SC UNDULATOR WITH VARIABLE STRENGTH AND POLARIZATION

A. Mikhailichenko, Cornell University, LEPP, Ithaca, New York

Abstract. We describe design of optimized undulator with SC windings able to generate magnetic field of opposite helicities, including elliptic and a linear ones oriented as desired. For undulator period 25*mm*, aperture 8*mm*, length ~10*m* (single piece), *K* factor could be changed from zero up to 1.5 by changing the feeding current.

1. OVERVIEW

Undulators are planned for installation in all SR facilities. Unique properties of Undulator Radiation (UR), make it being a powerful tool for exploration in wide sciences. Properties of UR are well known (see for example [1]-[3]). Important properties of UR are monochromaticity and polarization.

One unique peculiarity of installation such undulators at linear accelerators (ERL, FEL, ILC, etc) is that aperture of undulator could be small; no necessity to make it with varying aperture, as it required in cyclic machines for injection of beam in storage ring.

Total number of photons in all spectrum radiated by single particle in undulator having M periods and undulatority factor $K \leq 1$ goes to be

$$N_\gamma \cong \frac{4\pi}{3}\alpha \cdot M \cdot \frac{K^2}{1+K^2} \qquad (1)$$

where $K = \frac{eH_0\lambda_u}{2\pi mc^2}\left(\cong 93.4 \cdot B[T] \cdot \lambda_u[m]\right)$, H_0 stands for amplitude of magnetic field, λ_u is period of undulator. Cut-off energy (frequency) of first harmonics is a Doppler-shifted spatial frequency of undulator field

$$E_{\gamma\max} \cong \hbar\omega_c = \frac{\hbar\Omega}{1-\vec{n}\vec{v}} \cong \left.\frac{2\hbar\Omega\gamma^2}{(1+K^2+\gamma^2\vartheta^2)}\right|_{\vartheta=0} \cong \frac{2.48\cdot(\gamma/10^5)^2}{\lambda_u[cm](1+K^2)}[MeV], \qquad (2)$$

where $\Omega = 2\pi c/\lambda_u$, $\vec{n}$ stands for unit vector in direction to observer, $\vec{v}$ is instant particle's velocity. As period of undulator fixed by mechanical constrains, the only possibility to change energy of photon radiated in fixed direction remains by changing energy of particle, changing K factor, more precisely–the amplitude of magnetic field in undulator. Other possibility–to change observed photon energy by varying observation angle inevitably changes polarization, as [1]

$$\xi_{2n} = 2\left(\frac{\cos\vartheta - \beta_\parallel}{\beta_\perp \sin\vartheta}\right) \times \frac{J_n'(n\kappa)J_n(n\kappa)}{J'^2_n(n\kappa) + \left(\frac{\cos\vartheta-\beta_\parallel}{\beta_\perp\sin\vartheta}\right)J_n^2(n\kappa)}, \qquad (3)$$

where $\kappa = \frac{\beta_\perp \sin\vartheta}{1-\beta_\parallel \cos\vartheta} \cong \frac{2K\gamma\vartheta}{1+K^2+\gamma^2\vartheta^2}$, $\frac{\cos\vartheta - \beta_\parallel}{\beta_\perp \sin\vartheta} \cong \frac{1+K^2-\gamma^2\vartheta^2}{2K\gamma\vartheta}$, J_n and J_n' stand for Bessel function of first order and its derivative, $\beta_\perp$ and $\beta_\parallel$ are transverse and longitudinal components of normalized to c velocity of particle, n stands for harmonic number, ξ_2 is Stokes parameter. For $K \leq 1$, (3) could be expanded as

$$\xi_{21} = \xi_{22} = \frac{1-\gamma^4\vartheta^4}{1+\gamma^4\vartheta^4}, \tag{4}$$

So for observation angles $\vartheta \cong 1/\gamma$ polarization comes to zero. This means, that mechanism of changing polarization should work independently from observation angle.

Photon energy could be changed by changing the beam energy. Changing energy of beam in ERL type machines requires to keep a RF balance of powers arranged for some specific energy and current running in machine. All these parameters restrained by coupling of RF structure with external RF generator. Basically ERL is tuned to fixed energy, although, some energy change is possible even here.

Spectral distribution of intensity of radiation generated by particle in finite-length undulator can be represented as [2]

$$\frac{dI_\omega(\xi)}{d\xi} = \frac{e^4 H_0^2 \gamma^2}{\pi m^2} \int_{1/2}^{\infty} \frac{dy}{y^4}(1-2y+2y^2)(1+(2\xi y)^2)\frac{1}{\pi M}\frac{Sin^2(2\pi M\xi y)}{1-(2\xi y)^2}, \tag{5}$$

where $\xi = \hbar\omega/\hbar\omega_c$ is the energy of photon normalized to the cut-off frequency $\omega_c \cong \frac{2\Omega\gamma^2}{1+K^2}$, $y = \gamma^2(1-\vec{n}\vec{v}) \cong \frac{1}{2}(1+K^2+\gamma^2\vartheta^2)$, $\hbar = 1 = c$. For infinitely long undulator, $M \to \infty$, it yields

$$\frac{dI_\omega}{d\xi} = \frac{e^4 H_0^2\gamma^2}{\pi m^2}\xi\cdot(1-2\xi+2\xi^2) = I_0\cdot\xi\cdot(1-2\xi+2\xi^2) \tag{6}$$

We would like to attract attention to one circumstance here. According to (5) the shape of spectrum practically does not depend on the number of poles starting from M~10. So, for machines like ERL, where the beam has significant average power, there is no necessity for long undulator.

The only instance when the length (number of periods M) might play important role could be described as the following. During each period of oscillation with spatial frequency $\Omega = 2\pi c/\lambda_u$, the front of radiation goes ahead of particle to the distance $\cong \lambda_u \cdot (1+K^2)/2\gamma^2$. So if observer would like to see interference of radiation between head and tail particles in finite-size bunch having length $l_b \cong c\tau$, the number of periods should be at least

$$M \geq \frac{2\gamma^2 l_b}{\lambda_u\cdot(1+K^2)} \cong \frac{2\gamma^2 c\tau}{\lambda_u\cdot(1+K^2)} \ . \tag{7}$$

Substitute here for estimation the time duty of bunch $\tau \cong 1ps$, $\lambda_u \cong 2.5cm$, $\gamma \cong 10^4$, K=0 (weak undulator) one can obtain

$$M \geq \frac{2\cdot 10^8 3\cdot 10^{10} 10^{-12}}{2.5} \cong 2.4\cdot 10^6,$$

i.e. length $L = \lambda_u M \geq 6\cdot 10^6 cm$. For 1 *femtosecond* bunch-length the length of undulator goes to $L \geq 6\cdot 10^3 cm = 60m$ minimum. It is clearly seen that this type of observations are unrealistic.

The average distance between particles in the bunch as it is seen by observer in Lab system is

$$\delta \cong (V / N)^{1/3} \cong \left(\frac{\sigma_x \sigma_y l_b}{N} \right)^{1/3} \quad , \tag{8}$$

where N stands for the bunch population, σ_x, σ_y -are transverse bunch sizes for corresponding directions. Substitute here for estimations $\sigma_x \cong \sigma_y \cong l_b \approx 1 \mu m$ ($\tau \approx l_b / c \cong 3 \cdot 10^{-15} \sec \equiv 3\, fs$), $N \cong 10^8$, one can obtain $\delta \cong 2.15 \cdot 10^{-7} cm$, meanwhile the wavelength of radiation for the first harmonic is $\lambda_u / 2\gamma^2 \cong 2.5 / 2 \cdot 10^8 \approx 1.25 \cdot 10^{-8} cm \equiv 1.25 A^\circ$. One can see, that radiation, emitted by arbitrary chosen electron, will touch neighboring electron after $\cong \delta / (\lambda_u / 2\gamma^2) \sim 20$ periods. This "touch" will yield some splashes of coherence in radiation. So the length of undulator must be 20 periods or 50*cm* minimum for 5*fs* long bunch (for 5*ps* the length comes to 5*m* minimum). The upper limitation defined by engineering only; we could see, that undulator, where radiation interference of radiation of neighboring electrons in bunch of 2 picoseconds, occurs if undulator is at least 10*m* long.

Variation of *K*-factor allows control of spectrum produced by undulator for coherence purposes. Really, length of formation of UR coincides with undulator spatial period $l_f \cong \lambda_u$. After passage of l_f particle radiates $\alpha \cong e^2 / \hbar c$ photons (i.e. one photon after ~137 periods), so basically formula (1) could be interpreted as following. The number of formation lengths along the undulator length L is L / l_f. Number of photons is hence $N_\gamma \cong \alpha L / l_f = \alpha L / \lambda_u = \alpha M$. Factor K^2 reflects suppression of number of radiated photons due to Landau-Migdal-Pomeranchuk effect, which is in line with dependence of radiated power on square of magnetic field value. This is competition between length of formation of UR, l_f and length of formation for SR of main harmonic, which is $l_{SR} \cong \rho / \gamma$, where $\rho = mc^2 \gamma / eH_0 \cong \lambda_u \gamma / K$, so $l_{SR} \cong \lambda_u / K$. In number of photons this ratio appears as a square, factor K^2. Some interesting phenomena occurs when $K \to 0$. Formally, in this case formation length $l_{SR} \to \infty$, while UR formation length remains the same, $l_f \cong \lambda_u$. This means that for formation of radiation all undulator length plays role. Thus, the situation may happen, when energy radiated is so low, that it is not enough to build up quanta. In this case radiation is going statistically so in average, the energy radiated is the one calculated by classic formula. This effect might be important in process of Optical Stochastic Cooling at low intensity of radiation [8]. So one can imaging experiment at ERL, where beam power is high, the passage through weak undulator, $K \to 0$, could generate radiation which will manifest extreme statistical properties. That might be the situations, when passage of some bunches will generate much smaller power than the others.

2. NUMERICAL MODEL

The ability to change polarization and *K* factor are crucial factors for many experiments. Meanwhile the undulator able to generate radiation with variable polarization suggested a long time ago: [3], [4]. Basically undulator suggested there has two windings with opposite helicities, but the same period. By changing currents in windings it is possible to arrange any polarization – from linear to circular and vice versa. Third winding having the same helicity as one of already existing, but shifted in longitudinal direction by half period, could be added for change the orientation of linear polarization. Possibility of realization of such type of undulator with *K*~1 became open with usage of SC winding now.

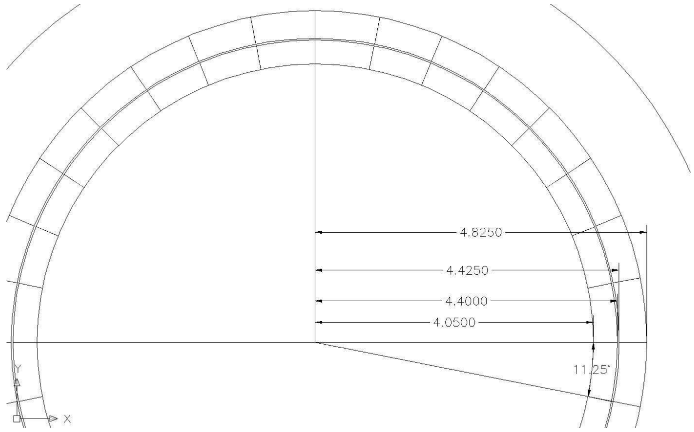

Figure 1: Dimensions of windings (in *mm*).

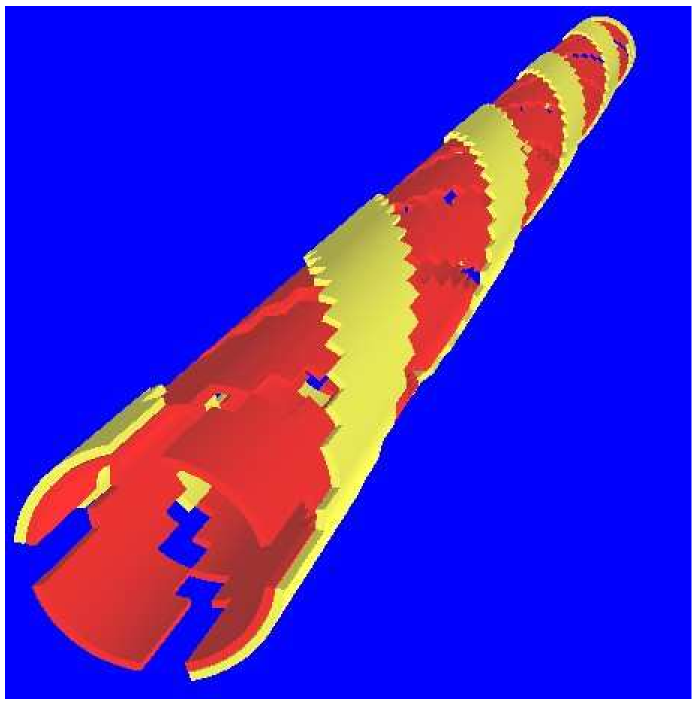

Figure 2: Scheme of windings as they appear in 3D MERMAID model. Two windings in upper layer (yellow) have left-handed helicity- inner windings are right handed ones.

We investigate technological possibilities for such undulator here. As helical undulator is pure 3D system, we used appropriate codes able to carry 3D calculations of helical structure. We made a model for calculation with 2D and 3D code MERMAID [6]. 2D calculation with MERMAID is going by substitution of *x, y, z* coordinates by *z*-dependent rotating system which angle is function of *z*

$$B_{\varphi}(\rho,\varphi,z) \propto B(\rho)Cos\left[\varphi-\varphi_0-\frac{2\pi z}{\lambda_u}\right].$$

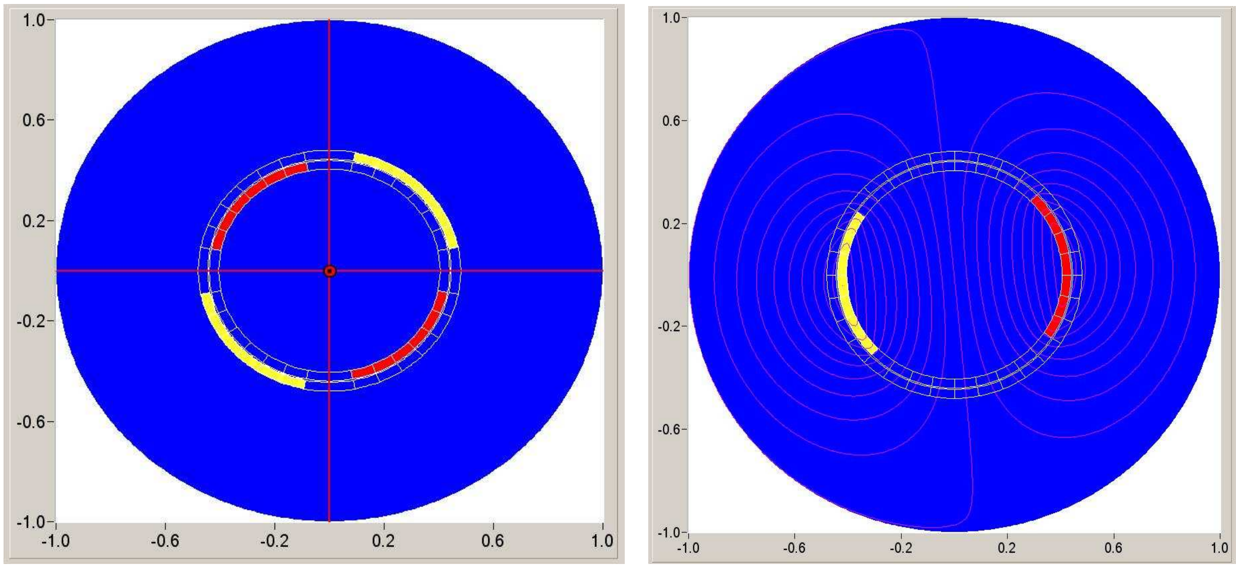

Figure 3: Cross section of two coils as they appear in MERMAID, at the left. Magnetic lines for circular polarization, at the right. Here by red and yellow marked the same-layer inner winding. The current is running in opposite directions in yellow and red conductors.

Usage of Iron yoke increases axial field ~2 times. Few examples of different polarization could be generated with these thee windings are presented in Figures below.

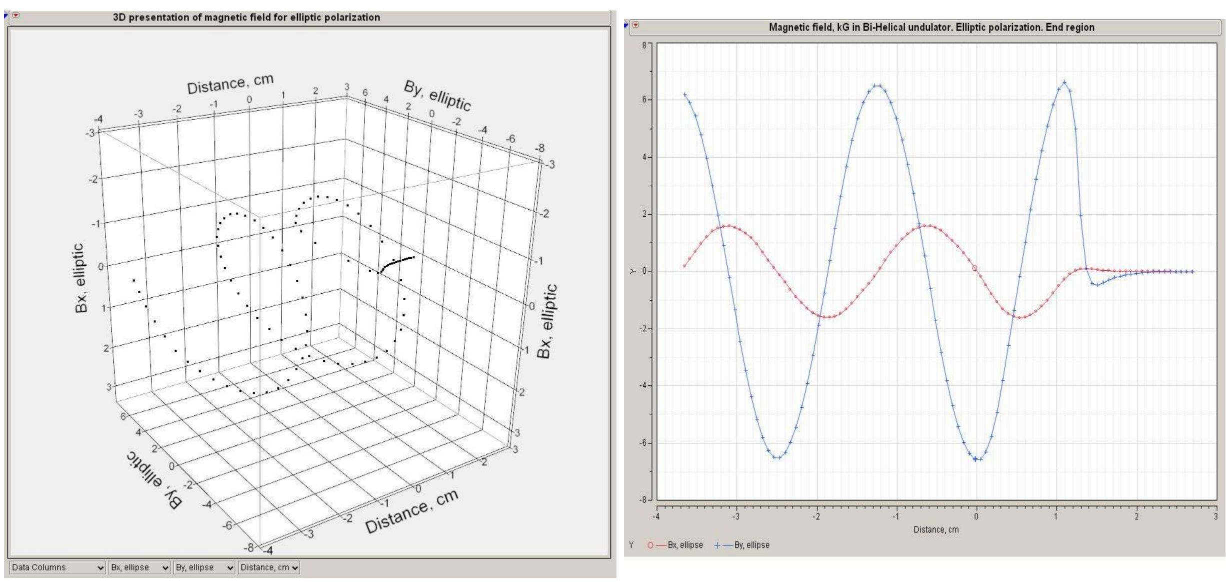

Figure 4: Elliptic polarization. At the left –3D presentation of fields represented as function of longitudinal coordinate; at the right –each component represented by individual plot. Field values are represented in kilo-Gauss units. (At the left the B_x *and* B_y axis have different scale).

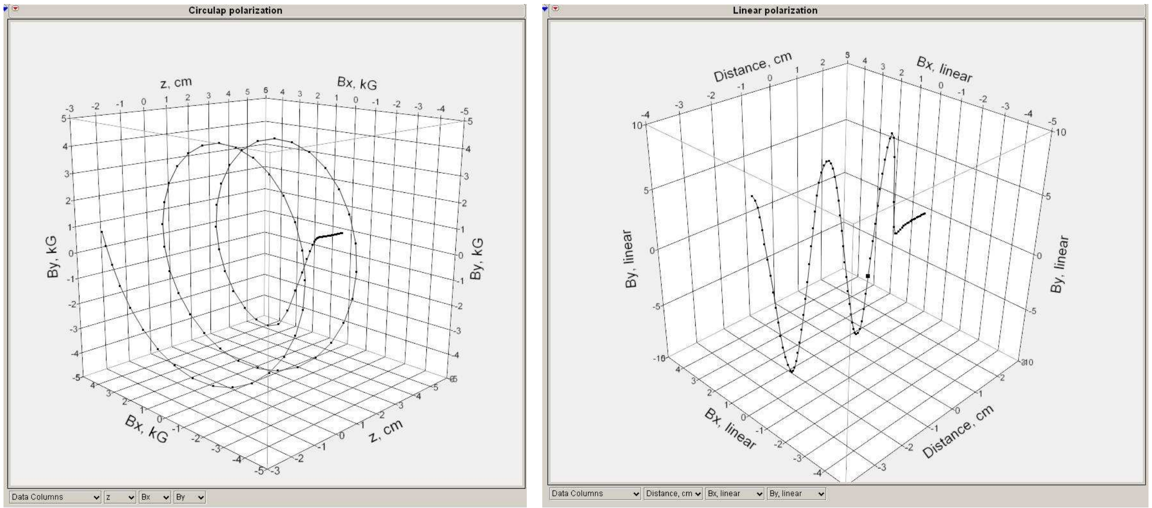

Figure 5: Circular polarization, at the left. The dots represent magnetic field values B_x, B_y as functions of longitudinal coordinate. Linear polarization, at the right.

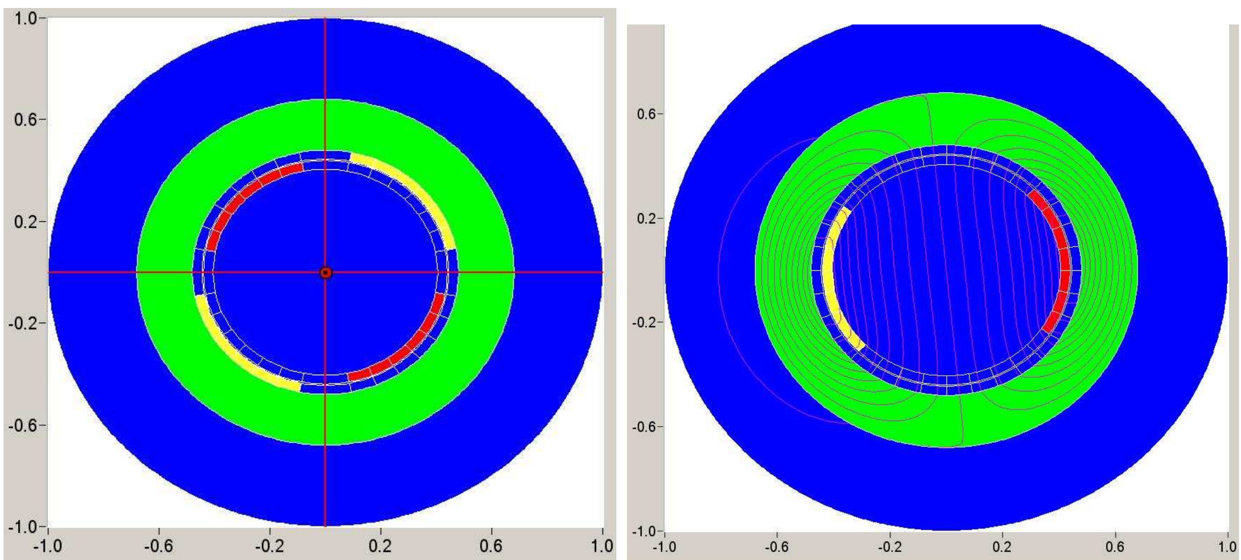

Figure 6: Coils enclosed by Iron yoke, left. Magnetic lines in undulator with Iron yoke, right.

Installation of coils in Iron tube ~doubles *K* factor. This might be a good margin. Utilization of thicker wire could also increase field value; all final parameters must be defined by user, although UR specifically associated with *K*<1.

3. PRACTICAL MODEL

We erected model of such undulator. This model fabricated just for test technology of windings and does not associated with any specific project. Winding was done on Oxygen free Copper thin-wall tube of 8 *mm* in inner diameter and period of ~25*mm*, having length ~45 c*m*. The tube wrapped by 0.5 mills-thick Kapton tape. This gives for 5 *GeV* ($\gamma \cong 10^4$) beam the cut-off energy for *K*=1

$$E_{\gamma\max} \cong \frac{2.48\cdot(\gamma/10^5)^2}{\lambda_u[cm](1+K^2)}[MeV] \approx 5keV \tag{5}$$

By changing K to a smaller value range, one can shift frequency an octave higher. This is going at fixed energy. Ability to change energy in some margins adds to the possibility for tuning ($\sim\gamma^2$).

We used ribbon-type flat 6 wire strands stick together with Formvar for winding. Bare wire diameter is 0.3 *mm*, insulated-0.33 *mm*, ratio of SC to Copper is 1.2:1. First layer has 48 wires total, the outer layer has 2x12 wires.

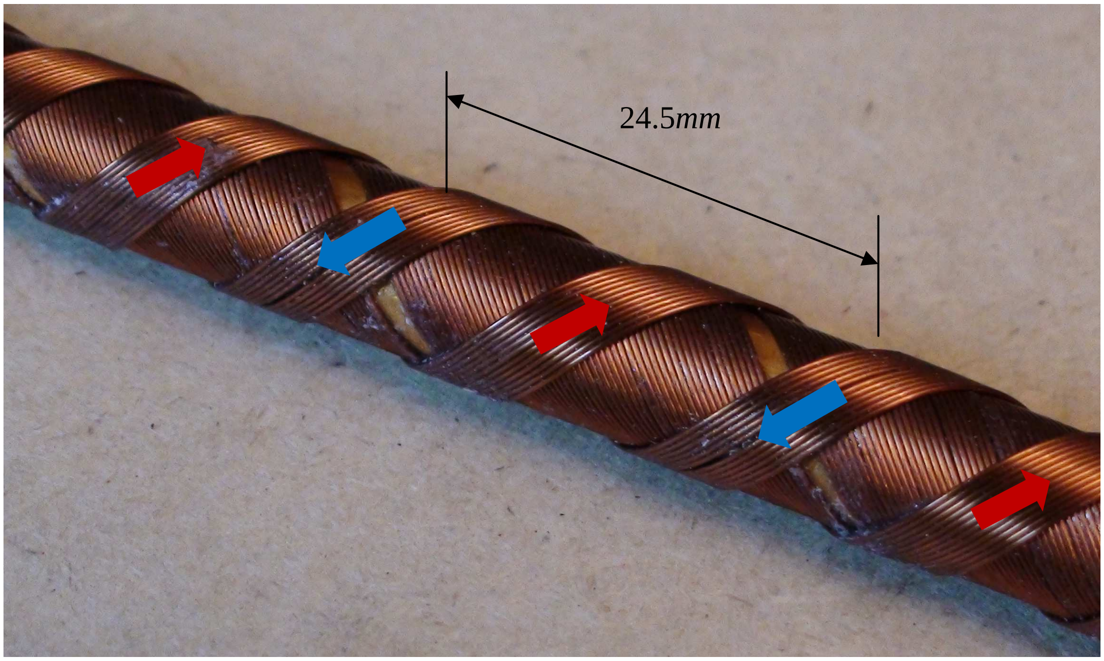

Figure 7: Coils wounded above Copper tube with six wire strands stick together in flat SC cable. Period 24.5 *mm*. Outer diameter is 10 *mm*, inner diameter clear for beam is 8 *mm*. Direction of current in corresponding coil is shown by arrow.

Example of installation of this kind of cold mass in cryostat one can find in [5]. Undulator, fabricated with technology tested could have the length up to 10 *m* as single piece. Vacuum is not a problem here due to cryogenic pumping of cold inner surface of vacuum chamber.

Model is ready for test in Dewar. No doubts that designed parameters could be reached, based on previous experience with short-period undulators, see [5].

For operation of undulator three set of windings and three power supplies required. One of them serves for rotation of plane of polarization in arbitrary direction.

4. SUMMARY

The type of undulator described allows easy manipulation by polarization and *K* factor independently. Ability to change *K* factor within 0-1.5 allows fine tuning of radiated spectrum. Clean aperture is 8*mm* wide, provided by smooth surface of Oxigen-free Copper tube For mostly experiments this ability to manipulate with *K* factor and polarization might be crucial.

Undulator is inexpensive, having small transverse outer size of cryostat (~3") and fits well in SC RF systems planned worldwide. This type of undulator could be recommended for ERL in first place. Large aperture (8 *mm* inner diameter), period 25 *mm*, single piece length up to 10 *m*, varying *K* factor *and* polarization allows wide range of experiments to be carried with this device. Period of undulator could be set to any specific value; increase of period allows exponential increase of *K* factor (indeed, decrease of period for fixed aperture reduces achievable *K* factor exponentially also). Ratio of aperture diameter to the length of undulator could reach 0.8*cm*/1000*cm*= $8\cdot10^{-4}$, what is the same as successfully operated pulsed undulator fabricated at Cornell for SLAC experiment (0.8*mm*/1000*mm*) [7].

Work supported by NSF

5. REFERENCES

[1] E.G.Bessonov, A.A.Mikhailichenko," *Some Aspects of Undulator Radiation Forming for Conversion System of the Linear Collider"*, BUDKER-INP-1992-43, Jun 1992. 26pp.; http://ccdb4fs.kek.jp/cgi-bin/img_index?9302032

[2] V.N.Bayer, V.M.Katkov, V.M.Strokhovenko, "*Electromagnetic Processes at High Energy in Oriented Monocrystalls*", Novosibirsk, Nauka, 1989, ISBN 5-02-028613-3.

[3] D.F.Alferov, Yu.A.Bashmakov, E.G.Bessonov, "*Generation of Circularly Polarized EM Radiation*", JETF, 1976, Vol.46, N 11, pp. 2392-2397.

[4] D.F.Alferov, Yu.A.Bashmakov, E.G.Bessonov, "*Device for Obtaining Polarized Electromagnetic Radiation*", Authors Certificate N^{o} 538508, USSR.

[5] A.Mikhailichenko, "*Test of Short Period SC Undulator*", EPAC 2008, Genoa, Italy, June 22-28, 2008, Proceedings, MOPP023.

[6] **MERMAID**- **ME**sh **R**outine for **MA**gnet **I**nteractive **D**esign, Sim Limited, 630058, Novosibirsk, P.O. Box 160, Russia.

[7] G. Alexander *et al*, " *Undulator-Based Production of Polarized Positrons*", Published in Nucl. Instrum. & Meth. **A**610: 451-487, 2009, e-Print: arXiv:0905.3066; see http://arxiv.org/PS_cache/arxiv/pdf/0905/0905.3066v1.pdf .

[8] E.G.Bessonov, M.V.Gorbunkov, A.A.Mikhailichenko, "*Muon Cooling* ", arXiv:0906.4624 (2009); see http://arxiv.org/ftp/arxiv/papers/0906/0906.4624.pdf .