CBN 10-9

ULTIMATE POLARIZATION FOR ILC

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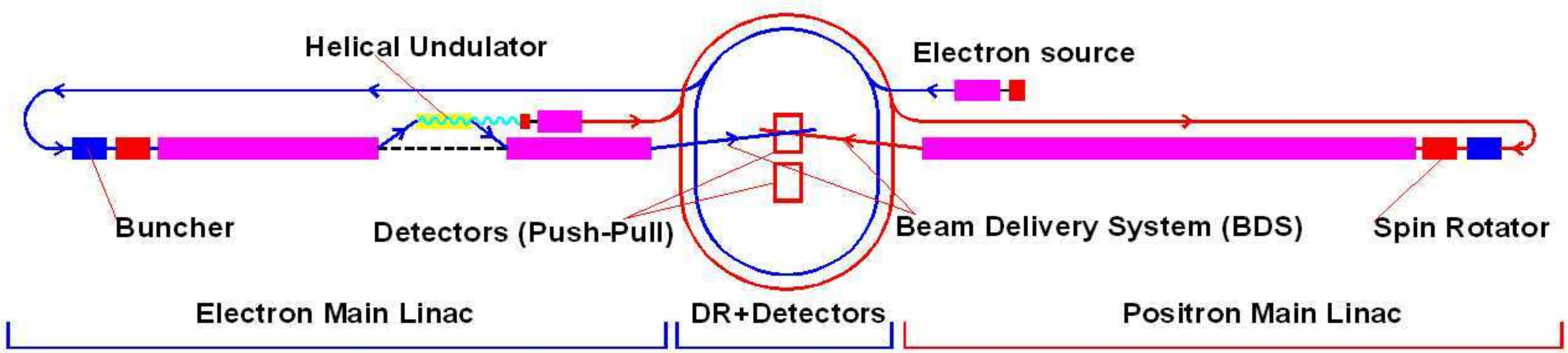
POSITRON SOURCE FOR ILC

The undulator scheme of positron production has been chosen as a baseline for ILC

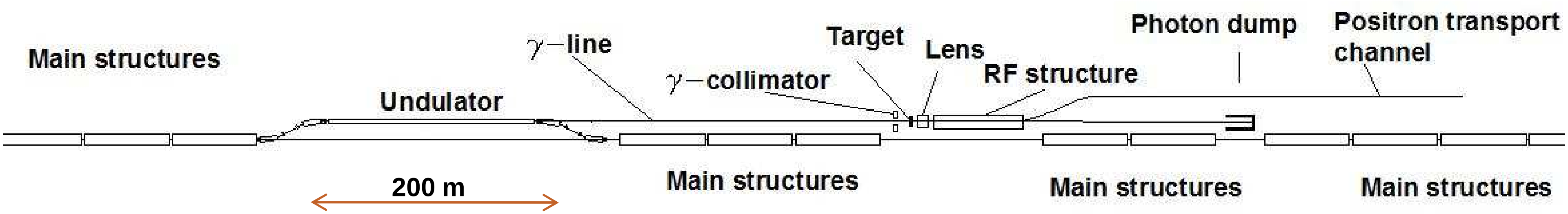


Main advantage of this scheme is that it allows **POLARIZED** positron production

In principle, positrons could be generated by positrons, so the linacs become independent

Positron source is a complex system which includes a lot of different components and each of these components could be a subject of a separate talk

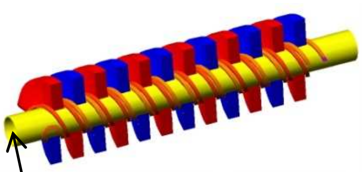
MORE DETAILED VIEW

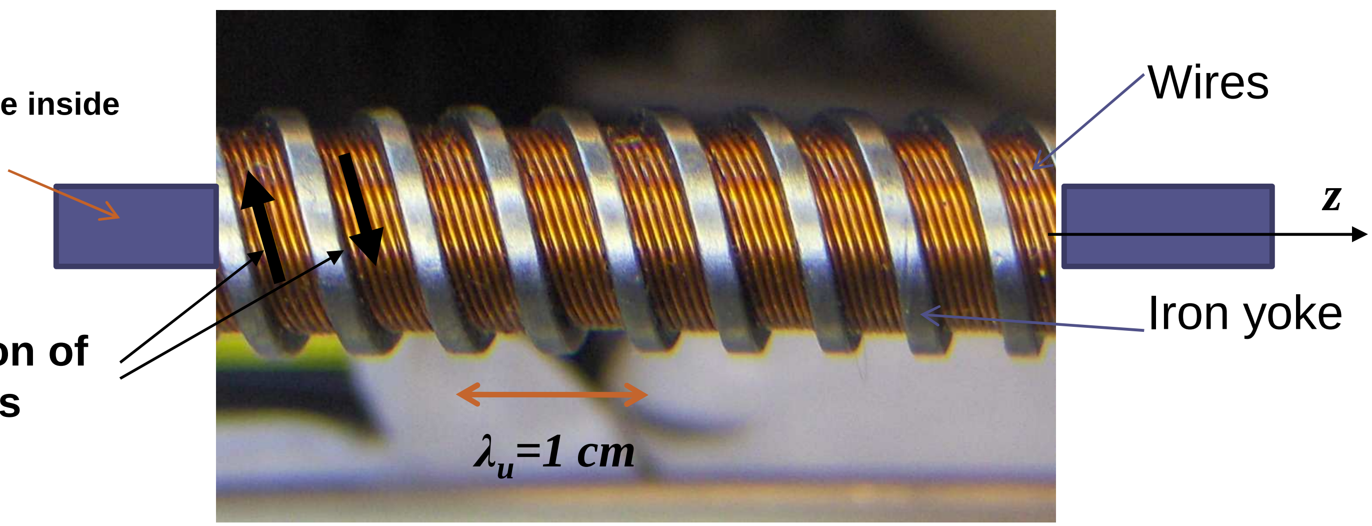


Helical undulator is a device for generation magnetic field of a type

$$\vec{H}_{\perp}(z) = \vec{e}_x H_{xm} Cos\frac{2\pi z}{\lambda_u} + \vec{e}_y H_{ym} Sin\frac{2\pi z}{\lambda_u}$$

Radius of particle's helix $a \cong eH_{\perp}\lambdabar_u^2 / mc^2\gamma$





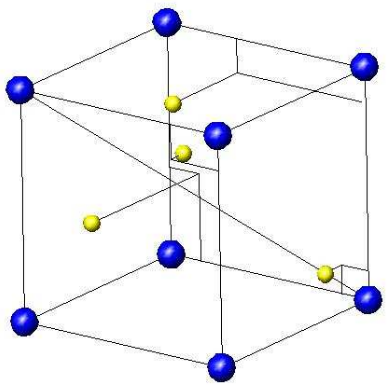
Fragment from publication of Balakin-Mikhailichenko, Budker INP 79-85, Sept. 13, 1979.

Circularly polarized photons are produced in helical fields of minimal period. Much more interesting is to obtain such fields with the help of the usual helical static fields and the electromagnetic waves. It may well be that the method of gamma production in helical crystals can be useful in future.

Scattering on the Laser radiation is the same process as the scattering on the electromagnetic wave.

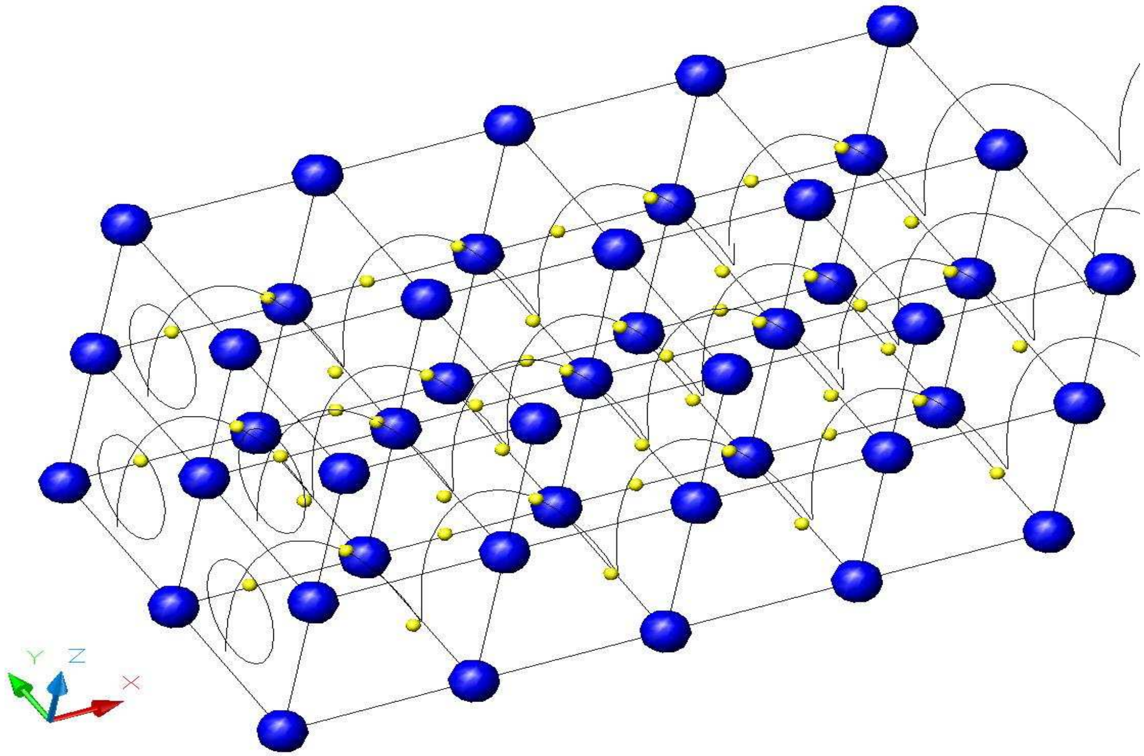
One comment about helical crystals first .

Helical (chiral) crystals



Crystal structure MnSi and FeGe

P.Bak, M.H.Jensen, J.Phys.C: Solid St.Physics, 13,(1980) L881-5



Helical structure demonstrates $CsCuCl_{3,,}$ FeGe, MgSi, Ba_2CuGe_2O7, MnS_2

Laser bunch as an undulator

The number of the quantas radiated by an electron by scattering on photons (real from the laser or virtual from the undulator) can be described as the following

$$K = eH\lambda_u / 2\pi mc^2 \cong 0.934 \cdot H[T] \cdot \lambda_u[cm] \qquad K = \beta_\perp \gamma$$

$$N_\gamma \cong 4\pi\alpha \frac{L}{\lambda_u} \frac{K^2}{1+K^2} = 4\pi \frac{e^2}{\hbar c} \frac{L}{\lambda_u} \left(\frac{eH\lambda_u}{2\pi mc^2} \right)^2 \approx \left(\frac{e^2}{mc^2} \right)^2 \frac{L\lambda_u}{2\pi\hbar c} H^2 \cong r_0^2 L \frac{H^2}{\hbar\Omega} \cong \sigma_\gamma n_\gamma L$$

Formation length in undulator $l_f \cong \lambda_u$

L- length of undulator

$$\sigma_\gamma \cong \pi r_0^2 \qquad n_\gamma \cong H^2 / \hbar\Omega \qquad \Omega = 2\pi c / \lambda_u$$

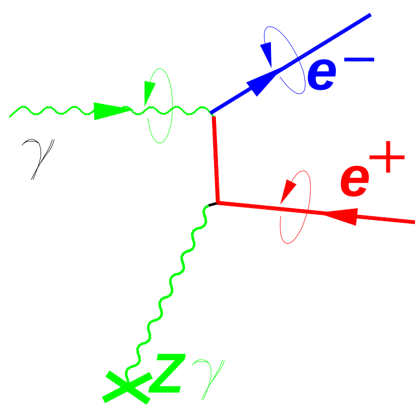
$l_\gamma \cong 1/\sigma_\gamma n_\gamma$ – Length of interaction

Written in this form it is clear that the photon back scattering (especially with 90º crossing angle) is a full equivalent of radiation in an undulator (as soon as the photon energy is much less, than the energy of particle).

$$E_{\gamma n} \cong \frac{n \cdot 2.48 \cdot (\gamma / 10^5)^2}{\lambda_u [cm](1 + K^2 + \gamma^2 \vartheta^2)} [MeV]$$

That is why undulator installed at >100 GeV line, where $E\gamma$ >10 MeV

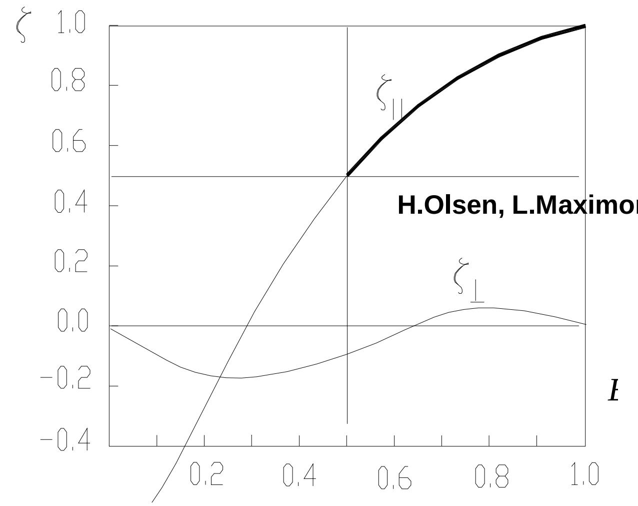
(POLARIZED) POSITRON PRODUCTION



+cross diagram

Only gamma quanta can create positron (with electron)

Longitudinal polarization as function of particle's fractional energy $E^+/(E_\gamma - 2mc^2)$



Photon polarization

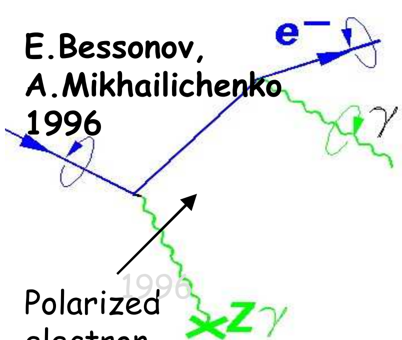
$$\vec{\varsigma} = \xi_2 \cdot \left[f(E_+, E_-) \cdot \vec{n}_\parallel + g(E_+, E_-) \cdot \vec{n}_\perp \right] = \vec{\varsigma}_\parallel + \vec{\varsigma}_\perp$$

Polarization is a result of selection positrons by theirs energy

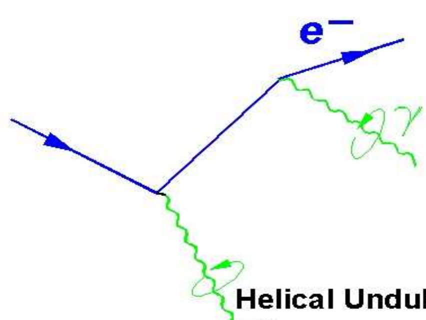
The ways to create circularly polarized photons in practical amounts

V.Balakin, A. Mikhailichenko, 1979

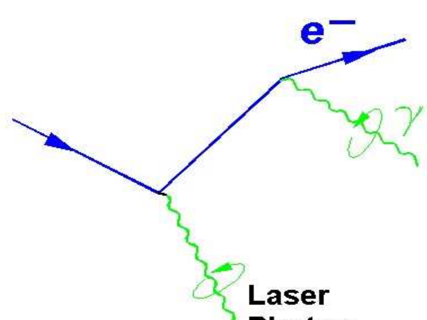
E.Bessonov, A.Mikhailichenko 1996



Polarized electron



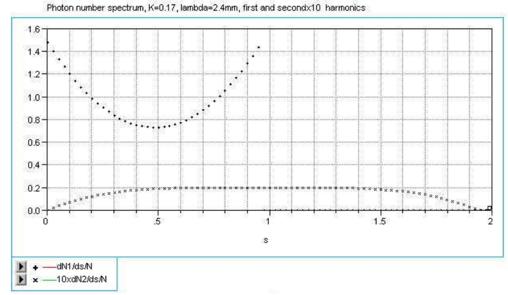
Helical Undulator, EM wave, Crystal with helical dislocations



Laser Photon

E.Bessonov 1992

Analytical calculations are possible



Spectral density of radiation

$$\frac{dN_\gamma}{dE_\gamma} = \sum_n \frac{dN_{\gamma n}}{dE_\gamma} = \frac{\alpha K^2 L}{2\gamma^2} \sum_{n=1}^{\infty} F_n(K, s)$$

where $s = E_{\gamma n} / E_{\gamma n\,\max}$ $E_{\gamma\max}$ **is the energy of photon radiated straightforward**

$$F_n(K,s) = J_n'^2(n\kappa) + \frac{1+K^2}{4K^2}\frac{(2s-1)^2}{s(1-s)} J_n^2(n\kappa) \qquad \kappa = 2K\sqrt{s(1-s)/(1+K^2)}$$

The number of positrons generated by a single photon in the target becomes

$$\frac{dN_+}{dE_+ d\tau} \cong 0.4 \frac{\alpha K^2 L}{\gamma^2 \hbar c} \frac{7}{9} (1 - E / E_{\gamma 1})(1 - e^{-7\tau/9})$$

For E_0=150 *GeV*, L=150 *m*, K^2=0.1, τ=0.5 (rad units)

$$\frac{1}{N_{tot}} \frac{dN_+}{dE_+} \cong 0.2\,[1/MeV]$$

Analytical formula taking into account finite length of undulator and finite diameter of target

E.Bessonov, A.Mikhailichenko, 1992

$$\Delta N_{+1} \cong 2 \cdot 10^{-2} \chi^2 \frac{L}{\lambda u} \delta \frac{K^2}{1+K^2} \frac{z_f}{z_i} \eta$$

For χ = ½ , L=200m , λ_u=1cm δ=0.5, K=0.35, η=0.3

$$\Delta N_{+1} \cong 3$$

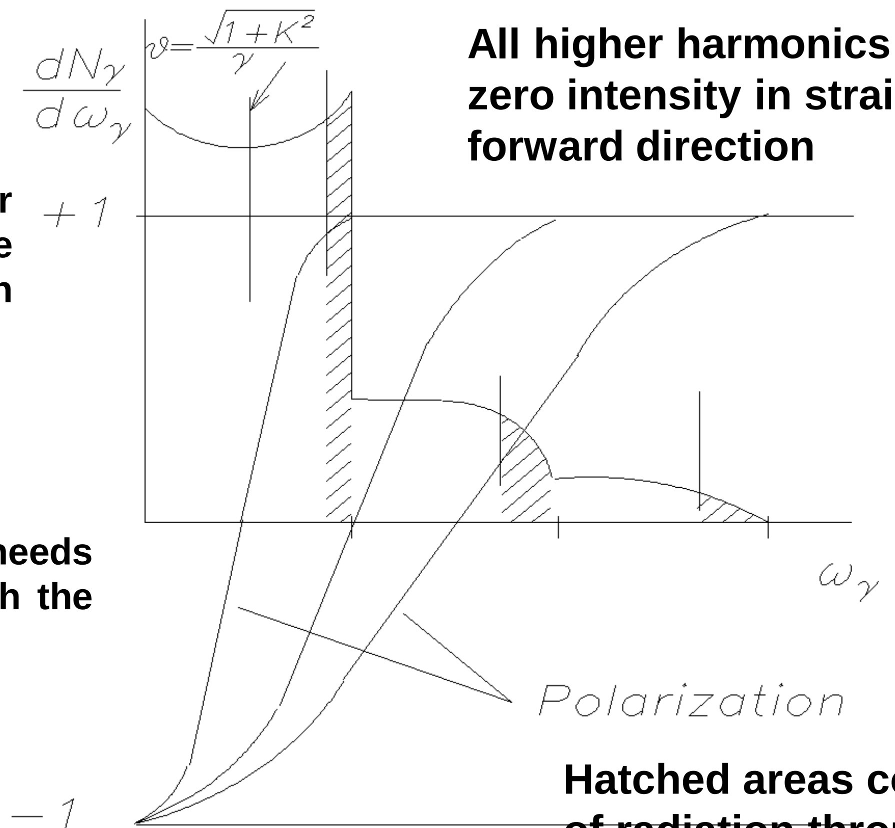
$Z_{f,I}$ - are the coordinates of undulator end and beginning calculated from the target position;

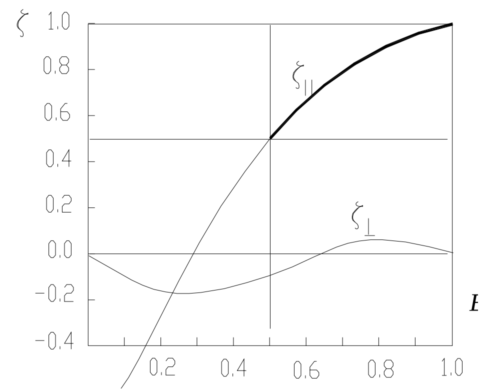
χ is a fraction of what is the target radius in respect to the size of the gamma spot at the target distance

Spectral distribution and polarization schematics

Angle to observer (radiation) and the energy of the photon are not independent

All higher harmonics have zero intensity in straight forward direction





This curve applied to each harmonics

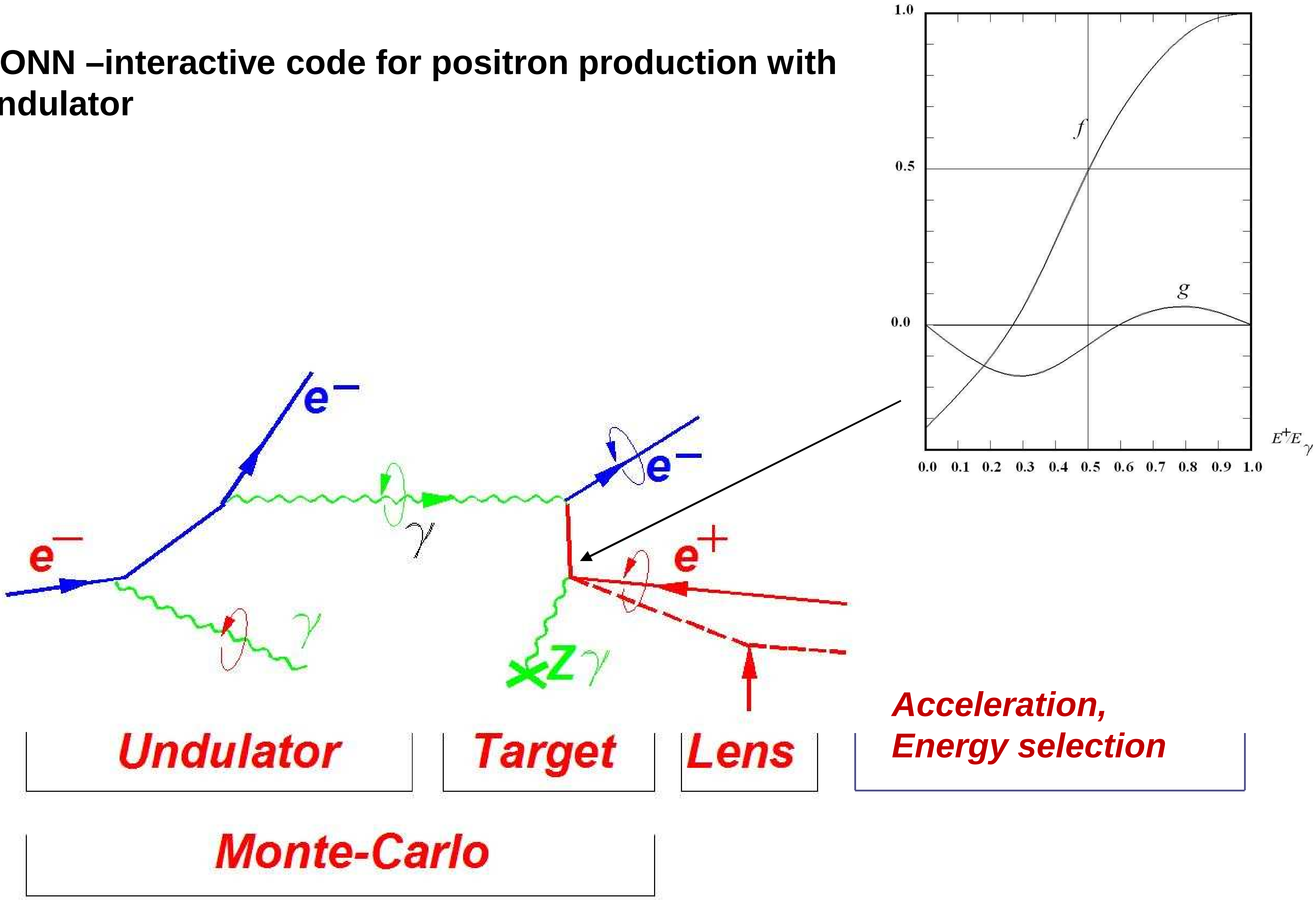
Polarization curve needs to be convolved with the photon density

Hatched areas correspond to a passage of radiation through the collimator

Collimation of photons helps to enhance integrated photon polarization as each harmonics carries the photon polarization as a factor

Detailed calculation should be carried

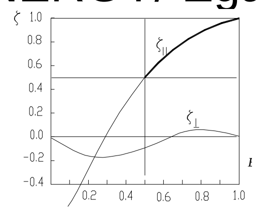
KONN –interactive code for positron production with undulator



```
!   POLARIZATION CURVE APPROXIMATION
!
      EP4=EP-0.4
      EP6=EP-0.6
      PP=0.305+2.15*EP4
      IF(EP.LT.0.4)PP=PP-0.05*EP4-2.5*EP4**3
      IF(EP.GT.0.6)PP=PP-0.55*EP6-2.65*EP6**2+0.7*EP6**3    ! PP=PP-0.55*EP6-2.6*EP6**2
      IF(PP.GT.1.)PP=1.
!
```

EP=POSITRON ENERGY/ Egamma-2mc^2

Sentinel



Depolarization occurs due to spin flip in act of radiation of quanta having energy $0 < \hbar\omega_\gamma \le E_1$ where E_1 stands for initial energy of positron. Depolarization after one single act

$$D = 1 - \left| \frac{d\sigma_{\gamma e}(\zeta_1, \zeta_1) - d\sigma_{\gamma e}(\zeta_1, -\zeta_1)}{d\sigma_{\gamma e}} \right|$$

Where $d\sigma_{\gamma e}(\zeta_1, \zeta_1)$ stands for bremstrahlung cross section without spin flip, $d\sigma_{\gamma e}(\zeta_1, -\zeta_1)$ –the cross section with spin flip and $d\sigma_{\gamma e}$ is total cross section.

$$D = \frac{\hbar^2\omega_\gamma^2 \cdot [1 - \frac{1}{3}\zeta_{1\parallel}^2\ ]}{E_1^2 + E_2^2 - \frac{2}{3}E_1 E_2}$$

Energy after radiation

$$L_{dep} \cong \frac{1}{n\int D(\vec{p}_1, \zeta_1)d\sigma} \longrightarrow L_{dep} \cong \frac{2X_0}{1 - \frac{1}{3}\zeta_\parallel^2} \cong 3X_0$$

Rad. length

Depolarization ~5%

PROGRAM KONN

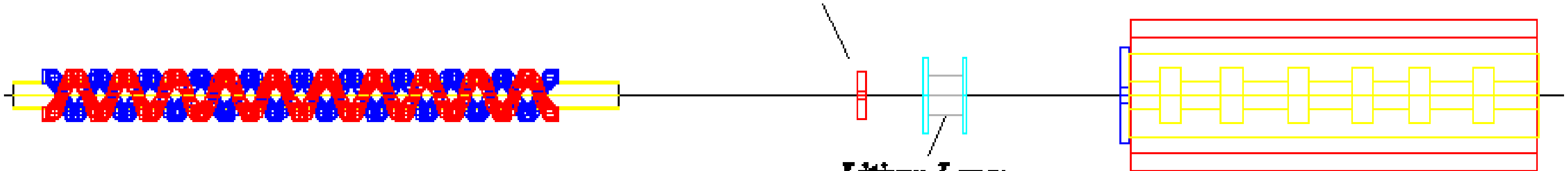
T.A.Vsevolozhskaya, A.A.Mikhailichenko

Monte-Carlo simulation of positron conversion

Energy of the beam;
Length of undulator;
Undulator period M=L/λ_u;
K-factor;
Emittance;
Beta-function;
Number of harmonics (four);
Number of positrons to be generated;

Target:
Distance to the undulator
Thickness;
Diameter of target;
Material;
Diameter of hole at center;
Step of calculation

Acceleration:
Distance to the lens;
Length of structure;
Gradient;
Diameter of collimator at the entrance;
Diameter of irises;
External solenoidal field;
Further phase volume captured;



Litium Lens:
Distance to the target;
Length;
Diameter;
Thicness of flanges;
Material of flanges;
Gradient;
Step of calculations;

CALCULATES at every stage:
Efficiency in given phase volume;
Polarization in given phase volume;
Beam dimensions;
Phase-space distributions;
Beam lengthening;
Energy spread within phase space;

Interactive code, now is ~3000 lines

Code has ~3000 rows;

Possibility for the file exchange with graphical and statistical Codes (JMP);

Possibility for the file exchange with PARMELA;

Few seconds for any new variant

"C:\MSDEV\Projects\POSITRON CONVERSION\Debug\POSITRON CONVERSION.exe"

Example

```
     CONVERSION        - C
     FOCUSING          - F
     ACCELERATION      - A

     WHAT TO DO?       -


 D0 =    .300   AL =    .400   DW0 =    .100   GG =    .070


 *** PARAMETERS OF ACCELERATION ***

DISTANCE TO RF STRUCTURE cm =  2.0000  :=
RADIUS OF DIAPHRAGM      cm =  3.0000  :=
LENGTH OF RF STRUCTURE  cm  =100.0000  :=
GRADIENT        MeV/cm      =   .5000  :=
LONGITUDINAL FIELD MGs      =   .0400  :=
INNER RADIUS OF DIPHRAGM cm =  3.0000  :=
FURTHER ACEPTANCE   MeVxcm  = 10.0000  :=

POSITRONS PASSED= 4051  POSITRONS ACCEPTED = 4011
 WW =     2.065  WWP =      .958
 F0 =      .553  BETA =    .384 DE/DT =    -1.19    EFF =    2.065

     PV0          AL0         ALMB        K        EPS         BT         RTG      GG

 150000.0     17500.0      1.00      .350     .000001     40000.0      .50     .070

 RMS =     .915   AMS =     .040  DEM = 136.344   EM =  58.225   D7 =18000.00
 PTM =    2.383   PZM =   58.176  DPZ =    5.001  PRM =   -.017  PVG =  19.071
  TM =  100.685   DTM =     .620   WW =    2.065   WP =    .464   N0 = 2400
  RF = 3.00      AL/Xo=  .40       H0 =   .040   EPSF = 10.00 MeVxcm

                     EFF(EX,CT)

  .0065    .0141    .0286    .0379    .0354    .1444

  .0602    .1703    .1959    .1583    .1233    .1462

  .0715    .1733    .1486    .1049    .0557    .0126

  .0248    .0843    .0700    .0255    .0106    .0007

  .0158    .0315    .0211    .0106    .0032    .0006

                     EFP(EX,CT)

  .0372   -.0222    .0644    .0730   -.0607    .0555

  .4200    .4642    .4093    .4150    .3323    .2957

  .7056    .6618    .6318    .6424    .6403    .5375

  .5951    .6645    .6523    .5436    .5939    .3507

  .6141    .6423    .6217    .6786    .6096    .7789

          EFF  = 1.610     EFP=   47.420 %
```

Period

K-factor

Efficiency and polarization

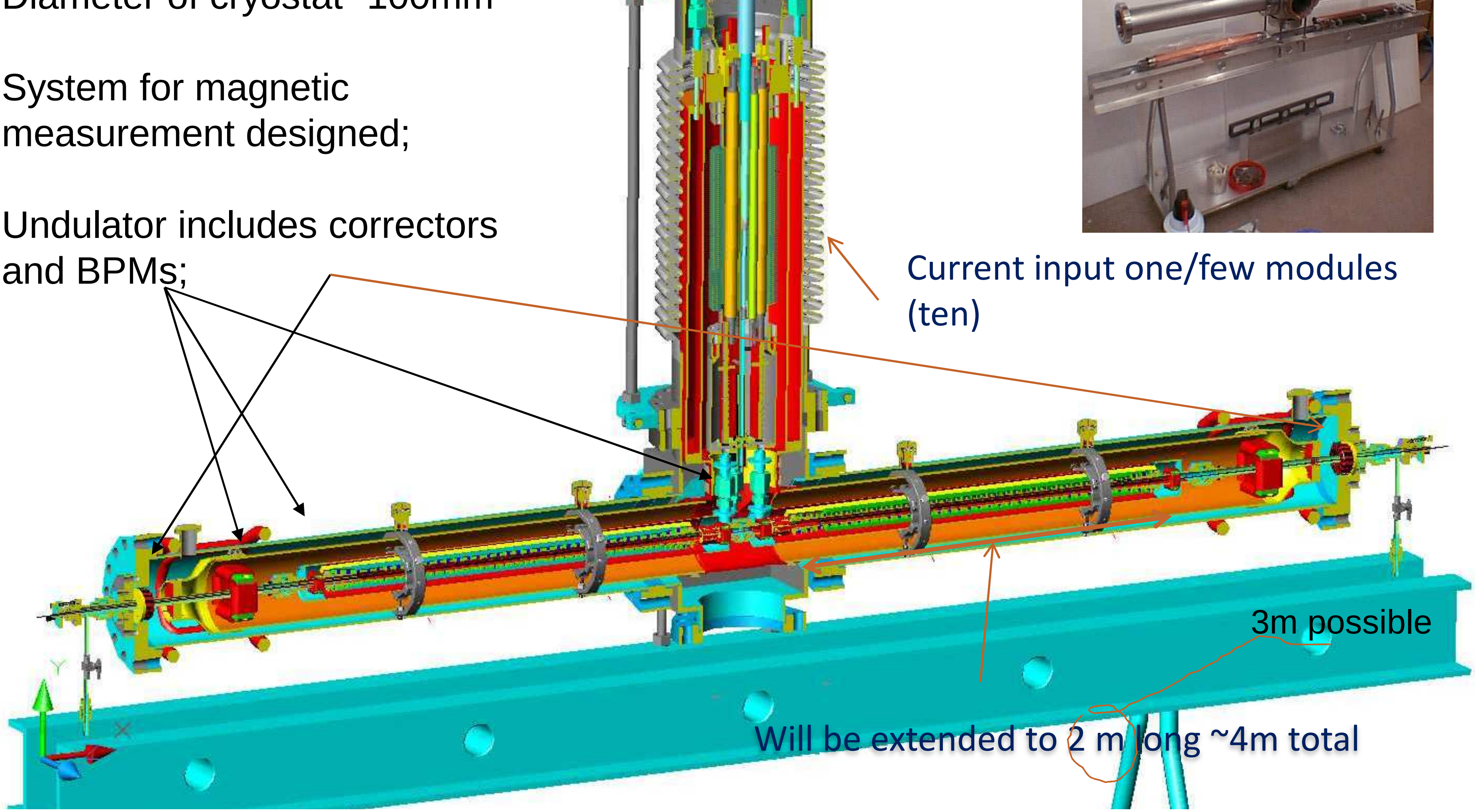
Before representing final results of calculation let us demonstrate how different elements of conversion system look like

UNDULATOR (Cornell design)

Complete design done;
Diameter of cryostat -100mm

System for magnetic measurement designed;

Undulator includes correctors and BPMs;

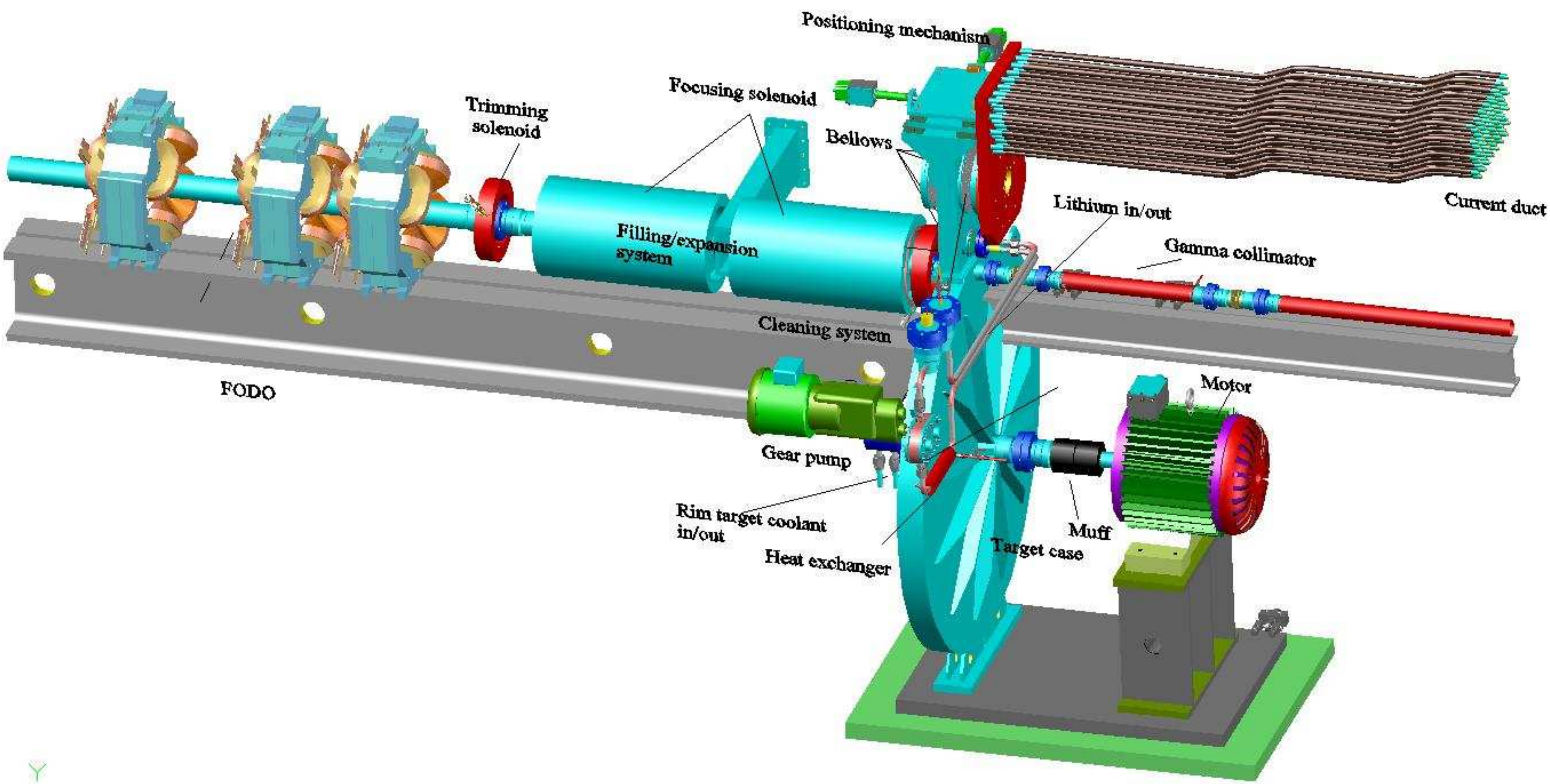


Current input one/few modules (ten)

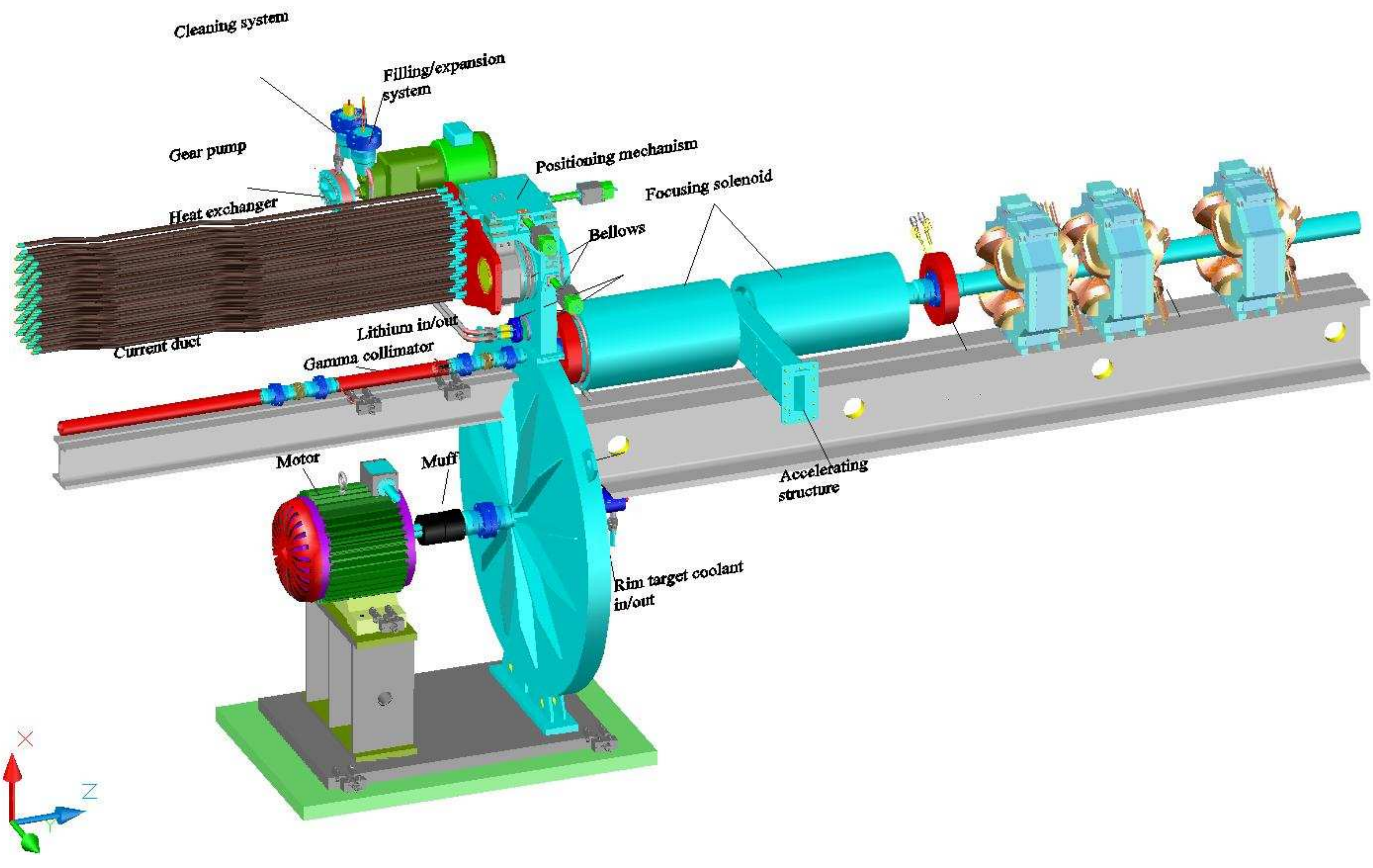
3m possible

Will be extended to 2 m long ~4m total

TARGET STATION FOR ILC



TARGET STATION



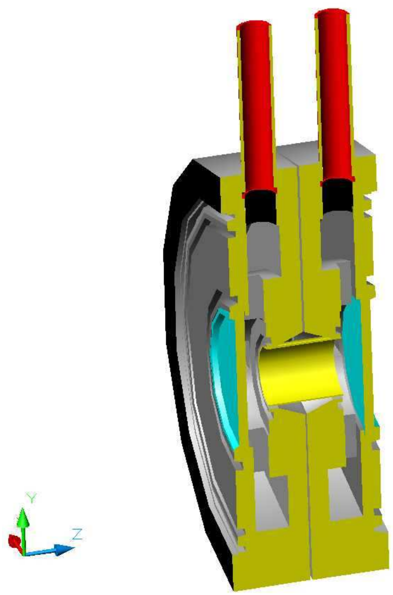
LITHIUM LENS

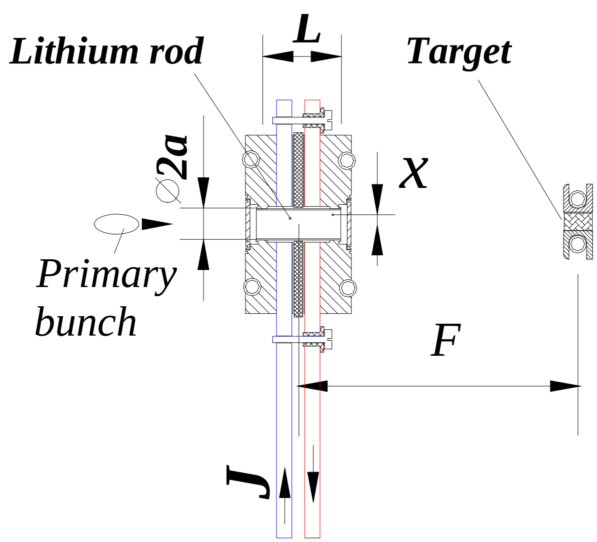
If steady current *I* runs through the round conductor having radius *a*, its azimuthal magnetic field inside the rod could be described as

$$H_\vartheta(r) = \frac{0.4\pi Ir}{2\pi a^2}$$

where magnetic field is measured in *Gs*, *a* –in *cm*, *I* –in Amperes. Current density comes to $j_s = I/\pi a^2$ The particle, passed through the rod, will get the transverse kick

$$\alpha \cong \frac{H(x)\cdot L}{(HR)} \cong \frac{0.2I\,Lx}{a^2\cdot(HR)}$$





This picture drawn for the focusing of electron beam to the target

So the focal distance could be defined as the following $F \cong \dfrac{a^2\cdot(HR)}{0.2IL}$

Li lens resume

Utilization of Lithium lens allows Tungsten survival under condition required by ILC with N_e~$2x10^{10}$ with moderate K~0.3-0.4 and do not require big-size spinning rim (or disc). Thin W target allows better functioning of collection optics (less depth of focusing).

Lithium lens (and x-lens) is well developed technique.

Usage of Li lens allows drastic increase in accumulation rate, low K-factor.

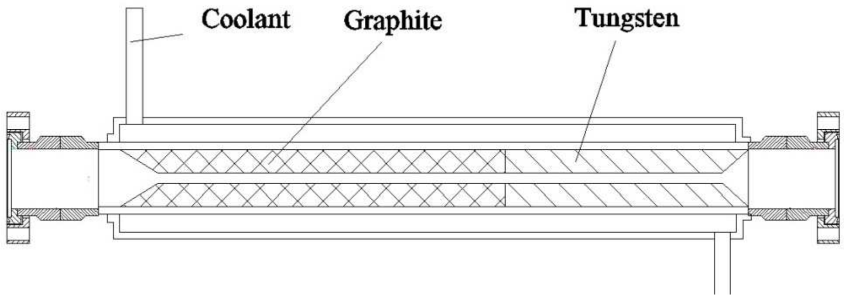
Field is strictly limited by the surface of the lens from the target side.

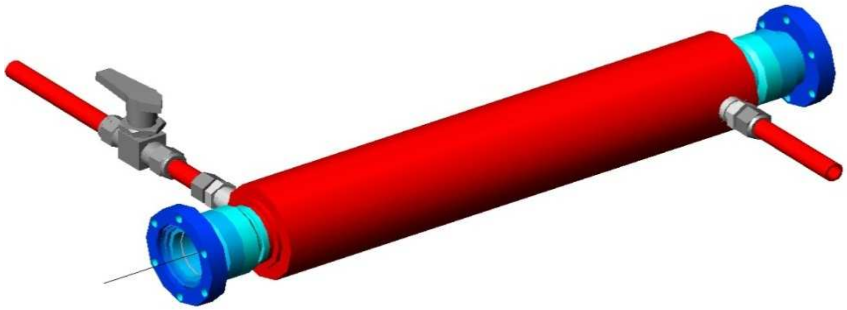
Liquid targets such as Pb/Bi or Hg allow further increase of positron yield.

Plan is to repeat optimization for cone-shape Lithium rod.

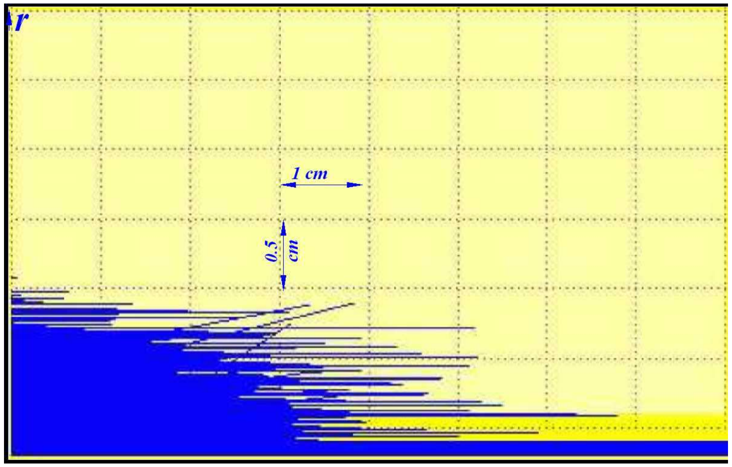
COLLIMATOR FOR GAMMAS

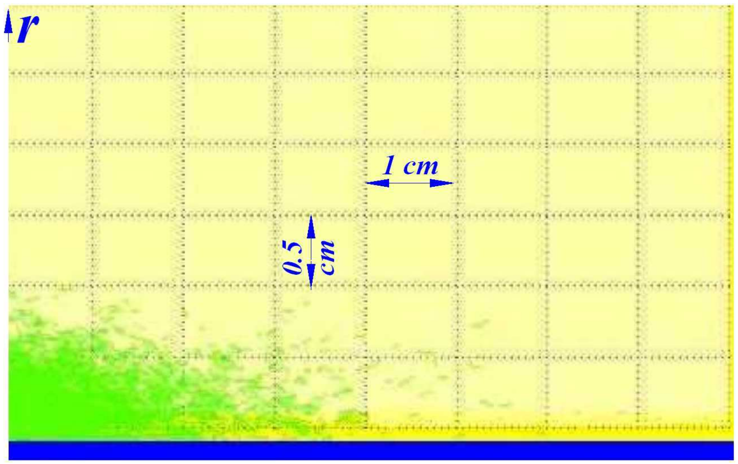
Pyrolytic Graphite (PG) is used here. The purpose of it is to increase the beam diameter, before entering to the W part. Vacuum outgassing is negligible for this material. Heat conductivity ~300 W/m-oK is comparable with meals. *Beryllium* is also possible here, depending on task.





Transverse dimensions defined by Moliere radius





Gamma-beam. $\sigma_\gamma =$ 0.5cm, diameter of the hole (blue strip at the bottom) *d*=2 mm. Energy of gamma-beam coming from the left is 20 *MeV*.

Positron component of cascade

PERTURBATION OF POLARIZATION

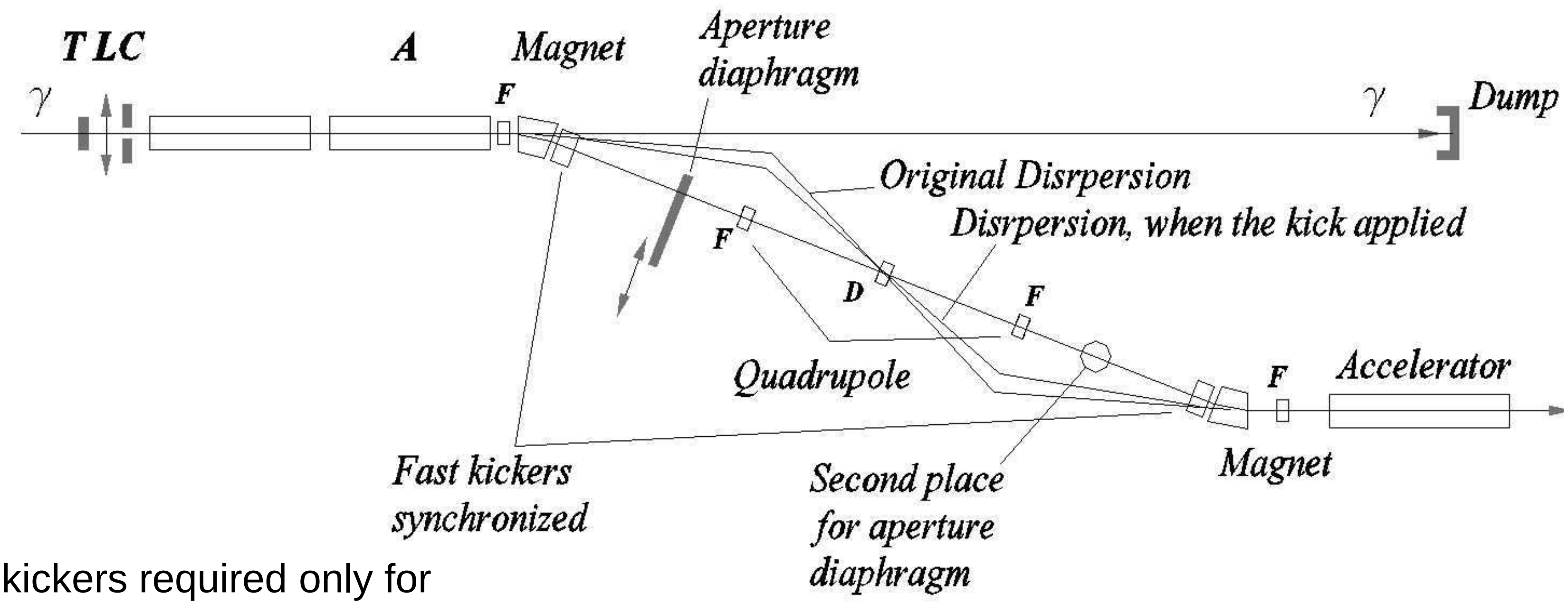
Perturbation due to multiple scattering	**- absent**
Spin flip in a target	**~5% (taken into account by KONN)**
Spin flip in an undulator	**- hardly mentionable effect**
Depolarization at IP	**<5%, for typical parameters <1%**
Cinematic depolarization in undulator	**-absent**

CONCLUSIONS ABOUT POLARIZATION

Perturbation of spin is within <5% total (from creation).

Polarization can be increased by increasing the length of undulator, making the target thinner (two targets) and beams more flat at IP.

SCHEME FOR PARTICLES SELECTION BY ENERGY

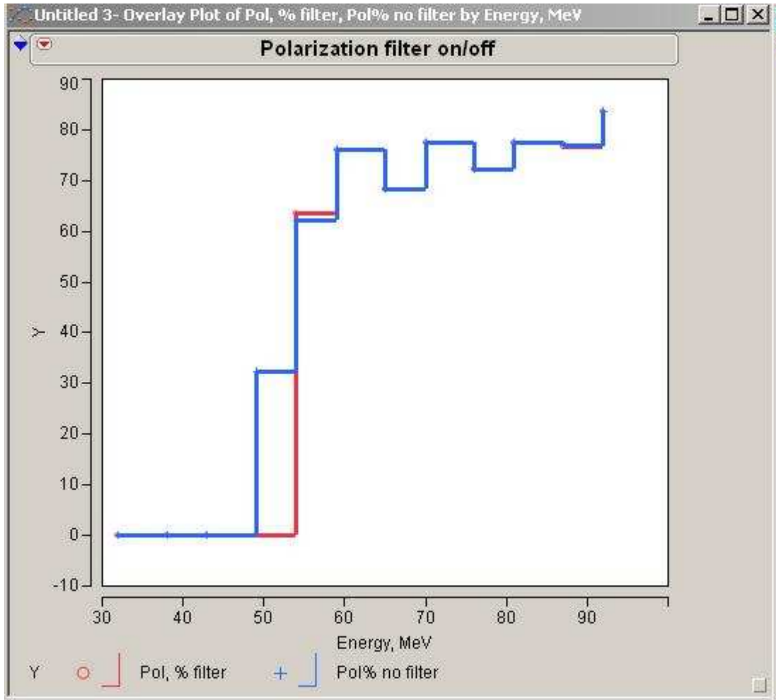


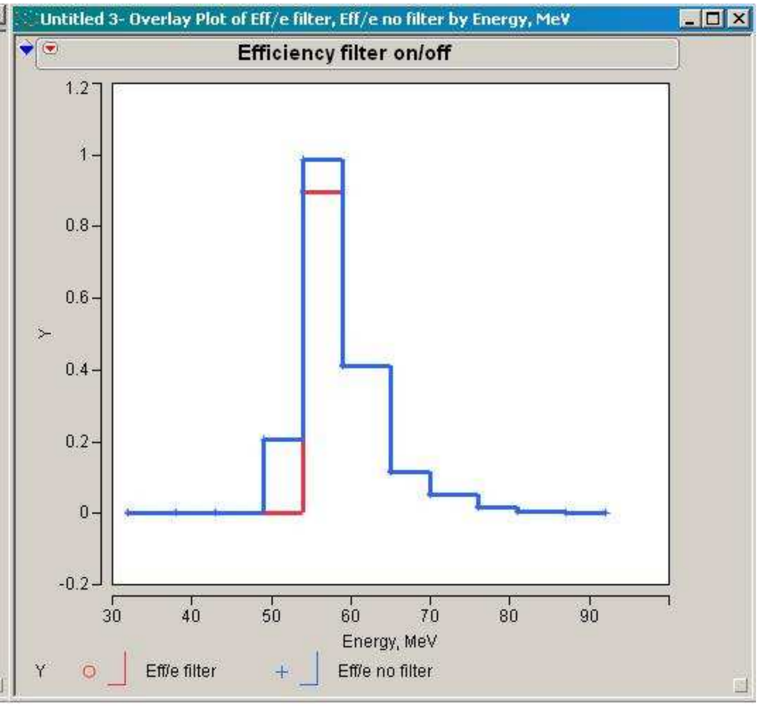
Fast kickers required only for
independent positron wing operation

Achromatic bend with aperture diaphragm. *T*–is a target, *L*–is a short focusing lens, *C*–stands for collimator. *F* and *D* stand for focusing and defocusing lenses respectively. *A* stands for RF accelerator structure.

A.Mikhailichenko, " Independent Operation of Electron/Positron Wings of ILC", EPAC 2006.MOPLS106.

ONE EXAMPLE





Average polarization
Filter on: 68.15%
Filter off: 62.78%

Efficiency
Filter on: 1.49
Filter off: 1.796

Resulting efficiency and polarization calculated with KONN

*readius of target is an equivalent of the radius of collimator

Beam energy, *GeV*	150	250	350	500
Length of undulator, *m*	170	200	200	200
K factor	0.44	0.44	0.35	0.28
Period of undulator, *cm*	1.0	1.0	1.0	1.0
Distance to the target, *m*	150	150	150	150
Radius of target*, *cm*	0.049	0.03	0.02	0.02
Emittance, *cm·rad*	1e-9	1e-9	1e-9	1e-9
Bunch length, *cm*	0.05	0.05	0.05	0.05
Beta-function, *m*	400	400	400	400
Length of target/X_0	0.57	0.6	0.65	0.65
Distance to the length, *cm*	0.5	0.5	0.5	0.5
Radius of the length, *cm*	0.7	0.7	0.7	0.7
Length of the length, *cm*	0.5	0.5	0.5	0.5
Gradient, *MG/cm*	0.065	0.065	0.08	0.1
Wavelength of RF, *cm*	23.06	23.06	23.06	23.06
Phase shift of crest, *rad*	-0.29	-0.29	-0.29	-0.29
Distance to RF str., *cm*	2.0	2.0	2.0	2.0
Radius of collimator, *cm*	2.0	2.0	2.0	2.0
Length of RF str., *cm*	500	500	500	500
Gradient, *MeV/cm*	0.1	0.1	0.1	0.1
Longitudinal field, *MG*	0.045	0.045	0.045	0.045
Inner rad. of irises, *cm*	3.0	3.0	3.0	3.0
Acceptance, *MeV·cm*	5.0	5.0	5.0	5.0
Energy filter, E > -*MeV*	54	74	92	114
Energy filter, E< -*MeV*	110	222	222	222
Efficiency, e^+/e^-	1.5	1.8	1.5	1.5
Polarization, %	70	80	75	70

One another way to increase the polarization associated with installation of a second target.

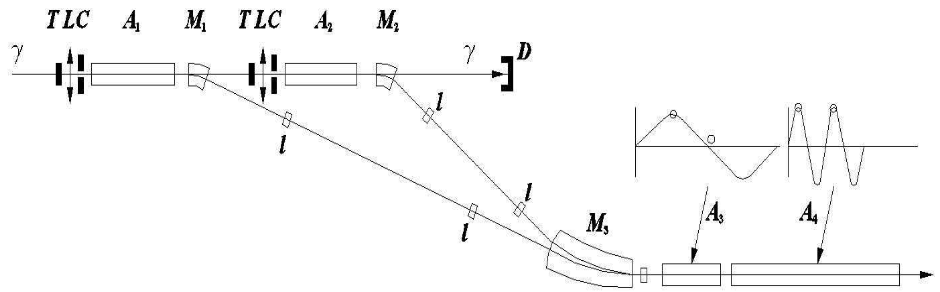
This is possible as the gamma beam loses its intensity after the first target ~13% only

Collection of positrons in the longitudinal phase space is possible with combining scheme

COMBINING SCHEME

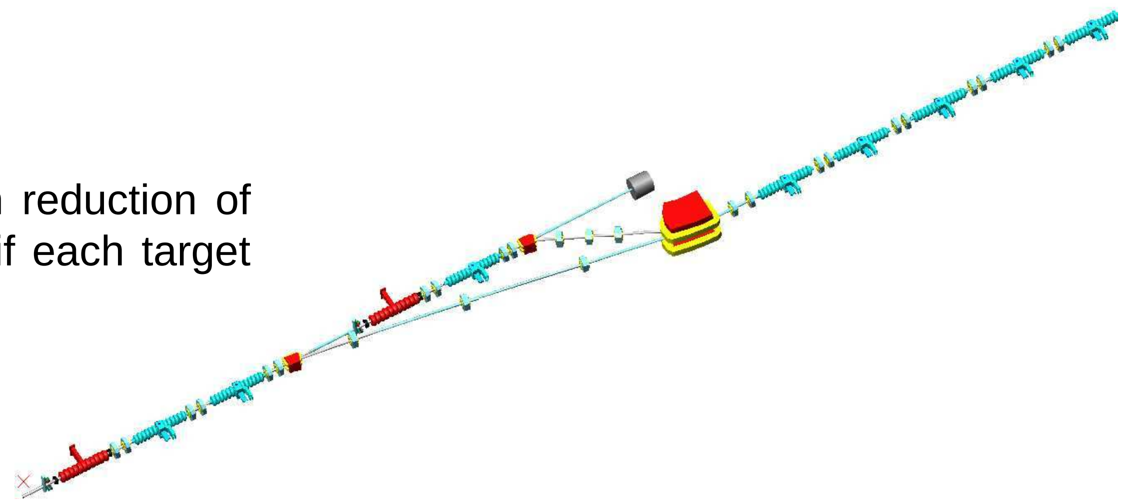
Combining in longitudinal phase space could be arranged easily in the same RF separatrix in damping ring.

Additional feed back system will be required for fast dump of coherent motion.



Energy provided by acceleration structures *A*1 and *A*2 are slightly different, *A*1>*A*2.

This combining can help in reduction of power deposition in target if each target made thinner, than optimal.



Combining scheme doubles the positron yield and cuts in half the length of undulator → increase of polarization

One more way to reconsider maximal polarization and luminosity achievable at IP

It is associated with the spin-spin interaction of particles in the same bunch while the bunch is moving towards IP

MINIMAL EMITTANCE

Fundamental restriction to the minimal emittance achievable in the electron/positron beam is

$$(\gamma\varepsilon_x)(\gamma\varepsilon_y)(\gamma\varepsilon_s) = (\gamma\varepsilon_x)(\gamma\varepsilon_y)(\gamma l_b(\Delta p / p_0)) \geq \tfrac{1}{2}(2\pi\lambda\!\!\!^{-}_C)^3 N$$

A.A. Mikhailichenko, *On the physical limitations to the Lowest Emittance (Toward Colliding Electron-Positron Crystalline Beams)*, 7th–Advanced Accelerator Concepts Workshop, 12-18 October 1996, Lake Tahoe, CA, AIP 398 Proceedings, p.294. See also CLNS 96/1436, Cornell, 1996, and in *To the Quantum Limitations in Beam Physics*, CLNS 99/1608**,** PAC99, New York, March 29- April 2 1999, Proceedings, p.2814.

This formula can be obtained from counting the number of states in the phase space of a Fermi gas :

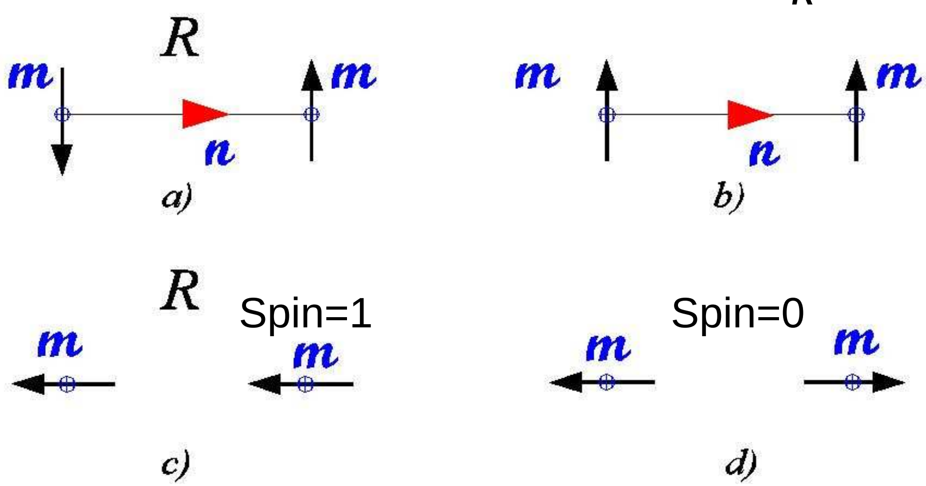
$$dn \cong 2\frac{dp_x dp_y dp_s \cdot V}{(2\pi\hbar)^3} \quad \longrightarrow \quad N = \int dn \cong 2\frac{p_x p_y \Delta p_\parallel \; S_\perp l_b \gamma}{(2\pi\hbar)^3} \cong 2\frac{\gamma\varepsilon_x \gamma\varepsilon_y \gamma l_b(\Delta p / p_0)}{(2\pi\lambda\!\!\!^{-}_C)^3} = 2\frac{\gamma\varepsilon_x \gamma\varepsilon_y \gamma\varepsilon_z}{(2\pi\lambda\!\!\!^{-}_C)^3}$$

The problem is that in the fully degenerated state polarization of beam is zero

SUPERCONDENSATION of ELECTRON GAS

Magnetic dipole $\vec{m}$ defines magnetic field around as

$$\vec{H} = \frac{3\vec{n}\cdot(\vec{m}\cdot\vec{n}) - \vec{m}}{R^3}$$



In case *a)* and *c)* there is attraction

by magnetic force.

In case *b)* and *d)* there is additional repulsion

Balancing attraction by magnetic force and repulsion by the same sign of charge, one can obtain

$$\frac{3(\vec{m}\cdot\vec{n})^2 - \vec{m}^2}{4R^4} \cong \frac{e^2}{R^2} \longrightarrow R \cong \frac{(2)\hbar}{4mc} = \frac{(2)}{4}\cdot \bar{\lambda}_C$$

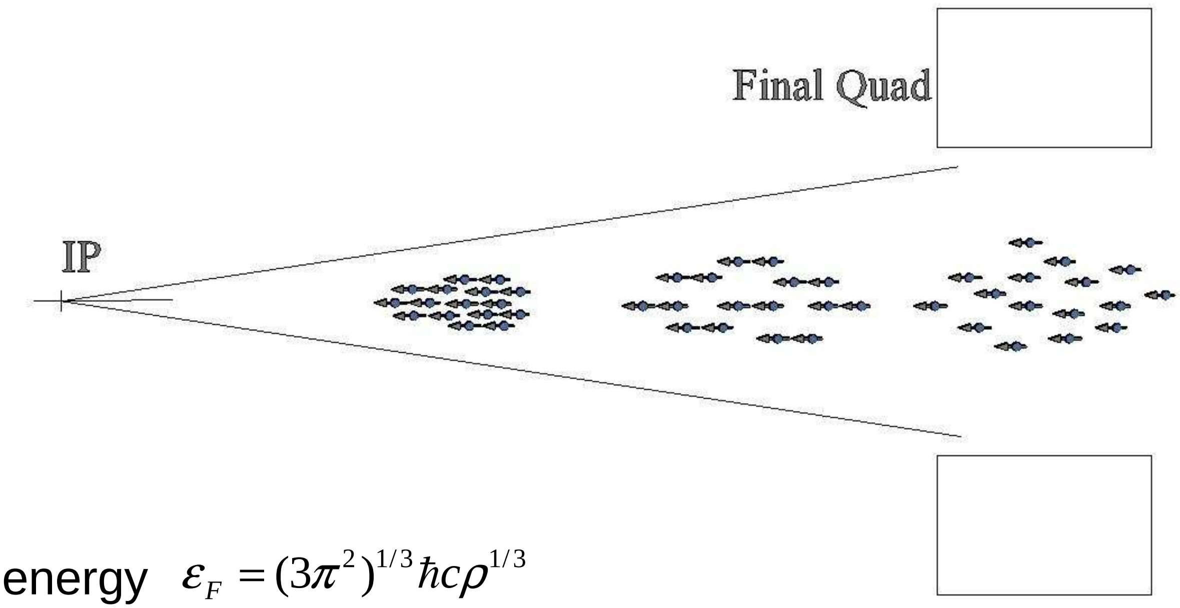
Energy required bringing two electrons to the distance of Compton wavelength is

$$\frac{e^2}{\bar{\lambda}_C} \cong \frac{e^2}{r_0}\cdot\alpha \cong \alpha\cdot mc^2$$

i.e. pretty small compared with energy of transverse motion at IP especially

SUPERCONDENSATION at IP – A WAY TO SUPER LUMINOSITY

While beam in running to IP its density increases



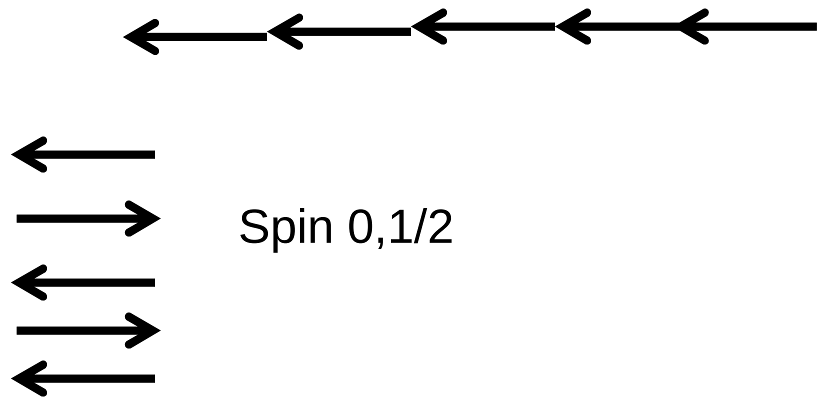
Spin=0
or spin=1
Is possible

Fermi energy $\varepsilon_F = (3\pi^2)^{1/3}\hbar c\rho^{1/3}$

The condition for degeneration $k_B T \leq \left(3\pi^2\right)^{1/3}\hbar c\rho^{1/3}$

Fermi gas becomes more degenerative while its density increased.
These conditions realizing better and better while beam traveling to IP.
Taking into account interaction through magnetic momentum, it will possible to condense beam below Fermi-limit (couple of fermions behaves as a boson).

What is the limit for the number of electrons in a cluster



The topic of super-condensation requires theoretical investigations

E-166 experiment at SLAC

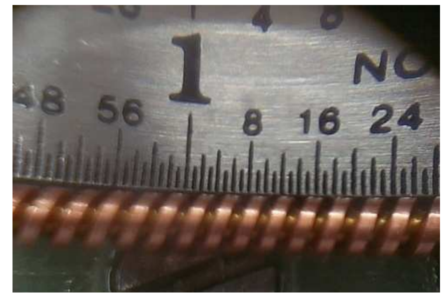
First suggested in 1992

Experimental test of polarized positron production

With gammas generated by high energy beam in undulator

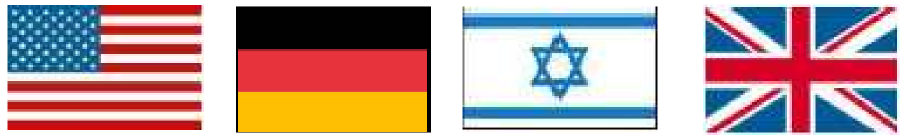
Beam chamber
0.8 mm in dia

Stretched wire to check
the straightness

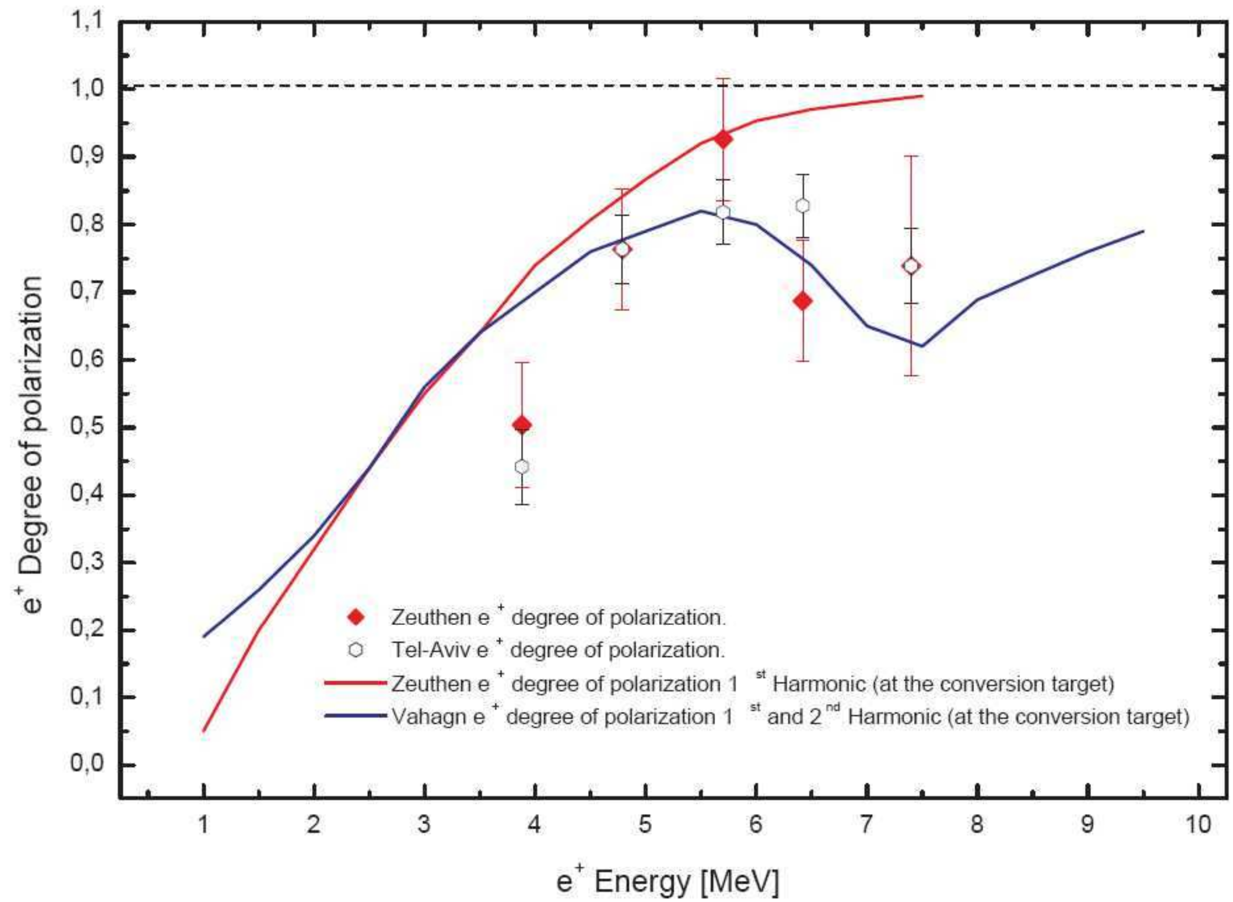


$$E_{\gamma n} \cong \frac{n \cdot 2.48 \cdot (\gamma / 10^5)^2}{\lambda_u [cm](1 + K^2 + \gamma^2 \vartheta^2)} [MeV]$$

Goes to 2.54 mm period for 50 GeV beam




E166 collaboration
Helical Undulator based polarized positron source for the ILC
50 GeV e-
E - 166
10 MeV γ
International Polarized Positron Collaboration



Degree of longitudinal polarization for positrons and electrons as measured by the E166 experiment .

Results published in PRL, NIM

CONCLUSIONS

Maximal polarization can be achieved not only by collimation of gammas, but with selection of positrons by energy after the target.

Separation of secondary particles (positrons or electrons) by energy is a key procedure in polarization gain. **80% polarization looks possible.**

Generation of polarized electrons is possible in this method as well.

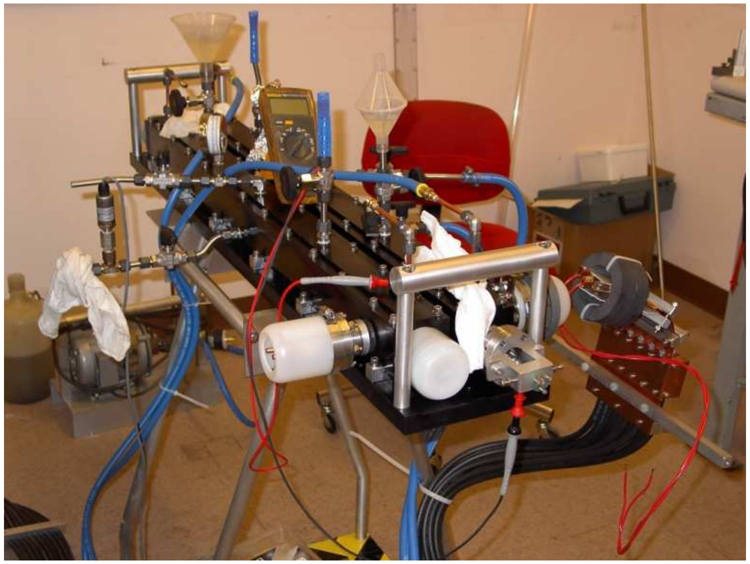
State with minimal emittance has zero overall polarization.

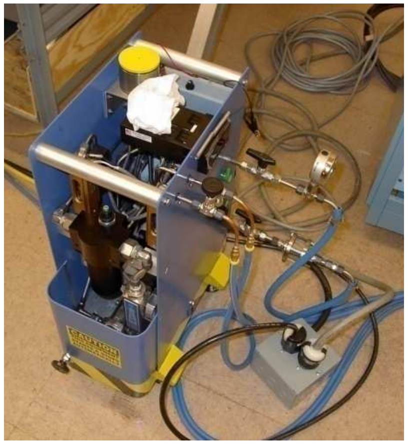
More detailed calculation required for process of squeezing the polarized beam to IP while spin-spin interaction are taken into account.

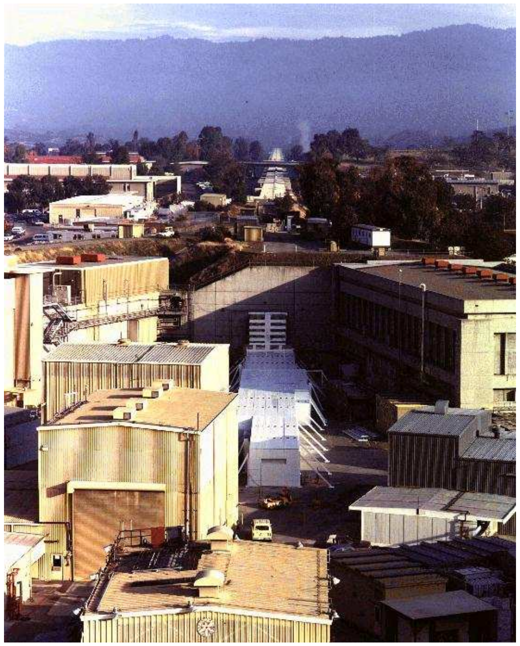
E-166 diffused any doubts about undulator-based positron production for ILC.
Measured polarization in E-166 goes to be 85% max.

Back-up slides

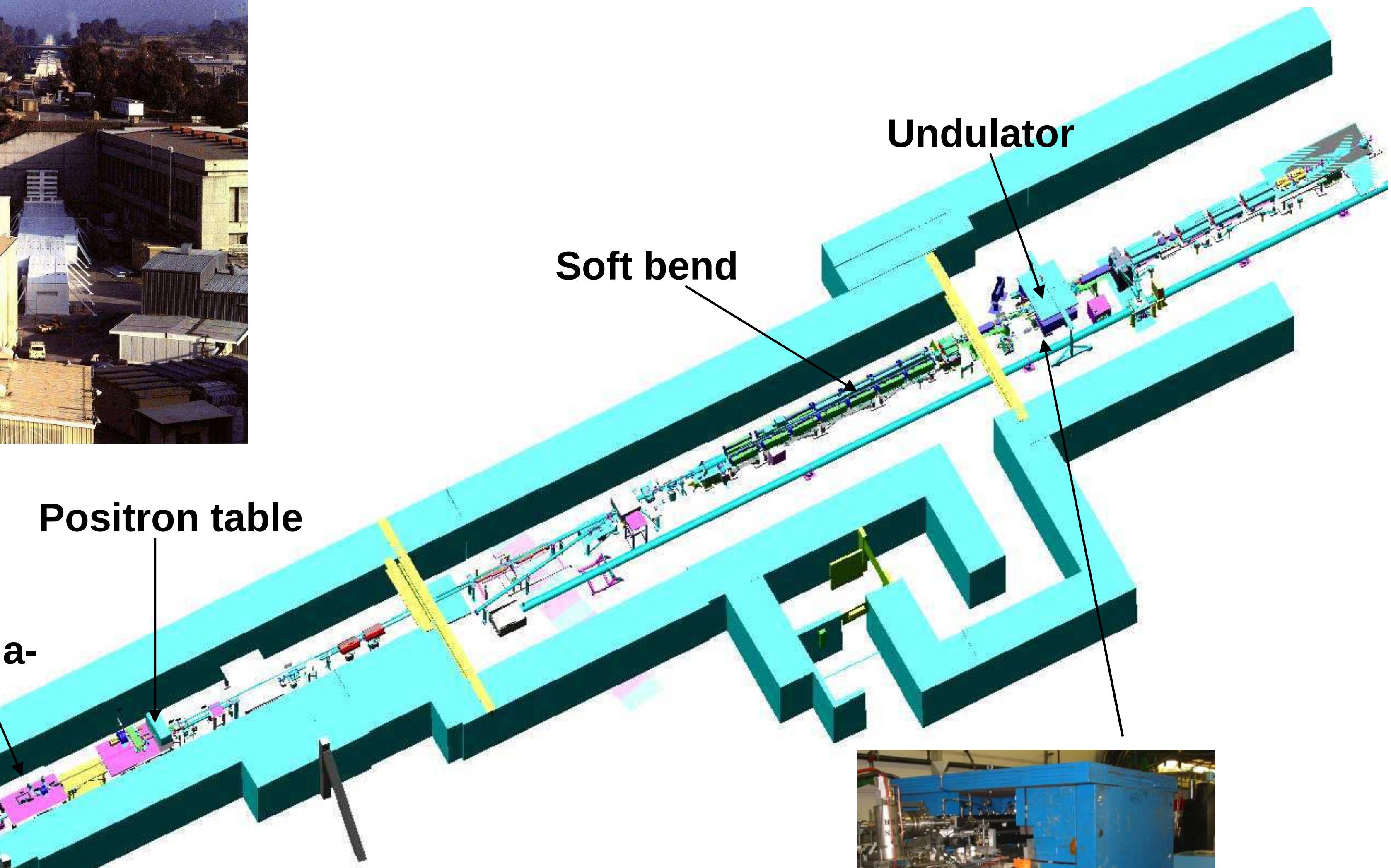
Helical undulator (Cornell)





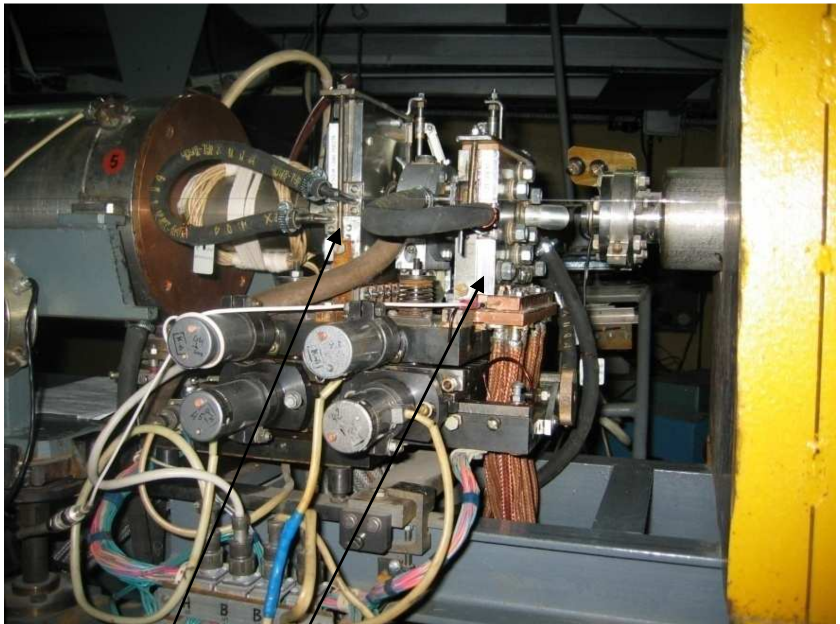


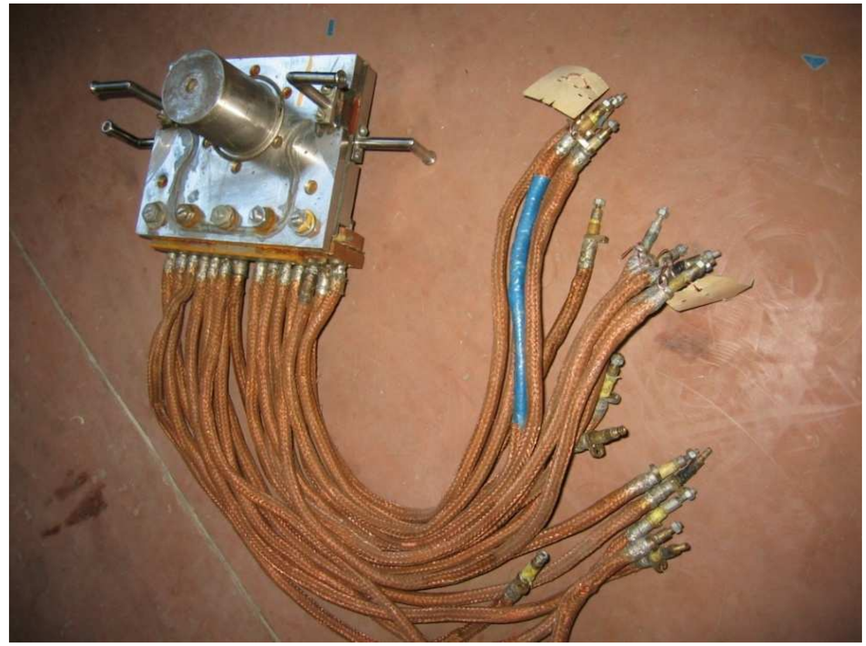
3D scope of E-166



Doublet of Lithium lenses in Novosibirsk BINP

Photo- courtesy of Yu Shatunov





First lens is used for focusing of primary 250 MeV electron beam onto the W target,

Second lens installed after the target and collects positrons at ~150MeV

Number of primary electrons per pulse ~$2 \cdot 10^{+11}$; ~0.7Hz operation (defined by the beam cooling in Damping Ring)

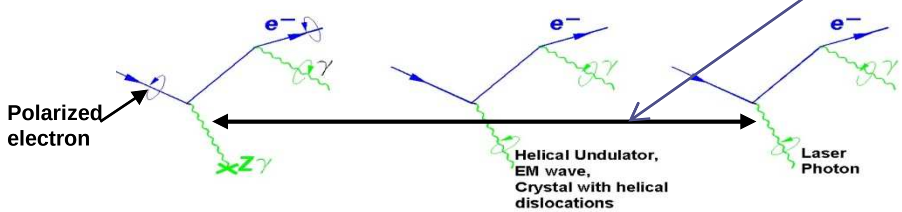
Lenses shown served ~30 Years without serious problem (!)

POLARIZED POSITRON PRODUCTION WITH STACKING IN DR

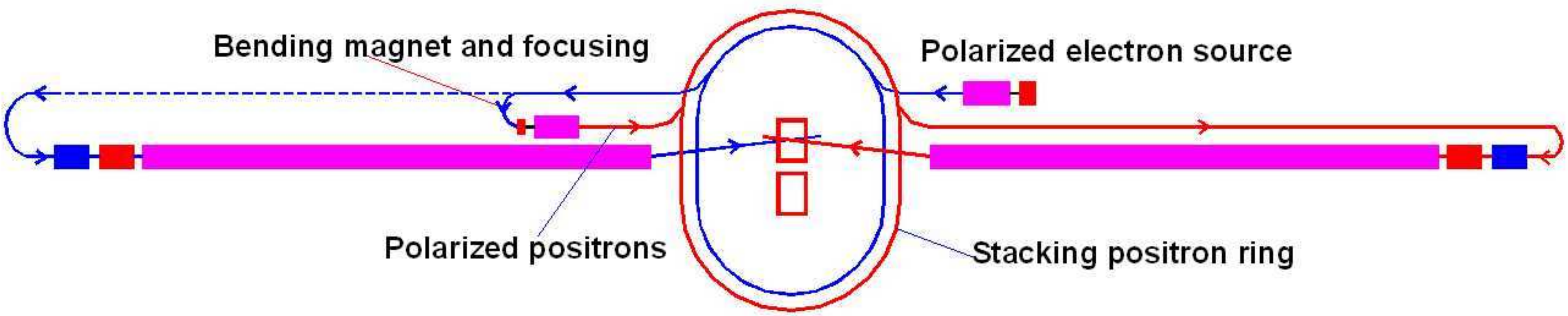
If stacking in DR is allowed, then there is one additional way to generate polarized positrons.

Calculations show that efficiency ~1.5% is possible for the first process with polarization ~75%

Efficiencies of these processes are ~the same



Realization of this method



No lasers !

However, the Undulator-based scheme remains more advantageous

Spin flip in undulator

Positron or electron may flip its spin direction while radiating in magnetic field. Probability:

$$\frac{1}{\tau}[\sec^{-1}] = w_{flip} = \frac{5\sqrt{3}}{16}\frac{r_0^2}{\alpha}\frac{\omega_0^3}{c^2}\gamma^5\left(1 - \frac{2}{9}\zeta_{\parallel}^2 - \frac{8\sqrt{3}}{15}\frac{e}{|e|}\zeta_{\perp}\right)$$

Probability of radiation:

$$w_{rad} \cong \frac{I}{\hbar\omega_0 2\gamma^2} = \frac{2}{3}\frac{e^4 H^2 \gamma^2}{m^2 c^3}\frac{1}{\hbar\omega_0 2\gamma^2} = \frac{1}{3}\alpha\gamma^2\omega_0$$

$$\lambda\!\!\!^{-}_c = r_0/\alpha = e^2/mc^2/\alpha \cong 3.8616\cdot 10^{-11}$$

The ratio

$$\frac{w_{flip}}{w_{rad}} = \frac{15\sqrt{3}}{16}\frac{\lambda\!\!\!^{-2}_c}{\lambda\!\!\!^{-2}_u}\gamma^3\left(1 - \frac{2}{9}\zeta_{\parallel}^2 - \frac{8\sqrt{3}}{15}\frac{e}{|e|}\zeta_{\perp}\right) \qquad \text{(K~1)}$$

Effect of spin flip still small (i.e. radiation is dominating).

KINEMATIC PERTURBATION OF POLARIZATION

- See A.Mikhailichenko, CBN 06-1, Cornell LEPP, 2006.

Fragment from CBN 06-1

Kinematical perturbations due to multiple scattering in a target

Let us consider the possible effect of *kinematical* depolarization associated with rotation of spin vector while particle experience multiple scattering in media of target before leaving. Typically polarized positron carries out ~$(0.5\text{-}1)\hbar\omega$ –energy of gamma quanta. As positrons/electrons created have longitudinal polarization, it is good to have assurance that during scattering in material of target polarization is not lost. Each act of scattering is Coulomb scattering in field of nuclei. So BMT equation describing the spin $\vec{\zeta}$ motion in electrical field of nuclei looks like

$$\frac{d\vec{\zeta}}{dt} = \frac{e}{mc^2\gamma}\left\{G\gamma + \frac{\gamma}{\gamma+1}\right\}\cdot\vec{\zeta}\times\left(\vec{E}\times\vec{v}\right), \qquad \text{(A16)}$$

where $\vec{E} \sim Ze\vec{r}/r^3$ stands for repulsive (for positrons) electrical field of nuclei, factor $G = \frac{g-2}{2} \cong 1.1596\times10^{-3} \approx \frac{\alpha}{2\pi}$. Deviation of momentum is simply $d\vec{p}/dt = e\vec{E}$.

So the spin equation becomes

$$\frac{d\vec{\zeta}}{dt} = \frac{1}{mc^2\gamma}\left\{G\gamma + \frac{\gamma}{\gamma+1}\right\}\cdot\vec{\zeta}\times\left(\frac{d\vec{p}}{dt}\times\vec{v}\right). \qquad \text{(A17)}$$

We neglected variation of energy of particle during the act of scattering, so $\frac{d\vec{p}}{dt} \cong m\gamma\frac{d\vec{v}}{dt}$ and vector $\vec{p}$ just changes its direction. Introducing normalized velocity as usual $\vec{\beta} = \vec{v}/c$, equation of spin motion finally comes to the following

$$\frac{d\vec{\zeta}}{dt} = \left\{G\gamma + \frac{\gamma}{\gamma+1}\right\}\cdot\vec{\zeta}\times(\dot{\vec{\beta}}\times\vec{\beta}) = \left\{G\gamma + \frac{\gamma}{\gamma+1}\right\}\cdot\vec{\zeta}\times\frac{d\vec{\varphi}}{dt}, \qquad \text{(A18)}$$

where $\boldsymbol{\varphi}$ stands for the scattering angle and the vector $d\vec{\boldsymbol{\varphi}}/dt$ directed normally to the scattering plane. For intermediate energy of our interest $\boldsymbol{\gamma} \sim 40$, so the term in bracket ~1 and, finally

$$\frac{d\vec{\zeta}}{dt} \cong \vec{\zeta}\times\frac{d\vec{\varphi}}{dt}. \qquad \text{(A19)}$$

The last equation means that spin rotates to the same angle as the scattering one, i.e. spin follows the particle trajectory.

Filling positron ring from electron source