Submitted to Proceedings of POSIPOL 2011, Aug.28-30, IHEP, China

WHY K-FACTOR IN ILC UNDULATOR SHOULD BE SMALL

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Abstract. We are analyzing the ILC positron source for the best polarization and efficiency. We represent the arguments why K-factor ($K = eH\lambda_u/2\pi mc^2$) should be ≤ 0.45 . Lower K-factor allows reduction the total power of radiation and dominance the first harmonic in radiation. As the first harmonic can be well collimated, hence more monochromatic, the energy separation of the secondary positrons/electrons at high edge of spectrum can be done easily. As a result ~70% polarization looks feasible for ~170 m undulator at 150-500 GeV and K factor ≤ 0.4 .

OVERWIEW

In ILC positron source the undulator scheme [1] is appointed as a baseline. Undulator radiation (UR) generated by particles with energy $E \sim 150-250 \ GeV$ serves as a source of circularly polarized gammas with energy $\hbar\omega \sim 10-20 \ MeV$. So called *K*-factor ($K = eH\lambda_u/2\pi mc^2 = \beta_{\perp}\gamma$ - where β_{\perp} stands for the transverse velocity of electron normalized by the speed of light, $\gamma = E/mc^2$, λ_u is a period of magnetic field in the undulator)—is the mostly important characteristic, which defines the properties of UR. These properties include the harmonic content, i.e. its spectrum and polarization. So the momentum associated with the transverse oscillation of the particle comes to be $p_{\perp} = mc\beta_{\perp}\gamma = mcK$. The total energy radiated by a particle in the undulator with total number of periods *M* is proportional to the K^2 ; more exactly to the square of magnetic field *B* averaged over period [2]-[4]

$$\Delta \varepsilon_{tot} = \frac{2}{3} r_e^2 \overline{B^2} \gamma^2 M \lambda_u = \frac{8\pi^2}{3} e^2 M K^2 \gamma^2 / \lambda_u \,. \tag{1}$$

One can see from (1) that the total intensity of radiation is not a function of undulator period. Dependence of intensity on magnetic field squared limits the maximal energy, which any particle might have after passage the region with the field (Pomeranchuk theorem [5]).

The energy distribution of undulator radiation emitted by a single particle in an undulator with the length $L_u = M\lambda_u$ (*M* –is the number of periods), during the time duration $\Delta t = 2\pi M / \Omega$ is defined by the expression [2]-[7]

$$\frac{d\varepsilon_n}{do} = \frac{dI_n}{do} \frac{2\pi M}{\Omega} = \frac{e^2 \omega_n^3 M}{cn\Omega^2} \left[\beta_{\perp}^2 J_n^{\prime 2} \left(\frac{n\beta_{\perp} Sin\vartheta}{1 - \beta_{\parallel} Cos\vartheta} \right) + \frac{(Cos\vartheta - \beta_{\parallel})^2}{Sin^2\vartheta} J_n^2 \left(\frac{n\beta_{\perp} Sin\vartheta}{1 - \beta_{\parallel} Cos\vartheta} \right) \right]$$
(3)

where $\beta_{\perp} = v_{\perp}/c = K/\gamma$, v_{\perp} is the transverse velocity. In the dipole approximation, K < 1, $\beta_{\parallel} = \sqrt{\beta^2 - \beta_{\perp}^2} \cong \beta$ and in the ultra-relativistic case $\gamma >> 1$, mainly the first harmonic radiated, n=1.

$$\frac{d\varepsilon_1}{do} = \frac{e^2 \Omega M}{c} \cdot F(\vartheta) \cong \frac{e^2 \Omega M}{c} \cdot \frac{\beta_{\perp}^2}{8\pi (1 - \beta_{\parallel} Cos\vartheta)^3} \left[1 + \frac{(Cos\vartheta - \beta_{\parallel})^2}{(1 - \beta_{\parallel} Cos\vartheta)^2} \right]$$
(4)

where the function $F(\vartheta)$ introduced accordingly. Indeed, for total radiation at all harmonics, expression (3) should be summarized over all indices *n* [6]

$$\frac{dI}{do} = \sum_{n=1}^{\infty} \frac{dI_n}{do} = \frac{e^2 \Omega^2 \beta^2}{32c} \cdot \left[\frac{4 + 3\beta^2 \sin^2 \vartheta}{\left(1 - \beta^2 \sin^2 \vartheta\right)^{5/2}} + \frac{\cos^2 \vartheta \cdot \left(4 + \beta^2 \sin^2 \vartheta\right)}{\left(1 - \beta^2 \sin^2 \vartheta\right)^{7/2}} \right]$$
(5)

So the number of photons can be obtained just by division of intensity of radiation within some spectral region (3) by the energy of the photon within this energy. The photon energy and its polarization for a given K factor depend on the observation angle ϑ measured from the longitudinal axis to the direction towards an observer

$$\hbar\omega_n = \frac{n\hbar\Omega}{1 - \beta_{\parallel} Cos\vartheta} , \qquad (6)$$

where n=1,2,3..., numerates the harmonics of frequency $\Omega = \beta_{\parallel} c/\lambda_u$, $\overline{\beta} = \overline{v}/c \cong \beta_{\parallel}$, \overline{v} is a particle's average longitudinal velocity in the undulator. Basically this formula represents the Doppler shift of radiation while the particle oscillates with frequency Ω .

$$\frac{dN_n}{do} = \frac{d\varepsilon_n}{\hbar\omega_n do} = \frac{dI_n}{do} \frac{2\pi M}{\hbar\omega_n \Omega} = \frac{e^2}{\hbar c} \frac{\omega_n^3 M (1 - \beta_{\parallel} Cos\vartheta)}{n^2 \Omega^3} \left[\beta_{\perp}^2 J_n^{\prime 2} \left(\frac{n\beta_{\perp} Sin\vartheta}{1 - \beta_{\parallel} Cos\vartheta} \right) + \frac{(Cos\vartheta - \beta_{\parallel})^2}{Sin^2 \vartheta} J_n^2 \left(\frac{n\beta_{\perp} Sin\vartheta}{1 - \beta_{\parallel} Cos\vartheta} \right) \right]^{(7)}$$

By introduction of functions [3]

$$F_n(K,\vartheta) = F_n^+ + F_n^-, \qquad F_n^{\pm}(K,\vartheta) = \frac{1}{2} \left(J_n'(n\kappa) \pm \frac{\cos\vartheta - \beta_{\parallel}}{1 - \beta_{\parallel}} \frac{1}{\cos\vartheta} \cdot \frac{1}{\kappa} J_n(n\kappa) \right)^2, \quad (8)$$

$$F_n(K,\vartheta) = J_n^{\prime 2}(n\kappa) + \left(\frac{\cos\vartheta - \beta_{\parallel}}{\beta_{\perp}\sin\vartheta}\right)^2 J_n^2(n\kappa), \qquad (9)$$

where $\kappa = \frac{\beta_{\perp} \sin \vartheta}{1 - \beta_{\parallel} \cos \vartheta}$, intensity and polarization can be expressed as [3], [6]

$$\frac{dI_n}{do} = \frac{dI_n^+}{do} + \frac{dI_n^-}{do} \tag{10}$$

$$\frac{dI_n^{\pm}}{do} = I_{tot} \cdot \left[\frac{3}{4\pi\gamma^4} \frac{n^2 F_n^{\pm}(K,\gamma)}{(1-\beta_{\parallel} \cos\vartheta)^3} \right] \equiv I_{tot} \cdot P_n^{\pm}(K,\vartheta,\gamma)$$
(11)

$$\xi_{2n} = \frac{F_n^+ - F_n^-}{F_n} \equiv \frac{F_n^+ - F_n^-}{F_n^+ + F_n^-} = \frac{\frac{\partial I_n^+}{\partial \vartheta} - \frac{\partial I_n^-}{\partial \vartheta}}{\frac{\partial I_n^+}{\partial \vartheta} + \frac{\partial I_n^-}{\partial \vartheta}} = 2 \left(\frac{\cos \vartheta - \beta_{\parallel}}{\beta_{\perp} \sin \vartheta}\right) \times \frac{J_n'(n\kappa)J_n(n\kappa)}{F_n(K,\vartheta)}$$
(12)

In formula (11) the angular dependence of intensity is represented as multiplication of two factors: the intensity of radiation I_{tot} and some function of angles, energy and *K*-factor $P_n^{\pm}(K, \vartheta, \gamma)$. As the integral over solid angle should coincide with the total intensity

$$\int_{4\pi} \frac{dI_n^{\pm}}{do} d\mathbf{o} = I_{tot} \cdot \int_{4\pi} P_n^{\pm}(K, \vartheta, \gamma) d\mathbf{o} = I_{tot}, \qquad (13)$$

so the integral $\int_{4\pi} P_n^{\pm}(K, \vartheta, \gamma) d\vartheta = 1$, hence $P_n^{\pm}(K, \vartheta, \gamma)$ can be treated as a *probability* of radiation

of spectral component with certain helicity in direction defined by the angle ϑ . The spectral distribution one can obtain from the distribution over the angles, as the energy of quanta registered by an observer under angle ϑ is strictly connected with this angle through the formula (6). From the formula (6) it is clear also, that narrowing the angle ϑ by collimation–narrows the energy spread in the photon beam. This peculiarity of presentation the radiation as a sum of *probabilities* at different harmonics used in a computer code KONN [8], see below.

Operation with low *K* factor which we advocated for a long time reduces the content of higher harmonics as the first harmonic power reaches the 50% of all radiated power at K=0.7 only; we believe that K<0.4 is the best choice, see Fig.1.



Figure 1. The ratio of power radiated at the first harmonic to the total power as a function of *K* factor.

One peculiarity of radiation in a helical undulator is that the intensity of radiation in a straight forward direction presented by the first harmonic only; the other ones have zero intensity in this (forward) direction. So basically there is some possibility to keep the K factor increased, if some collimator in front of the target is present.



Figure2. Orbital angular momentum appears as the particle radiates from the off-centered trajectory (helix) [10].

One should remember, that the photons radiated at harmonic #2 and higher, are carrying the orbital momentum, so the creation of positron pairs is going differently; this fact was missed so far. Orbital angular momentum in radiation appears as the particle radiates from the helical trajectory; simple physical explanation of this phenomenon is given in [10] (see Fig.2). So for reduction of possible negative sequences of the fact that created electron-positron pair should carry the orbital momentum it is better to have the content of second (and higher) harmonic as low as possible.

SOME TECHNICAL DETAILS [9]

Axial magnetic field generated by a pair of thin helical strips-like conductors caring opposite current values $\pm I$, wounded on cylindrical surface (see Fig.3) can be found from the following expression [11]

$$H_{\varphi}(\rho, \varphi, z) = -\frac{I}{\pi \rho} \cdot \left(\frac{2\pi a}{\lambda_{u}}\right) \cdot \frac{\sin(\alpha)}{\alpha} \cos\left(\varphi - \varphi_{0} - \frac{2\pi z}{\lambda_{u}}\right) \times I_{1}\left(\frac{2\pi \rho}{\lambda_{u}}\right) \left[K_{0}\left(\frac{2\pi a}{\lambda_{u}}\right) + K_{2}\left(\frac{2\pi a}{\lambda_{u}}\right)\right]$$
(14)

(SI units) where *a* stands for the radius of the windings, φ_0 is the local angle between the center of the strip and the axis *x*, ρ -is transverse radial coordinate, $K_0(2\pi a/\lambda_u)$, $K_2(2\pi a/\lambda_u)$, $I_1(2\pi a/\lambda_u)$ are the Bessel functions of the second kind, 2α represents the angle under which the strip is visible from the central axis. One would like to have period of undulator as small as possible $\lambda_u \rightarrow 0$, however $K_m(x) \approx e^{-x} \sqrt{\frac{\pi}{2x}} [1 - \frac{1}{8x} + ...]$ for x >> 1.

In typical case, the diameter of windings is not more, than the period of windings, so the ratio $2\pi a/\lambda_u \equiv \pi/2$, so $K_0(\pi/2) + K_2(\pi/2) \cong 0.71$.



Figure 3. B-helical coils (colored red and blue) that can generate a helical field. The beam is running inside the vacuum chamber colored yellow.

Using expansion of Bessel functions I_1 , (see for example [13]) $I_1(x) = \frac{x}{2} + \frac{x^3}{2^2 4} + \dots$ one can obtain dependence of magnetic field on the transverse coordinate, $x = 2\pi\rho/\lambda_u \cong \rho/\lambda_u$

$$H_{\varphi} = -\frac{0.71}{2} \frac{I}{\lambda_{u}} \cdot \cos\left(\varphi - \varphi_{0} - \frac{2\pi z}{\lambda_{u}}\right) \times I_{1}\left(\frac{2\pi\rho}{\lambda_{u}}\right) / \left(\frac{2\pi\rho}{\lambda_{u}}\right) \cong$$
$$\equiv -\frac{0.71}{4} \frac{I}{\lambda_{u}} \cdot \cos\left(\varphi - \varphi_{0} - \frac{2\pi z}{\lambda_{u}}\right) \times \left[1 + \frac{1}{8}\left(\frac{\rho}{\lambda_{u}}\right)^{2} + \frac{1}{768}\left(\frac{\rho}{\lambda_{u}}\right)^{4} + \dots\right]$$
(15)

So one can see that the period of undulator is restricted from the lower side by achievable current in a windings. For the undulator with period $\lambda_u \sim 1cm$ the typical *K* value is limited by a factor of one, *K*≤1. From the other hand- lower the *K* factor -larger the aperture in the undulator can be.

POLARIZATION AND THE ENERGY SEPARATION

The most valuable feature of the conversion using the undulator radiation is the possibility of generation of polarized positrons and electrons. Polarization of created positron as function of its energy E_+ can be expressed as [14]

$$\vec{\zeta} = \xi_2 \cdot [f(E_+/E_\gamma) \cdot \vec{n}_\parallel + g(E_+/E_\gamma) \cdot \vec{n}_\perp] = \vec{\zeta}_\parallel + \vec{\zeta}_\perp, \tag{16}$$

where the functions f and g describe the longitudinal and transverse polarizations shown in Fig.4, \vec{n}_{\parallel} – is the unit vector directed along initial direction of the gamma radiation, \vec{n}_{\perp} –is the unit vector normal to it.



Figure 4: Polarization as function of the fractional energy [14].

One can see from Fig.4 that polarization of the secondary particles is the result of the product of two factors: the polarization of the photon and the function defined by the details of electronpositron pair creation. Expression (16) should be accompanied by the differential cross section of the pair creation [15]. So the polarization of the photon beam is a maximal possible value of polarization of the secondary particles. Namely this first factor one can enhance by collimation of the photon beam as the



Figure 5. A scheme for the energy separation [16]. $A_{1,2}$ –accelerator structure, Q –is the focusing quadrupole M_1 –is the bending magnet, *OMD* stands for the Optical Matching Device.

It is interesting to underline, that in case if the first harmonic dominates in radiation, the role of collimator in front of the target is less important: if the energy selection tuned so that the only

energetic particles are coming through, then automatically this procedure will select the particles with highest degree of photon polarization as these photons will generate the mostly energetic positrons. This is because the specifics of formula (12): higher polarization-higher the energy of quanta is.

CALCULATIONS WITH KONN

To simplify the process of optimization of positron conversion system we developed the interactive start-to end simulation code KONN, see [8]. This code realizes the ideas about presentation of radiation formulas (11) and (12) in terms of probability and uses the Monte-Carlo algorithm describing the processes of radiation and further creation of positron in a target. Results of some optimizations with KONN represented in Table 1. Lithium lens is used in collection optics as focusing element.

Beam energy, GeV	150	250	350	500
Length of undulator, <i>m</i>	180	200	200	200
K factor	0.45	0.44	0.35	0.27
Period of undulator, <i>cm</i>	1.0	1.0	1.0	1.0
Distance to the target, m	150	150	150	150
Radius of collimator, cm	0.049	0.03	0.02	0.02
Emittance, <i>cm</i> · <i>rad</i>	1e-9	1e-9	1e-9	1e-9
Bunch length, <i>cm</i>	0.05	0.05	0.05	0.05
Beta-function, m	400	400	400	400
Thickness of the target/X ₀	0.57	0.6	0.65	0.65
Distance to the length, cm	0.5	0.5	0.5	0.5
Radius of the length, cm	0.7	0.7	0.7	0.7
Length of the length, cm	0.5	0.5	0.5	0.5
Gradient, MG/cm	0.065	0.065	0.08	0.1
Wavelength of RF, cm	23.06	23.06	23.06	23.06
Phase shift of crest, rad	-0.29	-0.29	-0.29	-0.29
Distance to RF str., <i>cm</i>	2.0	2.0	2.0	2.0
Radius of collimator [†] , <i>cm</i>	2.0	2.0	2.0	2.0
Length of RF str., <i>cm</i>	500	500	500	500
Gradient, MeV/cm	0.1	0.1	0.1	0.1
Longitudinal field, MG	0.045	0.045	0.045	0.045
Inner rad. of irises, <i>cm</i>	3.0	3.0	3.0	3.0
Acceptance, MeV·cm	5.0	5.0	5.0	5.0
Energy filter, $E > -MeV$	54	74	92	126
Energy filter, $E < -MeV$	110	222	222	250
	1			
Efficiency, e^+/e^-	1.5	1.5	1.5	1.5
Polarization, %	69	78	78	73

Table 1. Efficiency and polarization achievable with undulator scheme (KONN).

† Collimator at the entrance of RF structure

SUMMARY

Practical value of *K* factor for the undulator with 1-*cm* period is about unity, which is enough for successful operation of conversion system.

Selection of energy with dispersion optics and the scraper allows enhancement of polarization and operation with increased *K*-factor up to $K \sim 0.4$. Lowering the *K* factor allows larger aperture in undulator.

As the radiation of electron in a back- scattered radiation from a laser, can be described in a same way as the radiation in an undulator [17] while the energy of secondary photon is much lower than the energy of electron, so the recommendation for lowering K factor automatically fulfilled here due to limitation of power achievable in a laser system.

Operation with low *K*-factor in E-166 experiment (*K*~0.17) together with selection the energy of the secondary positrons by the spectrometer delivered the polarization measured of the order $\varsigma_{\parallel} \approx 85\%$ [18].

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