}	0.043	0.0133	0.0134	0.0032	0.0019
σ_{th}	0.033	0.0035	0.0028	0.0004	0.0002

Table 6: The theoretical errors on spectral moments computed from Reference [3].

	R_τ	D_τ^{10}	D_τ^{11}	D_τ^{12}	D_τ^{13}
R_τ	1	0.11	-0.13	-0.09	-0.06
D_τ^{10}	0.11	1	-0.34	-0.59	-0.49
D_τ^{11}	-0.13	-0.34	1	0.40	0.29
D_τ^{12}	-0.09	-0.59	0.40	1	0.90
D_τ^{13}	-0.06	-0.49	0.29	0.90	1

Table 7: The matrix of correlation coefficients of spectral moments.

The errors on the non-perturbative constants were conservatively taken to be twice the values quoted in the Ref. [2]. The errors, Σ_{th}, in the first row of Table 6 contain the contribution from uncertainties in non-perturbative parameters, and are used when the fixed values of non-perturbative parameters are used in the fits. The errors, σ_{th}, in the second row of Table 6 do not contain the contribution from uncertainties in non-perturbative parameters and are used in the global fit when the non-perturbative parameters are kept free.

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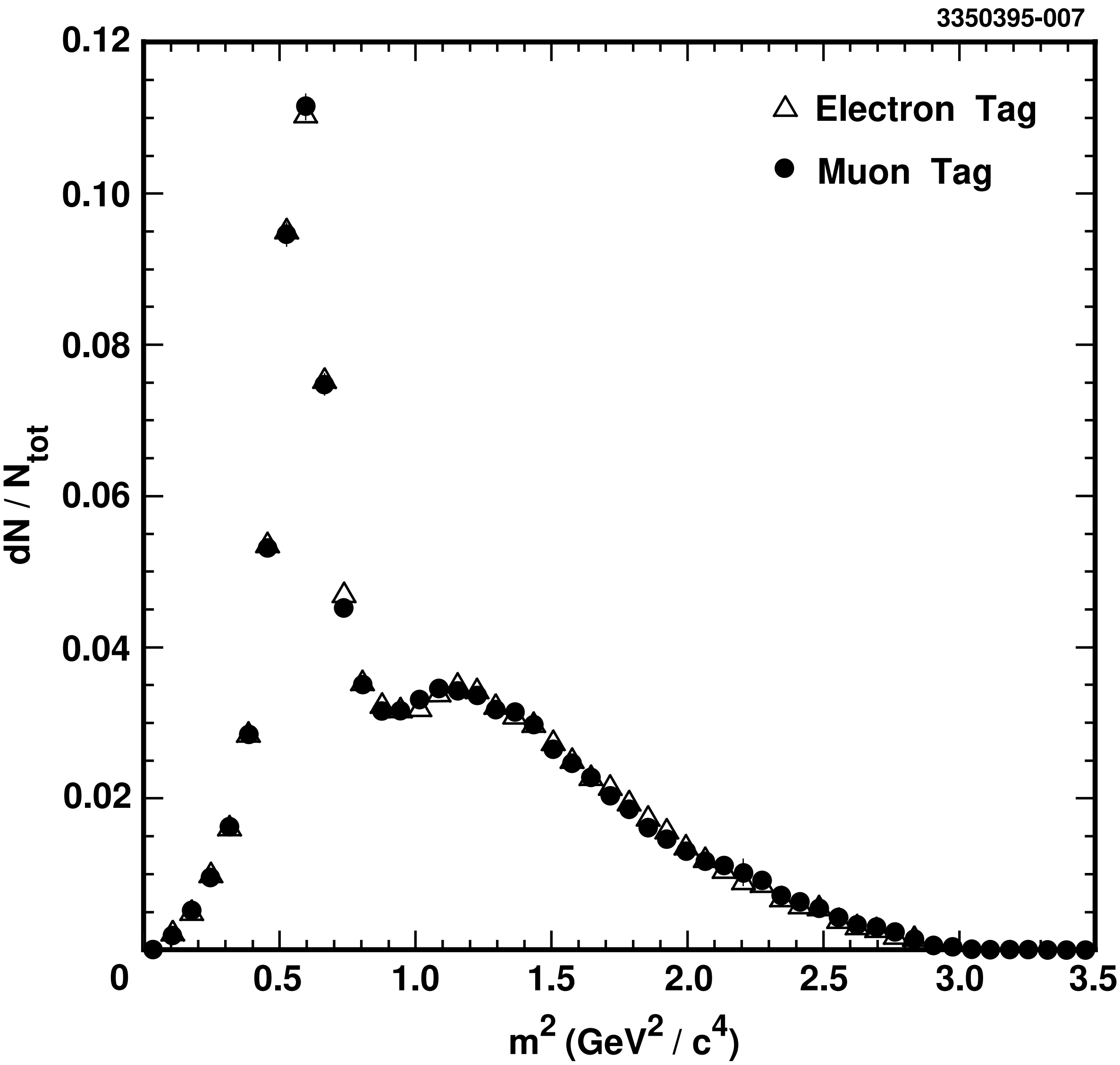

Figure 1: Distributions of mass squared for electron tagged and muon tagged events. The distributions from the individual channels have been combined, weighted by their branching fractions. The contribution of $\tau^- \rightarrow \pi^- \nu_\tau$ to this histogram is taken as a delta function and is not shown here. The corrections for backgrounds, bin migration and selection efficiency have been applied to the individual decay modes.