# CESR SXTUPOLE AS A SKEW QUADRUPOLE AND DIPOLE CORRECTOR 

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We represent here 3D calculations of effective integrated multipoles for the Dipole/Skew Quadrupole correctors incorporated into CESR Sextupole installed in close vicinity to the Quadrupole.

## INTRODUCTION

The Sextupole lens and its followed modifications described in [1]-[3]. Due to peculiarity of design it was possible to modify the poles of Sextupole with minor efforts. The coils remain the same as in original design. The sextupole coil has 160 turns/coil, which delivers 1.6 kA -turns for $10 A$ feeding current.

The Dipole corrector coil has 270 turns each and was run up to $20 A$ pick current during short operation tests [1]. Maximal operational current not exceed 14 A however, dissipating 536 W (at 38.3 Volt) and reaching $\sim 100^{\circ} \mathrm{C}$.


Figure1. The Sextupole. Dimensions are in inches.
One peculiarity of the Sextupole design is that the poles of Sextupole magnet are longer, than the length of Sextupole yoke (Fig.1, Fig2).


Figure 2. Sextupole location between the Quadrupole (at the left) and Dipole magnets (at the right).

The distance between Quadrupole and Sextupole yokes is 6.5 in . The space between the yokes occupied by return turns of the Quadrupole coils.

## MODEL

3D model was erected in MERMAID which includes $1 / 2$ of Sextupole and the fraction of the neighboring Quadrupole. Appropriate boundary conditions applied.


Figure 3. A $1 / 2$ of Sextupole with a fraction of Quadrupole yoke as they appear in MERMAID.

On the basis of these calculations, the integral of the transverse field $B_{y}(x, y, s)$ along the longitudinal axis could be found. Basically the integral is a function of two variables $x$ and $y$

$$
\begin{equation*}
I_{x}(x, y)=\int_{-35 c m}^{35 c m} B_{x}(x, y, z) d z \quad I_{y}(x, y)=\int_{-35 c m}^{35 c m} B_{y}(x, y, z) d z . \tag{1}
\end{equation*}
$$

$\pm 35 \mathrm{~cm}$ represent the total longitudinal length of the model in MERMAID.
The integrals $I_{x}(y)=\int_{-35 \mathrm{~cm}}^{35 \mathrm{~cm}} B_{x}(x=0, y, s) d s[k G \cdot \mathrm{~cm}]$ are represented as a functions of transverse displacement in the Table 1 below. The current is the same as in Fig. 3 what is 1.6 kA -turns, which corresponds to 10 A of a feeding current.

## DIPOLE CORRECTOR



Figure 4. The cross section of model across the center of Quadrupole yoke (at the left) and across the Sextupole yoke (at the right) as they appear in MERMAID.


Figure 5. The field lines in a case of dipole corrector arrangement.


Figure 6. $B_{x}(x=0, y=0, z)$ distribution along the $z$-axis through the Sextupole in a free space.


Figure 7. $\quad B_{x}(x=0, y=0, z)$ distribution along the $z$-axis through the Sextupole in a presence of the Quadrupole at the left side of this figure (compare with Fig.6).

From Fig. 6 and Fig. 7 one can see that the difference in graphs at the left side, where the quadrupole yoke located.

In the Table 1 Left integral taken along longitudinal coordinate $z$ from the side of quadrupole lens from outside to the middle of the Sextupole. The Right integral corresponds to the other side, i.e. the one not occupied by any magnetic obstacle. Integral defined as

$$
\begin{equation*}
I_{\text {left }}(y)=\left.\int_{\text {out }}^{\text {midof lens }} B_{x}(y, z) d z\right|_{\text {Quadside }}, I_{\text {right }}(y)=\left.\int_{\text {out }}^{\text {midof lens }} B_{x}(y, z) d z\right|_{\text {Free side }}, \quad L_{\text {eff }}=\frac{I_{\text {tot }}}{B_{y \text { mid }}} \equiv \frac{I_{\text {left }}+I_{\text {right }}}{B_{y \text { mid }}} \tag{2}
\end{equation*}
$$

where $B_{y}$ mid stands for the field value in the middle of Sextupole.

Table 1. The Dipole longitudinal integrals of $B_{x}(x=0, y, z)$ as functions of vertical displacement $y$ at $1 \mathrm{kA} /$ coil current ${ }^{1}$

| $y, \mathrm{~cm}$ | Left integral, $k G \times c m$ <br> (Quad side) | Right integral, $k G \times c m$ <br> (Free side) | $I_{\text {tot }}, k G \times c m$ | $B_{\text {mid }}, k G$ | $L_{e f f,} c m$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 3.483 | 3.727 | 7.210 | 0.229 | 31.48 |
| 0.5 | 3.483 | 3.726 | 7.209 | 0.229 | 31.48 |
| 1.0 | 3.480 | 3.723 | 7.203 | 0.229 | 31.46 |
| 1.5 | 3.469 | 3.710 | 7.179 | 0.228 | 31.48 |
| 2.0 | 3.439 | 3.702 | 7.141 | 0.226 | 31.60 |
| 2.5 | 3.379 | 3.615 | 6.994 | 0.221 | 31.64 |
| 3.0 | 3.272 | 3.504 | 6.776 | 0.214 | 31.66 |
| 3.5 | 3.104 | 3.329 | 6.433 | 0.202 | 31.84 |
| 4.0 | 2.856 | 3.073 | 5.929 | 0.184 | 32.22 |
| 4.5 | 2.509 | 2.713 | 5.222 | 0.161 | 32.43 |
| 5.0 | 2.048 | 2.238 | 4.286 | 0.131 | 32.72 |



Figure 8. Axial field distribution and its integral along $z$-axis from outside to the middle of Sextupole yoke (from left to the right) at $y=5 \mathrm{~cm}$ in a presence of Quadrupole. Integral is 2.0481 $k G x c m$; field at the center of Sextupole is $0.131 k G$, at end bumps $0.135 k G$ (should be compared with Fig. 7).

[^0]

Figure 9. Axial field distribution and its integral along $z$-axis from outside to the middle of Sextupole yoke (from left to the right) at $\mathrm{y}=5 \mathrm{~cm}$ in a free space. Integral is 2.238 kGxcm ; field at the center of Sextupole is $0.131 k G$, at the end bumps it is $0.135 k G$. (Compare with Fig. 8.)

From the Figs. 8 and 9 the influence of Quadrupole yoke is seen clearly. In [4] effective length of vertical corrector measures with Hall probe was identified as $L_{\text {eff }}=31,2 \mathrm{~cm}$. The difference between measured at that time and calculated now can be explained by earlier interruption of measurement as longitudinal function; the measurements stopped when the value of dipole field dropped to $10 \%$ of its maximum. Also, the poles of Sextupole was modified [2], [3], which significantly improved the sextupole field quality.


Figure 10. Integral $I_{\text {tot }}(y)=\int_{-o u t}^{\text {out }} B_{y}(x=0, y, z) d z$ as a function of vertical displacement (from Table1).

Integral $I_{\text {tot }}(y)=\int_{- \text {out }}^{\text {out }} B_{y}(y, z) d z$ can be approximated as the following (at 1 kA total current)

$$
\begin{equation*}
I_{\text {tot }}(y)=7.224-0.0121 \cdot y^{2}-0.00424 \cdot y^{4}[k G \times \mathrm{cm}] \tag{3}
\end{equation*}
$$

where $y$ measured in cm .
Dependence over other coordinate $x$ could be obtained by using equations

$$
\begin{equation*}
\frac{\partial B_{y}(x, y)}{\partial y}=-\frac{\partial B_{x}(x, y)}{\partial x}, \quad \frac{\partial B_{y}(x, y)}{\partial x}=\frac{\partial B_{x}(x, y)}{\partial y} \tag{4}
\end{equation*}
$$

only in 2D case, however these relations might be useful in 3D also.
In Table 2 represented integrals of $I_{\text {to }}(x, y)=\int_{\text {out }}^{\text {out }} B_{y}(x, y, z) d z$. The margins for transverse coordinates are defined by the clearance of space through the poles of Sextupole and Dipole.

Table 2. Integrals of $B_{x}(x, y, z), k G \times c m$, over longitudinal coordinate as functions of vertical and horizontal displacements at $1 \mathrm{kA} /$ coil current.

| $y^{x}$ | 0.0 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 7.210 | 7.209 | 7.203 | 7.179 | 7.112 | 6.994 | 6.776 |
| 0.5 | 7.209 | 7.211 | 7.211 | 7.198 | 7.150 | 7.040 | 6.839 |
| 1.0 | 7.203 | 7.215 | 7.233 | 7.249 | 7.240 | 7.177 | 7.026 |
| 1.5 | 7.179 | 7.202 | 7.255 | 7.326 | 7.387 | 7.402 | 7.333 |
| 2.0 | 7.141 | 7.155 | 7.260 | 7.307 | 7.577 | 7.714 | 7.760 |

Integral of other component- $B_{y}(x, y, z)$ is $\sim 10 \%$ of integrals of $B_{x}(x, y, z)$ at the edge of coordinate window; closer to the axis $y=0$ these integrals drop to zero.

The harmonics content in a central part calculated for the reference radius $r_{r e f}=2 \mathrm{~cm}$ in the middle of Sextupole yokeat $1 k A$-turns/coil, angle for analyses $180^{\circ}$ :

$$
\begin{equation*}
B_{x}\left(r, \vartheta,-i B_{y}(r, \vartheta)=(-i) \sum_{n=1}\left(a_{n}+i b_{n}\right)\left(\frac{r}{r_{r e f}}\right)^{n-1} e^{i n \vartheta}\right. \tag{5}
\end{equation*}
$$

|  | $a_{n}$ | $b_{n}, k G$ |
| :---: | :---: | :---: |
| 1 | 0.0 | -0.2287 |
| 3 | 0.0 | 0.000167 |
| 5 | 0.0 | 0.003338 |
| 7 | 0.0 | -0.000152 |
| 9 | 0.0 | $-7.0 \times 10^{-7}$ |
| 11 | 0.0 | $1.0 \times 10^{-7}$ |

where $\vartheta$ stands for the azimuthal angle.

One can see that the amplitude of Dipole field in this Table corresponds to the field in Table 1. In the first approximation this harmonics content can be applied to the whole integral, as the effective length weakly depends on the pass through (see Table 1).

Other type of calculations carried by MERMAID is the harmonics content for averaged value of the field between initial and final points separated in longitudinal direction ( $z$ axis in out case), i.e.

$$
\begin{equation*}
\frac{1}{L} \int_{0}^{L}\left[B_{x}(r, \vartheta, z)-i B_{y}(r, \vartheta . z)\right] d z \equiv \overline{B_{x}(r, \vartheta, z)-i B_{y}(r, \vartheta . z)}=(-i) \sum_{n=1}\left(\bar{a}_{n}+i \bar{b}_{n}\right)\left(\frac{r}{r_{r e f}}\right)^{n-1} e^{i n \vartheta} \tag{6}
\end{equation*}
$$

In the Table below the averaged harmonics represented for $L=70 \mathrm{~cm}$ ( $\pm 35 \mathrm{~cm}$ from the midplane center of Sextupole, at $1 \mathrm{kA} / \mathrm{coil}$

|  | $\bar{a}_{n}$ | $\bar{b}_{n}, k G$ |
| :---: | :---: | :---: |
| 1 | 0.0 | -0.10246 |
| 3 | 0.0 | 0.0000206 |
| 5 | 0.0 | 0.00132 |
| 7 | 0.0 | -0.00005 |
| 9 | 0.0 | 0.000000 |
| 11 | 0.0 | $2.0 \times 10^{-7}$ |

## SKEW QUADRUPOLE CORRECTOR

If the currents in the dipole coils commutated oppositely, the corrector generates skew quadrupole of moderate quality, see Fig. 9. For improvement of its quality a special pole cut was suggested [4]. However this cut affects the Sextupole harmonics content, so with sacrificing of quality of the Skew Quad, the corrector could be accepted; this is a matter of dynamic modeling.


Figure 9. The field lines in a case of Skew Quadrupole corrector arrangement.


Figure 10. The $B_{x}$ field of Skew Quadrupole corrector across the middle of Sextupole yoke in a midplane. Coordinate of 6 cm in this graph corresponds to the center of sextupole.

The integrals of vertical component of the field as functions of the vertical displacement calculated and represented in a Table 3 below.

$$
\begin{equation*}
I_{\text {left }}(y)=\left.\int_{\text {out }}^{\text {midof lens }} B_{y}(y, z) d z\right|_{\text {Quadside }} I_{\text {left }}(y)=\left.\int_{\text {out }}^{\text {midof lens }} B_{y}(y, z) d z\right|_{\text {Freeside }} L_{\text {eff }}=\frac{I_{\text {tot }}}{B_{y \max }} \equiv \frac{I_{\text {left }}+I_{\text {right }}}{B_{y \max }} \tag{7}
\end{equation*}
$$

Table 3. Skew Quad longitudinal integrals $B_{y}(x=0, y, z), k G \times c m$, as functions of vertical displacement at $1 \mathrm{kA} /$ coil current.

| $y, \mathrm{~cm}$ | Left integral, $k G \times c m$ <br> (Quad side) | Right integral, $k G \times c m$ <br> (Free side) | $I_{\text {tot }}, k G \times c m$ | $B_{\text {ymid }}, k G$ | $L_{\text {eff }}, c m$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 0 | 0 | $<0.0001$ | 0 | - |
| 0.5 | 0.284 | 0.287 | 0.571 | 0.0197 | 28.98 |
| 1.0 | 0.579 | 0.586 | 1.165 | 0.0403 | 28.91 |
| 1.5 | 0.897 | 0.909 | 1.788 | 0.0627 | 28.52 |
| 2.0 | 1.250 | 1.268 | 2.518 | 0.0878 | 28.68 |
| 2.5 | 1.648 | 1.674 | 3.322 | 0.1164 | 28.54 |
| 3.0 | 2.099 | 2.135 | 4.234 | 0.1490 | 28.42 |
| 3.5 | 2.605 | 2.654 | 5.259 | 0.1860 | 28.27 |
| 4.0 | 3.158 | 3.222 | 6.380 | 0.2267 | 28.14 |
| 4.5 | 3.721 | 3.801 | 7.522 | 0.2679 | 28.08 |
| 5.0 | 4.187 | 4.285 | 8.472 | 0.3015 | 28.10 |

The harmonics content in a central part calculated for the reference radius $r_{r e f}=2 \mathrm{~cm}$ in the middle of Sextupole yoke at $1 k A$-turns/coil:

$$
\begin{equation*}
B_{x}\left(r, \vartheta,-i B_{y}(r, \vartheta)=(-i) \sum_{n=1}\left(a_{n}+i b_{n}\right)\left(\frac{r}{r_{r e f}}\right)^{n-1} e^{i m \vartheta}\right. \tag{8}
\end{equation*}
$$

|  | $a_{n}$ | $b_{n}, k G$ |
| :---: | :---: | :---: |
| 2 | 0.0 | -0.07866 |
| 4 | 0.0 | 0.00954 |
| 6 | 0.0 | -0.000009 |
| 8 | 0.0 | -0.0000419 |
| 10 | 0.0 | $2.2 . \times 10^{-6}$ |
| 12 | 0.0 | $-1 \times 10^{-7}$ |

where $\vartheta$ stands for the azimuthal angle.
Averaged harmonics content

$$
\begin{equation*}
\frac{1}{L} \int_{0}^{L} B_{x}(r, \vartheta, z)-i B_{y}(r, \vartheta . z) d z \equiv \overline{B_{x}(r, \vartheta, z)-i B_{y}(r, \vartheta . z)}=(-i) \sum_{n=1}\left(\bar{a}_{n}+i \bar{b}_{n}\right)\left(\frac{r}{r_{r e f}}\right)^{n-1} e^{i n \vartheta} \tag{6}
\end{equation*}
$$

For the Skew Quadrupole represented in a Table below; $L=70 \mathrm{~cm}( \pm 35 \mathrm{~cm}$ from the midplane center of Sextupole, feeding current $1 \mathrm{kA} /$ coil, reference radius 2 cm , angle for analyses is $180^{\circ}$.

|  | $\bar{a}_{n}$ | $\bar{b}_{n}, k G$ |
| :---: | :---: | :---: |
| 2 | 0.0 | -0.0326 |
| 4 | 0.0 | 0.00369 |
| 6 | 0.0 | 0.0000154 |
| 8 | 0.0 | -0.0000132 |
| 10 | 0.0 | $1.2 \times 10^{-6}$ |
| 12 | 0.0 | $2.0 \times 10^{-7}$ |

## CONCLUSIONS

Presence of the Quadrupole in a close vicinity of the Sextupole arranged as a dipole or skew quadrupole corrector forces 3D calculations for accurate modeling of harmonics. Influence of the neighboring Quadrupole is mentionable, but not drastic, probably.

## REFERENCES

[1] D.Larson, L.Robers, R.Talman, "Sextupole Magnetic Measurements", CBN 78-1, Jan 10, 1978.
[2] A. Mikhailichenko," CESR Sextupole Upgrade", CON 96-05, March 9, 1996.
[3] A.Mikhailichenko, "Sextupole for CESR", CBN 98/02, Jan 29, 1998.
[4] A.Mikhailichenko, "The Poles for Skew Quadrupoles 47E/W", CON 96/13, Aug 9, 1996.


[^0]:    ${ }^{1}$ According to the specification of Dipole corrector coil [1], the total current allowable is $14 \times 270=3.78 \mathrm{kA}$.

