November 28, 2006 CBN 06-6

DAMPING RING FOR TEST THE OPTICAL STOCHASTIC COOLING

A Talk at Workshop on

Possible Directions for Advanced Accelerator R&D at the ILC test accelerator at FermiLab.

A. Mikhailichenko, Cornell University, LEPP, Ithaca, NY 14853

Abstract. A compact ring can be used to verify the OSC principles. Possible parameters of such ring described in this paper.

INTRODUCTION

After proposal to extend Stochastic Cooling method [1] to optical bandwidths [2], the idea about testing this method on specially designed compact damping ring was described soon after [3]. It was stressed there, [3], that under rates of cooling, deviled by OSC, the method can be interesting for protons, electrons/positrons, muons, as well as for multi charge ions. Later, application of OSC to the cooling of multi-charge ions was recognized as having advantage due to coherent radiation of ions ~Z^2 [4]. For electron and positron beams the method allows drastic reduction of final temperature with an optical amplifier of intermediate complexity. There was an idea to apply the OSC cooling to search for Fermi-degeneration in relativistic beam [5]. Complex introduced in [3] represented in Fig. 1 below.

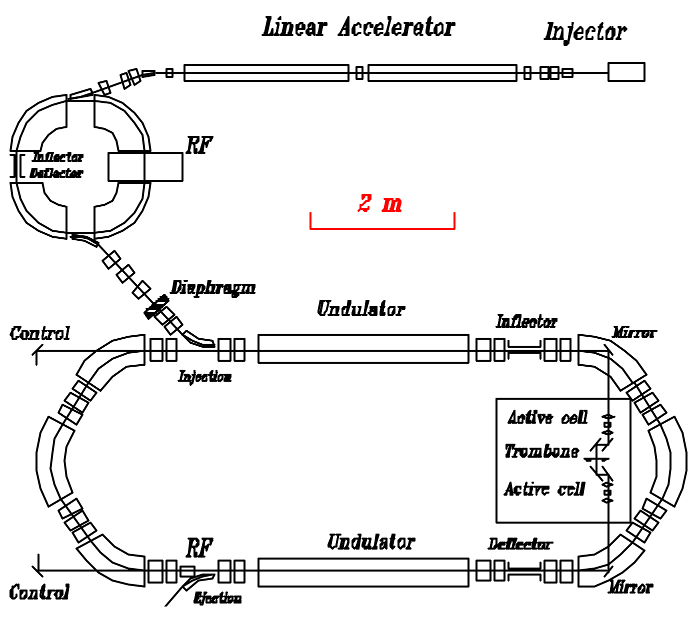


Figure 1: The damping ring with injection complex proposed for testing OSC [3].

One of the peculiarities was a possibility for isochronous bend with three-magnet system. Ten years later the similar bend was described in [6]. Here the influence of the quantum fluctuations in bends to the beam energy spread considered also.

Latest interest to this subject initiated by Vladimir Shiltsev.

OPTICAL STOCHASTIC COOLING BASICS

Hardware for stochastic cooling is a broadband feedback system. And it is working like any other feedback system, providing back reaction to the particles in accordance with instant deviation of bunch parameters from stationary ones at the pickup location. Decisive difference, however, is that this feedback system has wide bandwidth allowing to act to the particles distributed along the bunch length.

Hence, the bandwidth is important characteristic of the feedback system. Wider the bandwidth-more details along the bunch can be resolved and higher decrement can be obtained with the system. Migration to optical bandwidths, as it was proposed in [2], brings the feedback system to operation with optical signals instead of electrical ones, Fig. 2. For proper action, the structure of the bunch as it was identified at the pick-up location must be preserved by transport system up to the kicker location.

In Fig. 2 the Optical Stochastic Cooling scheme represented in general view. Here quadrupole wiggler, which is a series of strong quadrupole lenses serve as a pickup. Particle radiates here in accordance with its instant position with respect to the lens center. Resulting electromagnetic wave is a superposition of waves radiated by each particle of the bunch.

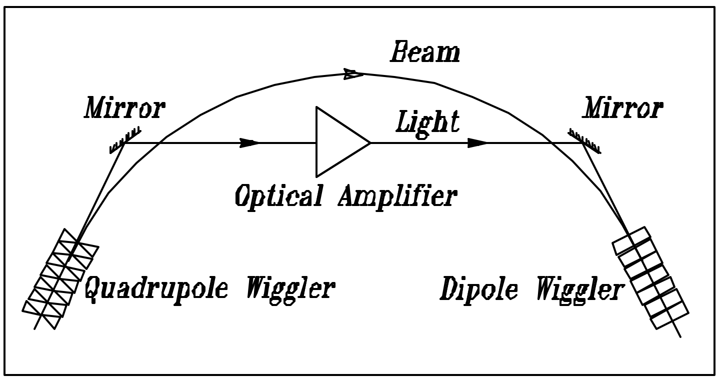


Figure 2: The general layout of the system [2].

Further, this wave amplified in optical amplifier with corresponding optical bandwidth and the signal applied to the co-directionally moving bunch down to the beam trajectory with appropriate delay. One peculiarity in action of optical signal to the particle is that optical plane wave and co-directionally moving relativistic particle can interact only if they are propagating with angle to each other, resulting *energy change*, not a transverse kick. The angle required between the wave and particle velocity arranged with a help of wiggler. The value of this angle is small, $\cong K/\gamma$, where $K = \frac{eH\lambda_u}{2\pi mc^2}$, λ_u –is spatial period of wiggler (undulator), but value of amplitude of this angle can be kept constant along significant length—the length of wiggler.

Few different types of OSC are represented in Fig. 3.

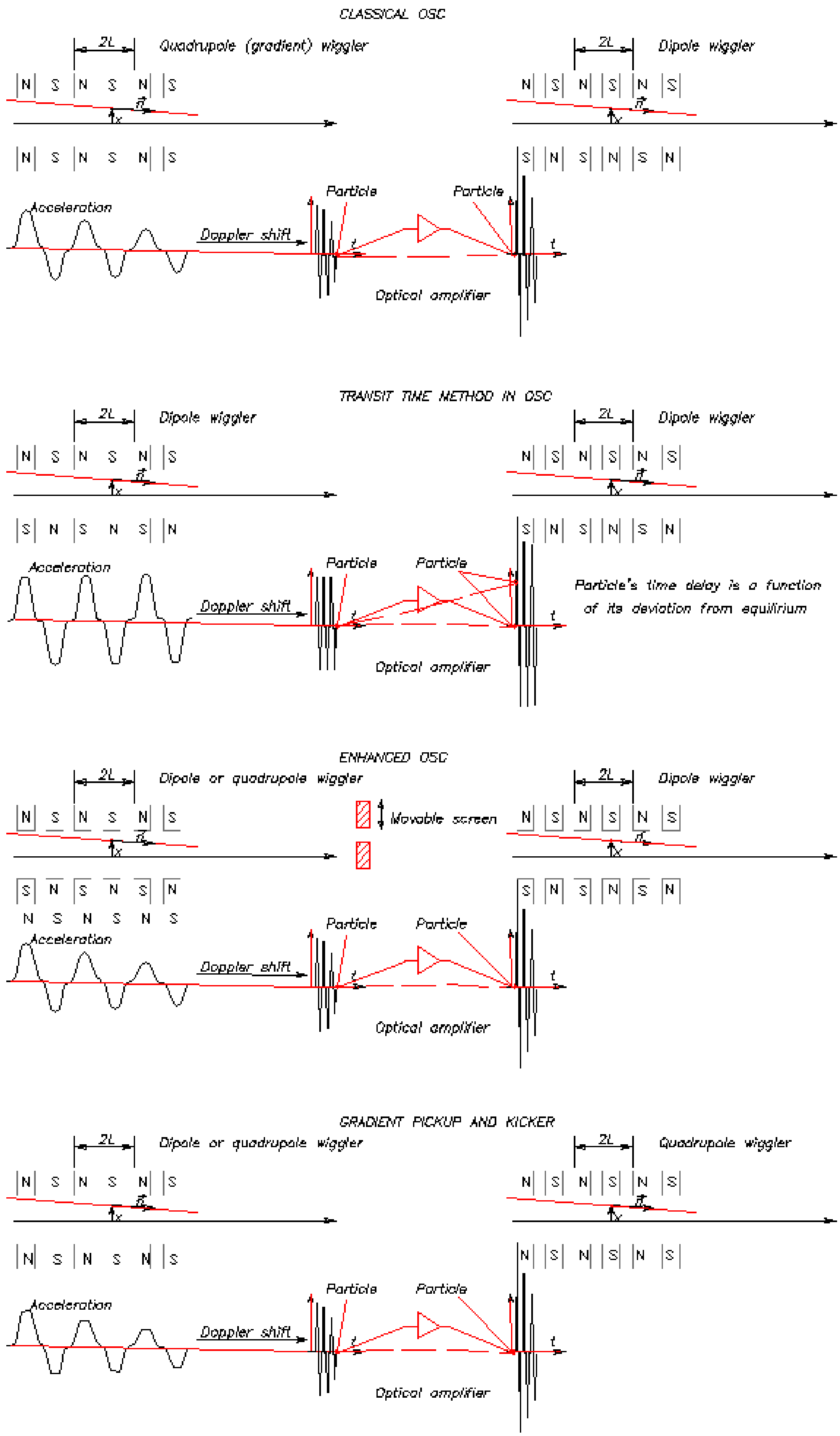


Figure 3: Types of OSC. In transit time method arriving to the second wiggler depends on parameters of particle at the pickup undulator [7]. Enhanced OSC [14] deals with screening of optical image for some parts of radiation from the beam.

So the stochastic cooling (SC) method deals with finite number of particles in the bandwidth directly [1]. Transverse fluctuation of the center of gravity position of N particles each having transverse coordinate x_j is

$$< x > \cong \frac{\sum_{j=1}^{N_S} x_j}{N_S} \approx \frac{A}{\sqrt{N_S}} \,, \tag{1}$$

where A is an effective amplitude of transverse oscillation[1], N_S --is the number of the particles in the bandwidth. One can see that the lower number of the particles in the bandwidth is better. This value of average displacement generates the electrical signal in differential pick up[2], proportional to $< x >$. Further, after amplification, this signal applied to the dipole kicker at the place with maximal slope of particle's trajectory, so the betatron *amplitude* of oscillation changes. Mathematically this could be described by transformation

$$x_i^{new} \cong x_i^{old} - G\frac{\sum_{j=1}^{N_S} x_j^{old}}{N_s} \to x_i^{old} - G\frac{x_i^{old}}{N_S} - G\frac{\sum_{j\neq i}^{N_S} x_j^{old}}{N_S} \,, \tag{2}$$

where i marks selected particle. The term, associated with this selected particle was detached from the sum also. G is a normalized amplification in the feedback.

One can see, that the damping, associated with the signal from the particle itself. The signal is $\approx G\frac{x_i}{N_S} \cong G\frac{A}{N_S}$ and the rest part $\approx G\frac{\sum x_j}{N_S} \cong G\frac{A}{\sqrt{N_S}}$ represents the heating for this particle selected.

Squaring (2) one can obtain for the dipole kicker [1]

$$\frac{d < x^2 >}{dn} + \frac{2G - G^2}{N_S} < x^2 > = 0 \,,$$

where n marks the turns. Maximal cooling rate corresponds to G =1, this means, that the kicker must eliminate the amplitude, corresponding to the picked averaged displacement in the pick-up. One can see that the cooling rate is simply associated with the number of the particles in the bandwidth. This number is $N_S \sim \frac{N}{\Delta f}\frac{c}{l_B}$, where Δf –is the bandwidth, l_B –is the bunch length[3], c–is the speed of light.

For typical proton cooling system with $\Delta f \approx 250\ MHz$, $N_S \approx 4\cdot 10^6$.

RADIATION FROM A QUADRUPOLE WIGGLER

Quadrupole wiggler is a series of magnets with alternating sign of gradient installed along straight line with a period $?_u$=2L. In simplest case that might be a series of dipole magnets (see Fig. 4). Series of quadrupoles with focal distance bigger, than the distance between can also serve as quadrupole wiggler. In this case proper polarization can be selected by polarizer. One can just use the usual dipole wiggler with switched polarities in all lower poles. Magnetic field in the aperture of quadrupole wiggler could be represented as $H_y(x) = g\cdot x$ or $H_x(y) = g\cdot y$, where g – is a gradient, depending on polarization of magnetic field or both, if series of quadrupoles with reversing polarities used as a

[1] This corresponds to a model, where all particles have the same amplitude, but random betatron phases. An effective beam emittance is represented as ellipse. This still exact for the beams with constant density also.

[2] What includes the pick up electrodes itself plus differential amplifier or any other element, where signals from opposite electrodes are subtracted.

[3] For continues beam the orbit perimeter substitutes the bunch length.

wiggler. For a particle, which is going off the central trajectory, the field looks like a field in ordinary dipole wiggler with magnetic field proportional to the displacement. Betatron motion modulates the field with the frequency of transverse oscillations [7]. Electrical field in far zone can be represented as

$$\vec{E}(t) = -\frac{e}{cR}\frac{\vec{n}\times((\vec{n}-\vec{b})\times\dot{\vec{b}})}{(1-\vec{n}\vec{b})^3}\Bigg|_{t'=t-R(t')/c} \quad , \qquad \vec{H} = \vec{n}\times\vec{E}$$

where $\vec{n}$ –is an unit vector in direction of observation, R–is the distance to the observer at the moment of radiation. In dipole approximation, $\vec{b}_{\perp} \cong \frac{K}{g}$, $K \leq 1$, where $K = \frac{egx2L}{2p\ mc^2} \leq 0.7$, $\vec{b} = \vec{b}_{|} + \vec{b}_{\perp} \cong \vec{k}b$, where $\vec{k}$ –is an unit vector in longitudinal direction. The field becomes

$$\vec{E}(t) = -\frac{e}{cR}\cdot\frac{\dot{\vec{b}}_{\perp}}{\left(1-b(\vec{n}\vec{k})\right)^2}\Bigg|_{t'=t-R(t')/c} \cong -\frac{e}{cR}\cdot\frac{4g^2\dot{\vec{b}}_{\perp}}{(1+K^2+g^2J^2)^2}\Bigg|_{t'=t-R(t')/c} \quad , \tag{3}$$

where J –is an angle between $\vec{k}$ and $\vec{n}$.
Transverse acceleration can be represented as

$$\left|c\dot{b}_{\perp i}\right| \cong \left|\frac{e\cdot\vec{k}\times\vec{H}_i}{mcg}\right| = \frac{e\cdot gx_i}{mcg} , \tag{4}$$

so (3) can be rewritten as

$$\left|\vec{E}_i(t)\right| \cong -\frac{e^2}{mc^2 g R}\cdot\frac{4g^2 g}{(1+K^2+g^2J^2)^2}\cdot x_i\Bigg|_{t'=t-R(t')/c} = E_o.x_i(t') . \tag{5}$$

In expression (5) we ignored the dependence $R(t')$ in denominator. Mostly important influence arises from $R(t')$ containing in x, reflecting the fact that the electrical field is proportional to the particle's displacement in the lens. In the same manner one can consider the radiation in *sextupole and so on wiggler*. This may give important information about *higher* moments in transverse beam distribution. To obtain the full radiation from the beam with finite transverse dimensions one needs to sum the fields (11) arisen from all particles. So the expression for the field becomes

$$\vec{E}_x(t) \cong -E_0\cdot\sum_{i=1}^{N} x_i(t')\Bigg|_{t'=t-R(t')/c} , \quad \vec{E}_y(t) \cong -E_0\cdot\sum_{i=1}^{N} y_i(t')\Bigg|_{t'=t-R(t')/c} .$$

The sum could be transformed into convolution (5) with normalized transverse distribution, what basically gives a *dipole* component of transverse distribution. Other types of wigglers (sextupole and so on) could be used for obtaining higher moments. Spectral angular density can be calculated in a usual way $\P^2 E/\P S\P w = c\left|\vec{E}(w)\right|^2$.

One can see from (3), that the time dependence of electric field strength $\vec{E}(t)$ copy the time dependence of the particle's acceleration in a time scale $1/(1-b(\vec{k}\vec{n}))$, what is basically a Doppler's factor, Fig.4.

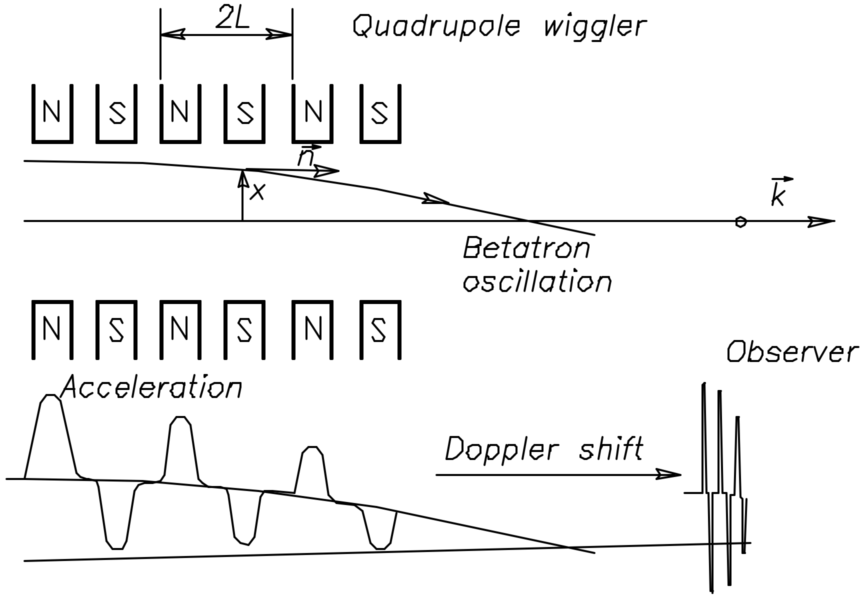


Fig. 4. Radiation from a quadrupole wiggler [7].

For transverse kick in a quadrupole wiggler, the time sequence in the picture must be reversed[4].
For the beam passing the quadrupole wiggler, for each particle with positive off axis position $+x$, there exists a particle with negative off axis position $-x$, so in a quadrupole field they have opposite acceleration, Fig.5, so the radiation in the forward direction interferes *destructively*.

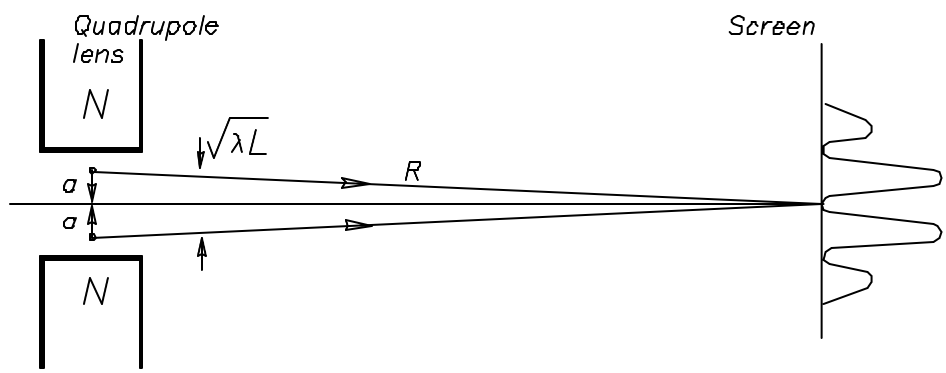


Figure 5: Destructive interference of radiation in a quadrupole lens from two particles.

The size of coherence $\sqrt{\lambda\,2ML}$, where M – is a number of periods, is bigger than the transverse dimension of the beam

$$\sqrt{\lambda 2ML} \geq \sqrt{\varepsilon\beta}\ , \tag{6}$$

where $\lambda \cong L(1+K^2+\gamma^2 J^2)/\gamma^2$ is the wavelength of the wiggler radiation, ε, β -- emittance and envelope function. So, radiation in *forward* direction occurs only due to fluctuation of the gravity center in a wiggler, similarly to the microwave pick up, described above. Using the electrical field representation (3), one can obtain the spectral density of photons as

$$\frac{dN_\gamma}{d(\omega/\omega_{\max})} \cong 4\pi\alpha\,\frac{K^2}{1+K^2}\cdot\frac{1}{2}\cdot\left[1-2\frac{\omega}{\omega_{\max}}+2\left(\frac{\omega}{\omega_{\max}}\right)^2\right], \tag{7}$$

[4] So the radiation and the particle are moving to the left side in Fig.1. At the wiggler position the dispersion must be a nonzero one.

where $\omega_{\max}=\dfrac{2\pi\gamma^2 c}{L\cdot(1+K^2)}$ is the maximal frequency in the spectrum corresponding to the first harmonic. In the spectrum region $\omega_{\max}\div\omega_{\max}\cdot(1-1/M)$, the number of photons becomes $N_\gamma\cong 4\alpha K^2/(1+K^2)$, with $K^2\sim x^2$. So the number of photons radiated by the particle with amplitude A in the relative bandwidth $\Delta f/f\cong 1/M$ becomes

$$N_\gamma\cong\alpha\left(\frac{A}{A_0}\right)^2=\alpha\frac{\varepsilon}{\varepsilon_0}, \tag{8}$$

where A_0 is a normalization amplitude, corresponding the beginning of the cooling, when K~0.7 and A – is an actual amplitude, ε_0 , ε -- are the corresponding emittances.

If N_S is the number of the particles in the bandwidth, then total number of photons, radiated by these particles due to fluctuation of center of gravity position becomes $\Delta N_\gamma\cong\alpha N_S\varepsilon/\varepsilon_0$. Total number of the photons, radiated by the beam with N particles, becomes

$$\Delta N_\gamma\cong\alpha N\varepsilon/\varepsilon_0. \tag{9}$$

Each of these photons has energy

$$\hbar\omega\cong\frac{2\pi\hbar c\gamma^2}{L\cdot(1+K^2+\gamma^2\vartheta^2)}. \tag{10}$$

So the total energy carried by these photons becomes $W_{pick}\cong\hbar\omega\,\Delta N_\gamma$.

One specifics of radiation from quadrupole wiggler is that the central frequency is a function of betatron amplitude, as one can see from (10). For maximal amplitude $K\cong 1$ and full change of frequencies cover full octave, as at the end $K\rightarrow 0$. This peculiarity can be used for cooling the peripheral particles only in addition to the fact that they radiate more.

TRANSVERSE KICK OVER ENERGY CHANGE

According to the idea of stochastic cooling the electromagnetic wave radiated must be applied to the particle in congruence with initial longitudinal distribution. This could be done if the photons and the beam are moving in the same direction. As the direct transverse kick is not possible under this condition, the transverse kick arranged through the *energy change* in the place, where dispersion of trajectory has some value $\eta(s)$. That means the amplitude of transverse betatron oscillations around new closed[5] orbit will change on $\Delta x\cong\eta(s)\dfrac{\Delta E}{E}$, where $\dfrac{\Delta E}{E}$ -- is a relative energy change, see Fig.3. According to previous considerations,

$$\Delta x\cong A/\sqrt{N_S}\cong\eta(s)\Delta E/E. \tag{11}$$

If the dipole wiggler has the same period and undulatority factor[6], then relative energy change will be

$$\frac{\Delta E}{E}\cong\frac{eE_\perp KLM}{E\gamma}=\frac{A}{\eta\sqrt{N_S}}, \tag{12}$$

[5] Corresponding to new energy.

[6] Typical magnetic field value in the wiggler is about 5-10*G* for the parameters under discussion. For quadrupole wiggler as a kicker, period must be the same. The fields in the quadrupole wiggler will be the same as in pick up wiggler for transformation matrix between pick up and kicker equals to ***-I***.

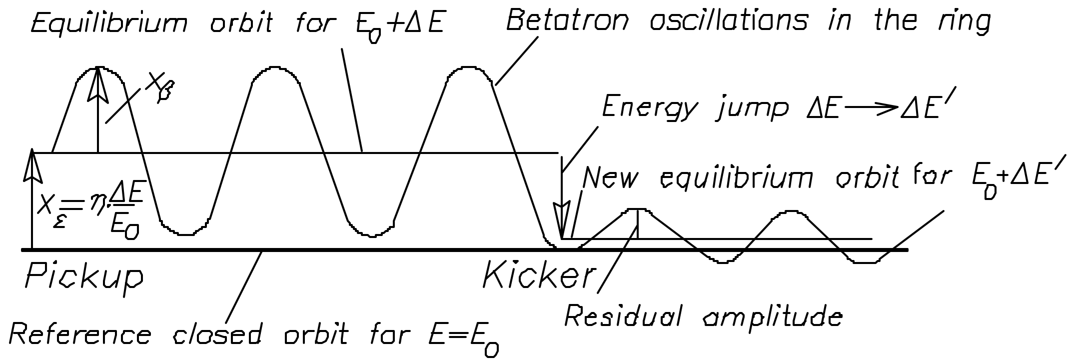


Figure 6: Transverse kick through the change of energy. This arrangement delivers cooling of longitudinal *and* transverse emittances simultaneously.

where $E_\perp$ -- is the electrical field strength in the electromagnetic wave after amplification. The last expression allows calculation

$$E_\perp \cong \frac{AEg}{eKLM\, h\sqrt{N_S}} \tag{13}$$

AMPLIFICATION

So the total energy delivered by optical amplifier must be

$$W_{kick} \cong \frac{1}{4p} E_\perp^2 V\,, \tag{14}$$

where V – is an effective volume. One can estimate

$$V \cong p\, A_0^2 l_B \cong p\, 2LMl\ \ l_B\,, \tag{15}$$

where transverse cross section chosen corresponding to the size of coherence (6). Combining (9), (16), (13), (14), (15) and estimating $h \cong A_0 \cdot (D E / E)^{-1}$ and $D f / f \cong 1/M$, one can obtain the amplification required from the optical amplifier as [2]

$$k = \sqrt{\frac{W_{kick}}{W_{pick}}} \cong \frac{g\, l_B}{r_0} \frac{\Delta E}{E} \frac{1}{N} \frac{\Delta f}{f} = \frac{g e_{\|}}{r_0} \frac{1}{N} \frac{\Delta f}{f}\ \ , \tag{16}$$

where $r_0 = e^2 / mc^2$, $g e_{\|} = g\, l_B \cdot (\Delta E / E)$ –is an *invariant longitudinal emittance*.

We would like to mention, that amplification, like described in (16) corresponds to G = 1 in (4). So the equation for the emittance change will be

$$\frac{de}{d\,n} = -\frac{e}{N_S}\,, \tag{17}$$

where $N_S \sim \frac{N}{D f} \frac{c}{l_B} \cong MN \frac{l}{l_B} \cong MNL \frac{l_B}{g^{\ 2}}$. This gives the cooling times $t \cong N_S T$ for a single cooling system. Cooling finishes when emittance reaches its final value like

$$e_{fin} \cong e_0 \frac{1}{a\ N_S}\,, \tag{18}$$

what corresponds to one photon in the coherence volume.

GRADIENT(QUADRUPOLE) WIGGLER AS A KICKER

In our considerations we used the dipole wiggler for the energy change. For the same reason the second *quadrupole* wiggler could be used as well. In this case one needs to have the beam optical transformation from pickup to kicker with magnification module equal to one. In this case one can avoid the necessity to control the frequency shift arising from changing the amplitude, and, hence, K parameter in (10). All formulas for optical amplification still valid, as the principle of operating through energy change in a wiggling motion accompanied by the amplified radiation remains the same. Philosophically speaking, this is a fully reversible picture if the particle in the kicker is following a reversed trajectory. This could be arranged by transforming matrix equal to minus unity. One can see, that in a quadrupole kicker particle will receive a kick, proportional it's current transverse position. So the particles in the bandwidth having opposite amplitudes in a pickup (and in kicker) will obtain the kicks of opposite polarities, what is necessary. Indeed, in a dipole wiggler all particles in a sample will obtain the same kick. So the particles, for example, with opposite displacement will be overheated. As one can see from (2), that particle's decrement arising only from interaction with it's self radiated field, but amplified only to the level, required to eliminate *average* [7] displacement in the pickup. Mathematically the action of quadrupole wiggler can be expressed as

$$x_i^{new} \cong x_i^{old} - x_i^{old}\cdot\left(G_Q\cdot\frac{\sum_{j=1}^{N_S} x_j^{old}}{N_s}\right) \to x_i^{old} - G_Q\cdot\frac{(x_i^{old})^2}{N_S} - G_Q\cdot x_i^{old}\cdot\frac{\sum_{j\neq i}^{N_S} x_j^{old}}{N_S},$$

Where G_Q is a normalized amplification. One can see from here, that the heating arising from the second term could be decreased significantly, as $G_Q\cdot x_i^{old}\cdot\frac{\sum_{j\neq i}^{N_S} x_j^{old}}{N_S} \to 0$.

BEAM PASS

The necessity to keep the bunch length unperturbed from pickup to the kicker within the level of the wavelength of the optical system is obvious. With the same accuracy the congruence of the light amplified and particle's beam must be arranged in the kicker as well.

However, the transverse kick to the particles moved with relativistic speed co-directionally with the light, could be arranged through the energy change in the place of the particle's orbit with nonzero dispersion function only. This requires some functional relations between parameters of beam optics on the way between pickup and kicker. So, simultaneous cooling of transverse and longitudinal emittances yield some threshold value in the invariant emittance level as $\gamma\varepsilon_x^{th} \cong \lambda\gamma\cdot(\Delta E/E)$, where $\gamma\varepsilon_x$ --is the invariant radial emittance, λ -- is a central wavelength of the optical amplifier (and all system), $\gamma = E/mc^2$, $\Delta E/E$ -- is the initial relative energy spread in the beam. The above value is of the order $\gamma\varepsilon_x^{th} \cong 3\div 5\cdot 10^{-4}\ cm\cdot rad$ for $\lambda \cong 1\mu m$, what slightly exceeds the typical value for the damping ring for Next Generation of Linear Collider, $\gamma\varepsilon_x \cong 3\cdot 10^{-4} cm\cdot rad$ [3].

In [4] some developments in OSC method made in attempt to eliminate dependence of initial emittance.

---

[7] For all sample, or for all particles within the bandwidth.

From the other hand, the investigations of all possible types of beam coolers as a damping ring, reside so called Kayak-paddle type racetrack[5] as a promising candidate for a machine with emittance of the order even $g\,e_x \cong 3\cdot 10^{-8}\,cm\cdot rad$. The reason for this kind of investigations was encouraged by desire to come to degeneration of the Fermi gas, what are the particles of electron or positron beam [5]. The emittance like just mentioned above, simply eliminates the troubles about threshold emittance at all.

Quadrupole wiggler as a kicker also rejects this problem.

As one needs to cool both longitudinal and transverse emittance simultaneously, the system with dipole wiggler as a second kicker must satisfy some certain conditions.

Let us represent the transverse position of the particle as a sum of two eigenvectors called sine-like *S(s)* and cosine-like *C*(*s*) trajectories. Basically these vectors describe the trajectory with initial conditions like $x_0'(s_0)=0;\ x(s,s_0)=x_0\cdot C(s,s_0)$ and $x_0(s_0)=0;\ x(s,s_0)=x_0'\cdot S(s,s_0)$, where s_0 corresponds to the longitudinal position of pick up. So transverse position of the particle becomes [11]

$$x(s)=x_0\cdot C(s,s_0)+x_0'\cdot S(s,s_0)+D(s,s_0)\cdot(\Delta E/E)\,, \tag{19}$$

where dispersion $D(s,s_0)$, describes the trajectory for the particle with different value of energy and zero initial conditions for coordinate and it's derivative. We also omitted index *i*, numbering each particle. Basically

$$D(s,s_0)=-S(s,s_0)\cdot\int_{s_0}^{s}\frac{C(t)}{r(t)}dt-C(s,s_0)\cdot\int_{s_0}^{s}\frac{S(t)}{r(t)}dt\quad, \tag{20}$$

where $r(s)$ –is a current bending radius. In description of particle's location it is accepted to separate the coordinate arisen from the betatron motion itself and from the energy offset like

$$x=x_b+h\cdot(D E/E)\,,$$

where function $h(s)$ was used earlier, eq. (11). So (19) can be expanded

$$x(s)=x_{0b}\cdot C(s,s_0)+x_{0b}'\cdot S(s,s_0)+[h_0\cdot C(s,s_0)+h'_0\cdot S(s,s_0)+D(s,s_0)]\frac{\Delta E}{E}\ . \tag{21}$$

One can see from (20), (21), that in absence of magnets, $r\rightarrow\infty$, dispersion is absent also. Expression (21) can be also rewritten as

$$x^{kicker}=x_b^{kick}+[h^{kick}+D^{kick}]\frac{D E}{E}\,, \tag{22}$$

while the same for the pickup
$$x^{pick}=x_b^{pick}+h^{pick}\frac{D E}{E} \tag{23}$$

Simultaneous cooling requires, that $h^{kick}+D^{kick}=-h^{pick}$, so $D^{kick}\cong -2\cdot h^{pick}$ and at least one of the integrals in (20) must be not equal to zero. Meanwhile the longitudinal motion of the particle with nonzero initial conditions and the energy offset could be described by [3]

$$\Delta l=-\int_{s_0}^{s}\frac{x}{r}dt=-x_0\int_{s_0}^{s}\frac{C}{r}dt-x_0'\int_{s_0}^{s}\frac{S}{r}dt-\frac{\Delta E}{E}\int_{s_0}^{s}\frac{D}{r}dt=-x_0M_{51}(s,s_0)-x_0'M_{52}(s,s_0)-\frac{\Delta E}{E}M_{56}(s,s_0)\,. \tag{24}$$

For example, $\grave{\mathrm{0}}\frac{S}{r}dt \; = M_{56} = g\,\frac{¶l}{¶g}\;@\;2h_{\,0}$, then the lengthening becomes

$$D\;l \cong -x_0' 2h_{\,0} = -[x_{0b}' + h_0' \cdot (D\;E\,/\,E)] \cdot 2h_{\,0}\,. \tag{25}$$

For the ring under consideration it is easier to have the dispersion function at the kicker positive and the same as in the pick up. In this case one needs to have $x^{pick} = -x^{kick}$ [8]. One can see from (21), that in this case $h^{kick} = -h^{pick}$, because of same C and S –functions involved in transformation for x and h. In this case one need to keep $D^{kick} \cong 2 \cdot h^{\;pick}$. The absolute value for the lengthening remains the same. As we suggested, that $\sqrt{e\,B} \approx h\frac{D\,E}{E}$, so the lengthening can be expressed as

$$D\;l \cong -x_0' 2h_{\,0} \cong -\left[\sqrt{\frac{e}{B}} + h'_{\;0}\cdot\frac{D\;E}{E}\right] \cdot 2 \cdot \frac{\sqrt{e\,B}}{(D\;E\,/\,E)} = -2\left[e \cdot (D\;E\,/\,E)^{-1} + h'_{\;0}\cdot\sqrt{e\,B}\right]. \tag{26}$$

As the lengthening must remain below the wavelength of radiation, $\left|D\;l\right| \le l$, (26) yields the threshold emittance value $ge_{\,th} \cong g\,\cdot\,l\cdot\frac{\Delta E}{E}$ if the derivative of h-function in the pick up wiggler is chosen close to zero $h_0' \cong 0$. The last expression rewritten for *invariant* emittance becomes

$$ge_{\,th} \cong l \cdot g\frac{\Delta E}{E} \cong \frac{l}{l_B}\cdot g\,l_B\,\frac{\Delta E}{E} = \frac{l}{l_B}\cdot g\,e_{\|}\;, \tag{27}$$

where again $ge_{\|} = g l_B \cdot (\Delta E / E)$ –is an *invariant longitudinal emittance*. For $l \approx 1mm = 10^{-4}\,cm$, $D\;E\,/\,E \cong 10^{-3}$, $g \cong 10^3$ (500 *MeV*), numerical value for the threshold emittance becomes $g\;e_{\;th} \cong 10^{-4} cm \cdot rad$. The damping ring for linear collider has typical values of emittances $g\;e_{\;x} \cong 3 \cdot 10^{-4} cm \cdot rad$, $g\;e_{\;yx} \cong 3 \cdot 10^{-6} cm \cdot rad$ [3]. For Kayak-Paddle type ring [5] equilibrium emittances can reach $g\;e_{\;x} \cong 1 \cdot 10^{-7} cm \cdot rad$, $g\;e_{\;yx} \cong 1 \cdot 10^{-9} cm \cdot rad$, so there is no limitations for this type of ring at all.

One can also arrange the cooling system so that it is able to cool only the particles with lower derivatives of trajectory that satisfy the condition (25). For example one can screen the radiation from some parts of the beam to avoid heating the core by peripheral parts of emittance. One positive property of quadrupole wiggler as a pickup is that the central parts of the beam, what have higher values of slope x_0' are moving *closer* to the center of the lens and, hence, radiate less. One can see also, that the particles with higher slope x_0' in the pick up (and having maximal lengthening), will be at the center in the quadrupole wiggler used as a kicker if transformation matrix ***-I*** (or for cosine trajectory $C(s \to kick) = -1$). That will reduce the heating from interaction with amplified radiation.

General conclusion is that the threshold emittance is not a problem for OSC in many interesting cases. The method described in [7] uses two dipole wigglers for manipulation with the beam emittance. As the dipole wiggler *does not* give any information about beam emittance, this information acquired from (24). So the lengthening of the trajectory, proportional to the beam emittance (coordinate or derivative) is used to put the selected particle onto proper phase of amplified radiation. This changes the energy of the particle in accordance with its initial emittance. The difficulty of this method is obvious: cooled

[8] This corresponds to the betatron transformation matrix equal to ***-I***, or *C(at kicker)*= -1.

beam radiates the same amount of radiation as initial one. So the amplification must be changed so the cooling process runs steady. Indeed, in quadrupole wiggler cooled particles have lower amplitudes and, hence, move in lower field (zero at the axis) and radiate less. The same done in enhanced optical cooling.

As far as de-bunching due to quantum fluctuations, this effect can be estimated as the following. Total number of quants radiated in a bending magnet can be estimated as

$$N_{\gamma m} \cong \alpha \gamma \varphi \,, \tag{28}$$

where $\alpha = 1/137$ is a fine structure constant, φ is the bending angle. As each quant carries out energy $\sim \hbar\omega \cong \frac{3}{2}\hbar c \gamma^3 / \rho$, where ρ stands for the bending radius, then the energy spread induced by bend will come to $\Delta E \cong \hbar\omega\sqrt{N_{\gamma m}}$ and

$$\left(\frac{\Delta E}{E}\right)^2 \cong \tfrac{9}{4}\alpha\gamma\varphi \frac{\hbar^2 c^2}{e^4}\gamma^6 \frac{e^4}{\rho^2 (mc^2\gamma)^2} \cong \tfrac{9}{4}\frac{r_0^2}{\alpha}\gamma^5 \frac{\varphi}{\rho^2}\,, \tag{29}$$

which is negligible in our case. Dispersion after passage the angle φ in a magnet goes to be $D(\varphi) \cong \rho \cdot (1 - Cos(\varphi))$, so the matrix element M_{56} can be estimated as

$$M_{56}(s,s_0) = \gamma \frac{\partial\lambda}{\partial\gamma} \cong \int_{s0}^{s} \frac{D}{\rho} dt \cong \int_0^{\varphi} D(\varphi) d\varphi \cong \rho \cdot (\varphi - Sin(\varphi)) \cong \frac{\rho\varphi^3}{3!} \tag{30}$$

So total estimation for the spread of the bunch length according to (24) comes to

$$(\Delta l)^2 \cong \left(\frac{\Delta E}{E}\right)^2 M_{56}^2(s,s_0) \cong \tfrac{9}{144}\frac{r_0^2}{\alpha}\gamma^5\varphi^7 \cong r_0^2\gamma^5\varphi^7\,. \tag{31}$$

Exact formula for the spread of the bunch length, after averaging over spectrum of SR, obtained in [6]. It contains factor $\frac{55}{24\sqrt{3}}\frac{1}{126} \cong 0.01$ instead of ours $\frac{9}{144} \cong 0.06$. So if one fixes the spread of length, the maximal possible bending angle in a magnet comes to

$$\varphi \le \sqrt[7]{\frac{(\Delta l)^2}{r_0^2\gamma^5}} \tag{32}$$

For operation at wavelength of optical amplifier around $\lambda \approx 1mm$, the (Δl) must be a fraction of wavelength λ. For $\Delta l \cong 0.1\lambda$ the bending angle according to (32) goes to be 57.6 degrees, which is close to 60^o in symmetric three magnet bend, although two edge magnets can be made slightly different; or the energy could be shifted to a bit lower value (~193 MeV).

RING

Series of damping rings suitable for the OSC test represented in Fig. 3 below.
Optical amplifier installed at stabilized platform. For the fine phase adjustments one can use the dual prism system.

Initial phase adjustments made by movement the table (trombone). Optical telescopes at both ways used for proper conjunction the radiation from the wiggler and amplifier.
The big path difference between the light and beam is possible in this cooler, what makes it possible to neutralize delay in materials of amplifier and windows easily. Active media three-four stages with Dye

could be placed on the table symmetrically. The necessary optics for optical pumping also installed on the same table.

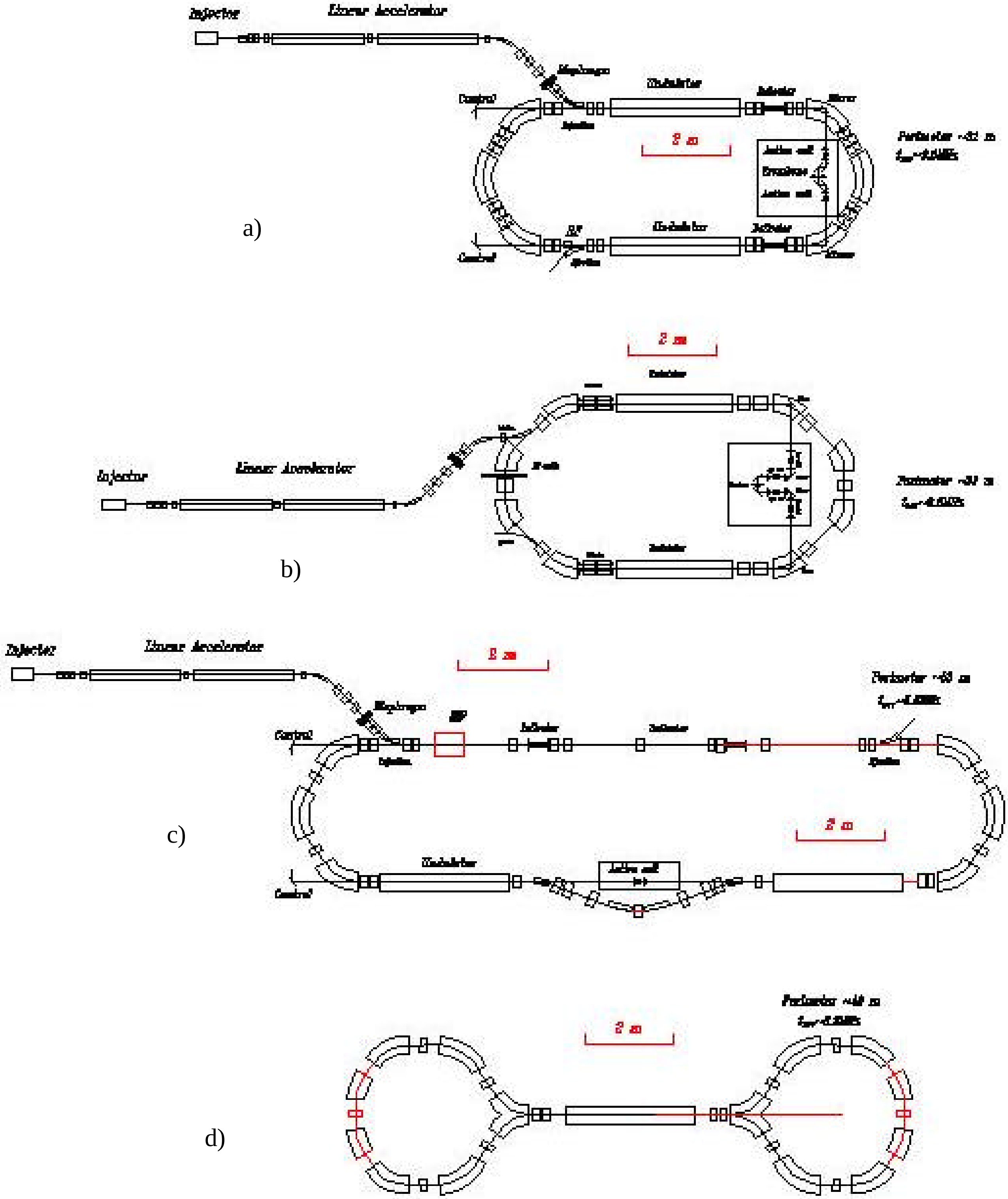


Figure 7: Possible ring shapes. Ring with shape c) does not require mirrors; the bypass can be arranged in *vertical* direction. This allows generation of local vertical dispersion in second wiggler, if then the dispersion is removed with the pair of magnets before the horizontal bend. It requires more space, however. Rings a), b) ask for less space, although OSC in these rings require mirrors. Generation of vertical dispersion also requires some space and vertical bends.

Diaphragm serves for cut the particles with large deflections from central ones, or simply speaking for lowering emittance at injection (this procedure keeps the phase density the same).

One important peculiarity is the beam temperature expressed in terms of beam parameters. The transverse momentum is invariant and the transverse kinetic energy is

$$\frac{p_x^2}{2m}+\frac{p_y^2}{2m}\cong\frac{1}{2m}m^2c^2\gamma^2\frac{\varepsilon_x}{\beta_x}+\frac{1}{2m}m^2c^2\gamma^2\frac{\varepsilon_y}{\beta_y}. \tag{28}$$

Full energy is a sum of kinetic and potential energy of motion in a focusing system. According to the virial theorem for harmonic oscillator the average potential energy equals to the kinetic one. So the temperature of the beam could be represented as the following

$$\tfrac{3}{2}Nk_BT\cong N\cdot mc^2\gamma\left[\frac{\gamma\varepsilon_x}{\beta_x}+\frac{\gamma\varepsilon_y}{\beta_y}+\gamma\left(\frac{1}{\gamma^2}-\left\langle\frac{D^2}{\beta_x}\right\rangle\right)\left(\frac{\Delta p_\parallel}{p_0}\right)^2\right] \tag{29}$$

One can easily recognized terms in brackets as Piwinski invariant. So this invariant is nothing else but temperature. Basically the input to the temperature from longitudinal motion connected with the fact that longitudinal mass is

$$\frac{1}{m_\parallel}=\frac{1}{m\gamma}\left(\frac{1}{\gamma^2}-\alpha\right).$$

In free space dispersion is zero D=0 and formula for temperature comes to its usual appearance

$$\tfrac{3}{2}Nk_BT\cong N\cdot mc^2\gamma\left[\frac{\gamma\varepsilon_x}{\beta_x}+\frac{\gamma\varepsilon_y}{\beta_y}+\frac{1}{\gamma}\left(\frac{\Delta p_\parallel}{p_0}\right)^2\right] \tag{30}$$

One can see that the longitudinal temperature has negative value above transient energy. Namely this circumstance yields the beam heating due to intra-beam scattering. The IBS is nothing else but thermalization; in mostly rings longitudinal degree of freedom has negative temperature and stationary distribution is not possible. So that is why we are suggesting operation with negative α value, where longitudinal mass is positive, hence longitudinal temperature is positive too.

These temperature considerations are important in a view of inevitable Intra Beam Scattering (IBS). From our point of view the role of IBS might be useful if the ring tuned to operate with positive longitudinal mass. *In this case the IBS yields thermalization to equilibrium and one can cool the one degree of freedom only.* We just remind here, that limitation for the threshold emittance (27) arisen from requirements of simultaneous cooling in transverse and longitudinal directions. If the cooling is going for one degree of freedom there is no such limitations at all.

Ring from Fig. 8 a) can be considered as a primer candidate for the OSC test if space is limited. It requires area ~4x7 m^2. Although, if there is no limitation in area, the ring from Fig. 8 c) becomes preferable. It requires ~ 4x12m^2, i.e. ~twice the first one.

In case of commutation for gradient wiggler, it will be sensitive to radial oscillations generating the electromagnetic wave with vertical polarization. Vertical polarization is better for mirrors. Vertical dispersion can be generated with pair of magnets and removed before horizontal bend with additional pair of magnets if necessary. Parameters of the ring in one particular configuration are represented below. For wider possibilities the up-down (in Fig. 8) symmetry of the ring can be broken. This allows having in one straight (wiggler) section zero dispersion and nonzero one in opposing wiggler section.

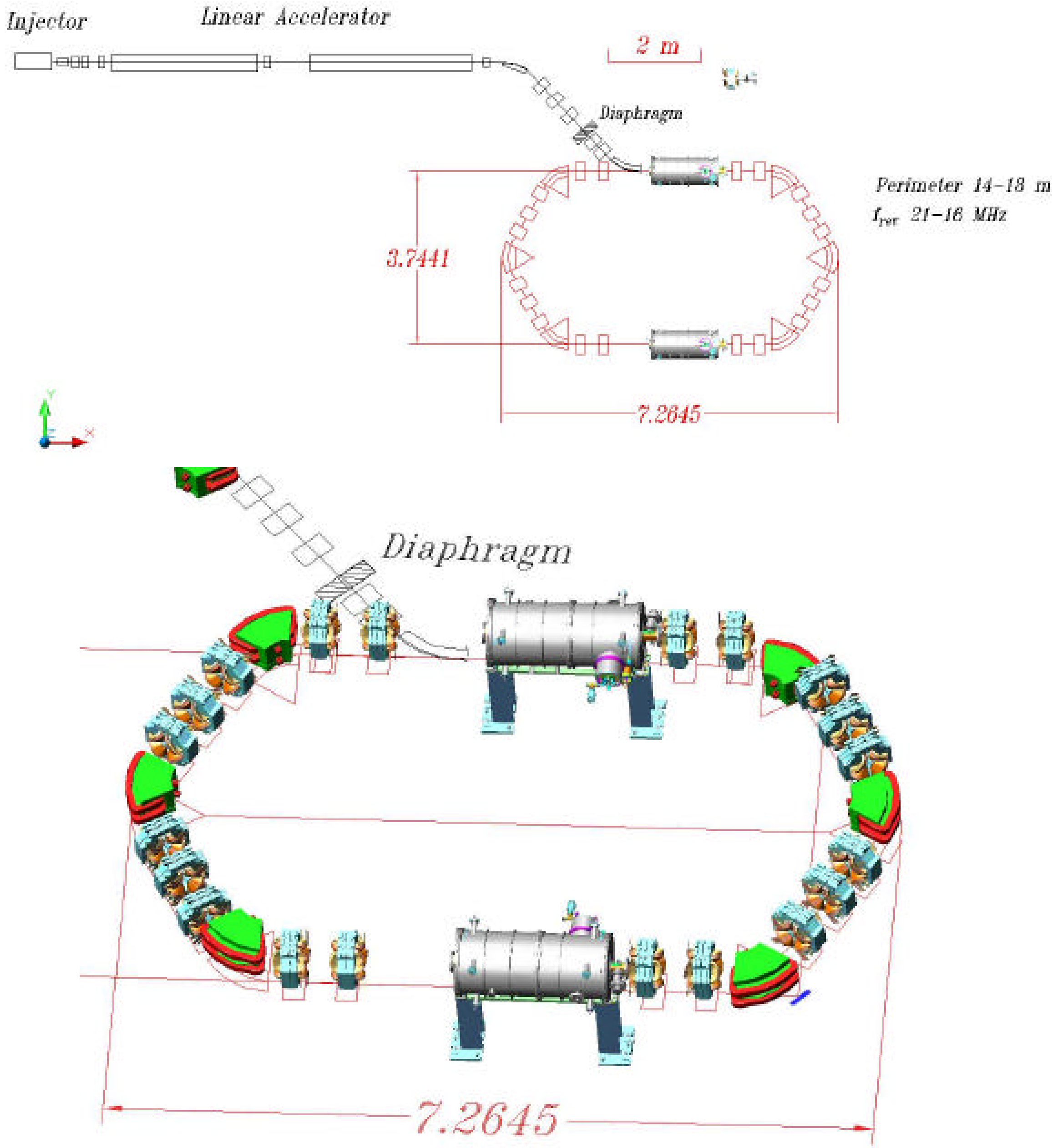


Figure 8: Ring suggested. RF cavity, sextupoles, vacuum chamber, inflector are not shown here. The straight sections with wigglers can be extended up to 4 meters. Wigglers are shown rotated 90^{o} along longitudinal axis so that magnetic field has *horizontal* polarization. Ring can be assembled so that it's plane oriented vertically. In this case wigglers have usual (vertical) field polarization.

```
# THE LENGTH L1 CAN BE VARIED UP TO 4.5 METERS GIVING STAIGHT SECTIONS
#UP TO~9 M; positive k[1/m^2] means y focusing
P=(
L1 = DR 1.34  Q1 = QU 0.2 -6.877      L2 = DR 0.3 Q2 = QU 0.2   11.7339
L3 = DR 0.15  M1 = DI 0.6 0.573 0 0 0 L4 = DR 0.1 Q3 = QU 0.25 -14.995
L5 = DR 0.35  Q4 = QU 0.25 13.0296    L6 = DR 0.2 Q5 = QU 0.25 -10.7434
L7 = DR 0.224 M2 = DI 0.6 0.573 7.18 0 0
L7 Q5 L6 Q4 L5 Q3 L4 M1 L3 Q2 L2 Q1 L1) P
```

In proton machines the beam exists well below critical energy.
For the reference we remind the values of more important integrals in the following Fig.9 and Fig.10

$$I_1[m]=\oint\left(D_x/r_x+D_y/r_y\right)ds\,;\quad I_2[m]=\oint\left(1/r_x^2+1/r_y^2\right)ds\,;\quad I_3[m^{-2}]=\oint\left(1/\left|r_x\right|^3+1/\left|r_y\right|^3\right)ds$$

$$I_{4x,y}[m^{-1}]=\oint\frac{D_{x,y}}{r_{x,y}^3}\left(1\pm 2\,r_{x,y}^2k\right)ds\,;\; I_{5x,y}[m^{-1}]=\oint\frac{H_{x,y}}{\left|r_x\right|^3}ds\,;\; I_{6x,y}[m^{-1}]=\oint k^2D_{x,y}^2ds$$

$$H_{x,y}=b_{x,y}D_{x,y}'^2+2a_{x,y}D_{x,y}D_{x,y}'+g_{x,y}D_{x,y}^2\,;\quad k=(\partial B_y/\partial x)/(BR)$$

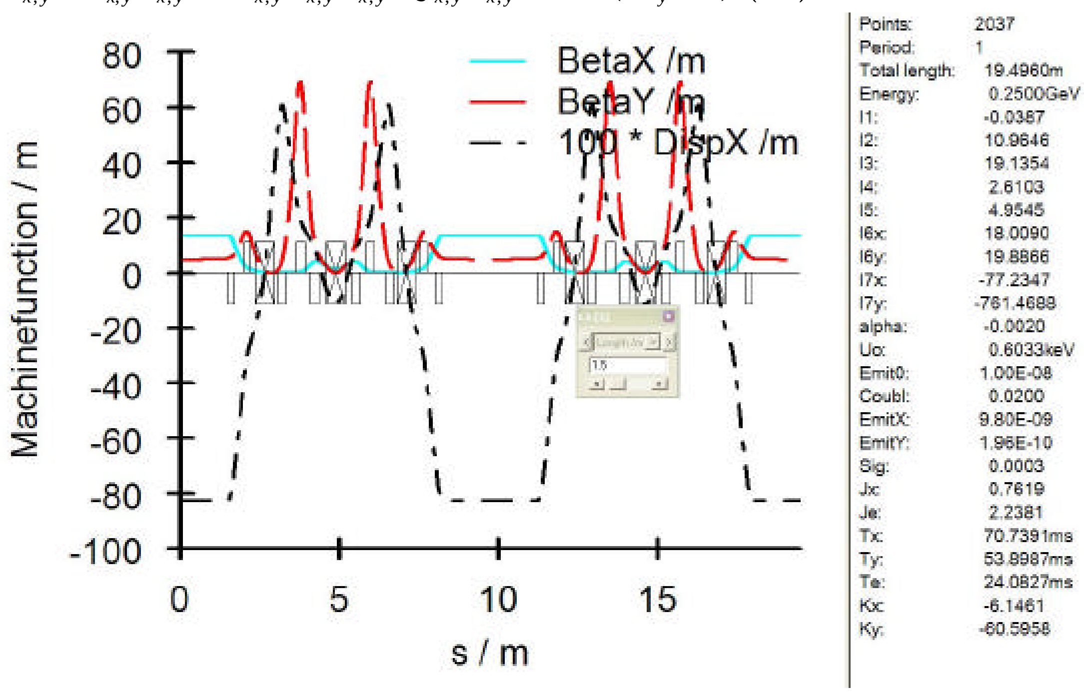


Figure 9: Machine functions for DR with negative alpha.

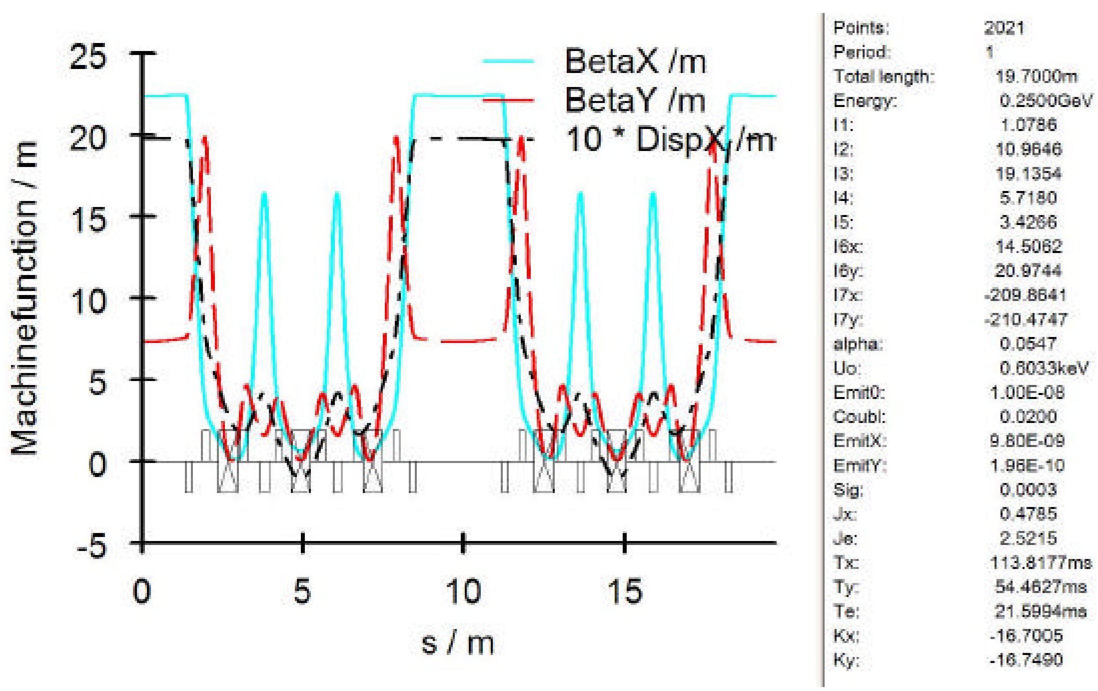


Figure 10: Machine functions for DR with positive alpha.

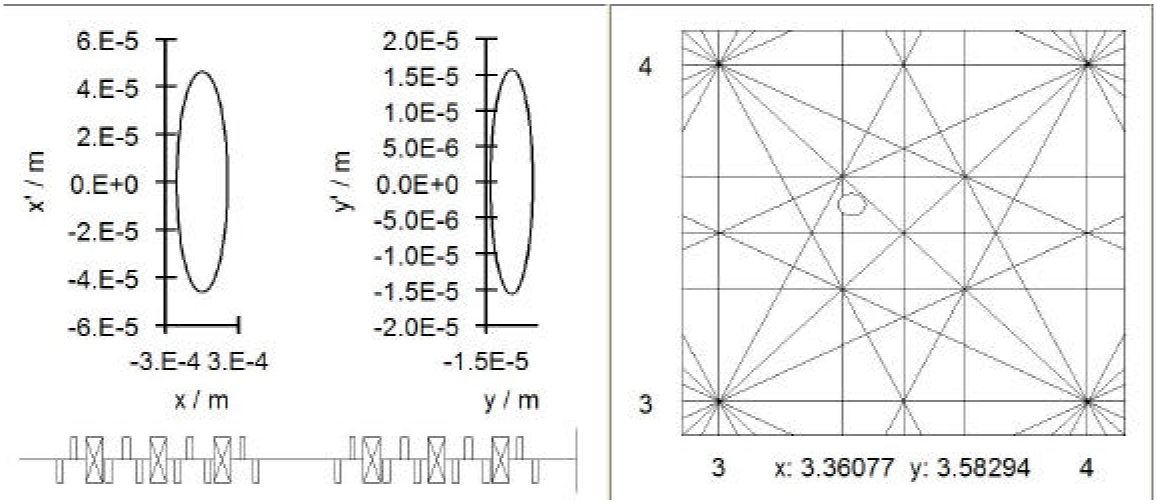


Figure 11: Tune plane and phase ellipses.

One interesting possibility we would like to mention here is that the ring can be assembled so, that the plane of the ring oriented vertically, see Fig.12. This allows usage of SC wigglers installed vertically and saves significant amount of room.

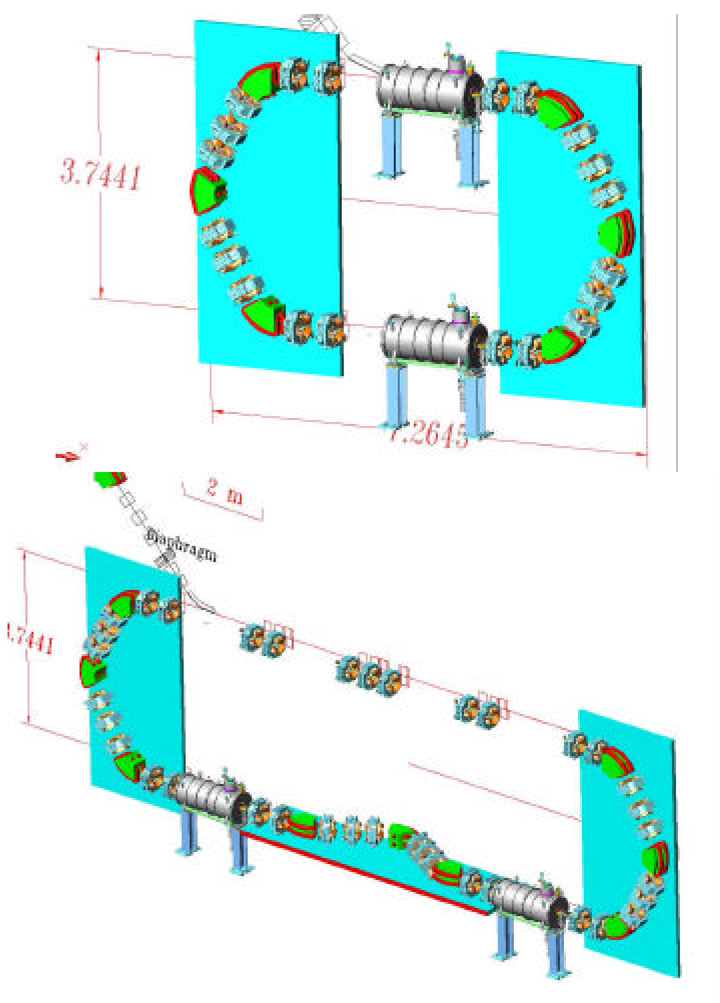


Figure 12: Vertical orientation of the ring's plane.

Ring from Fig.7 c) can be also oriented in vertical plane. In this case two wigglers can stand on the same floor, Fig. 12.

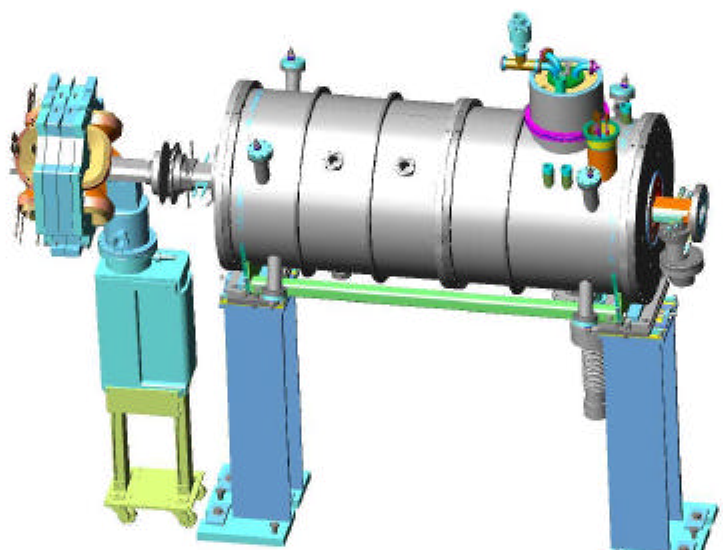
The magnets and lenses can be scaled also, so the ring might fit practically in any place. Typically the experimental halls have lot of free space in vertical direction.

WIGGLER

Energy of the cooler needs be keep at the low level because of quantum excitations. The energy of the beam and the period of undulator restrained by the formula

$$l_u \cong 2g^2 l/(1+K^2/2)$$

Undulator available from Cornell has period l_u=0.4 m what brings resonant energy to 273.8 MeV for the optical amplifier with central wavelength $l \approx \sim 1\ mm$ and K~1. More hard radiation from the dipole wigglers installed in straight section has different wavelength and does not interfere with optical amplifier operating at lower wavelength around $1 mm$. Lower energy of the beam required wiggler with shorter period according to~ $1/g^2$.




CORNELL

Wigglers give CESR a new charmed life

Figure 13: SC wiggler available from Cornell.

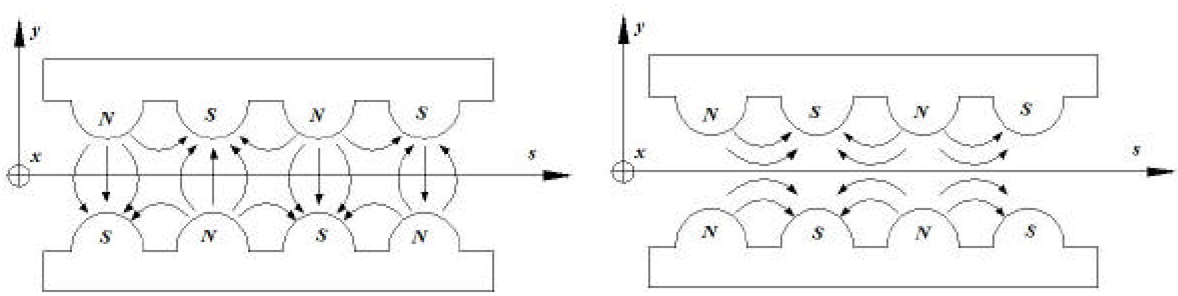


Figure 14: Dipole wiggler, at the left and gradient undulator/wiggler, at the right. Beam is moving along s –axis. In normal to the drawing's plane direction (x –direction), the poles are much wider, than the period in s direction.

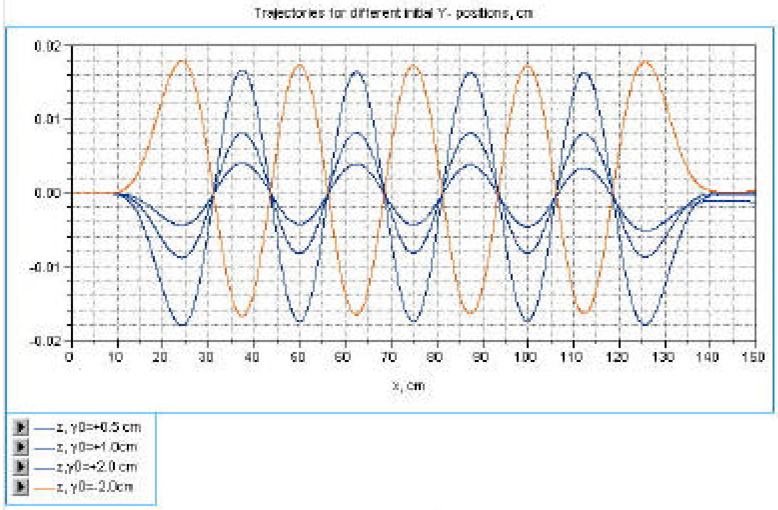


Figure 15: Trajectories in a gradient undulator. 11-pole wiggler is taken for this example.

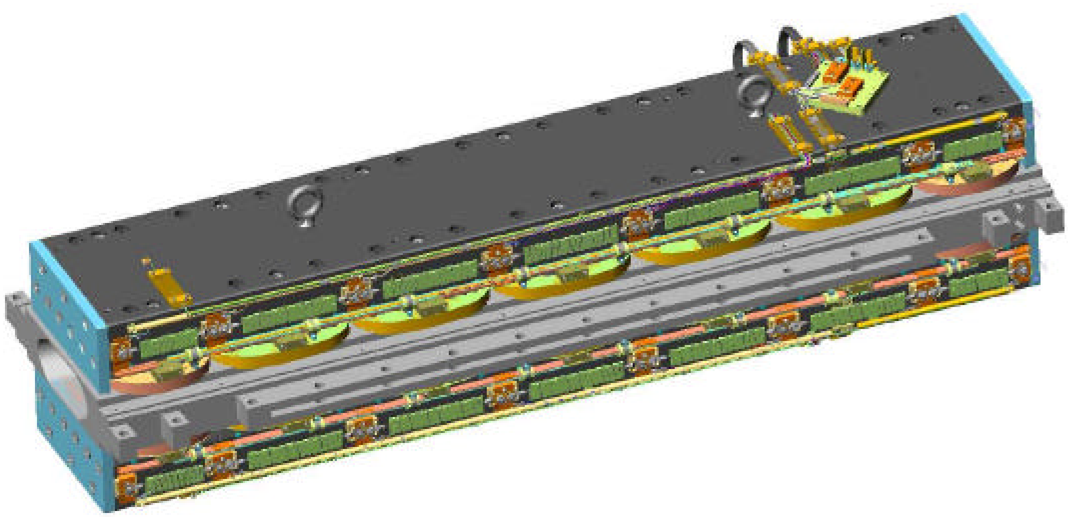


Figure 16: Cold mass of the wiggler. Upper and lower halves connected by two SC wires, so the polarity in all lower (upper) poles can be swapped by reconnecting of these two wires. Upper and lower halves each are feed in series.

The cold mass represented in Fig.16, has two halves. Each of these halves has independent Helium vessels, joint through the side tubing. Once again, in the mode with gradient field it is desirable to have strong gradient, so in principle one needs to keep the possibility of operation with liquid Helium open. Operation of wiggler rotated 90^o degrees requires some minor modifications. More radical might be vertical installation of the ring, Fig. 12.

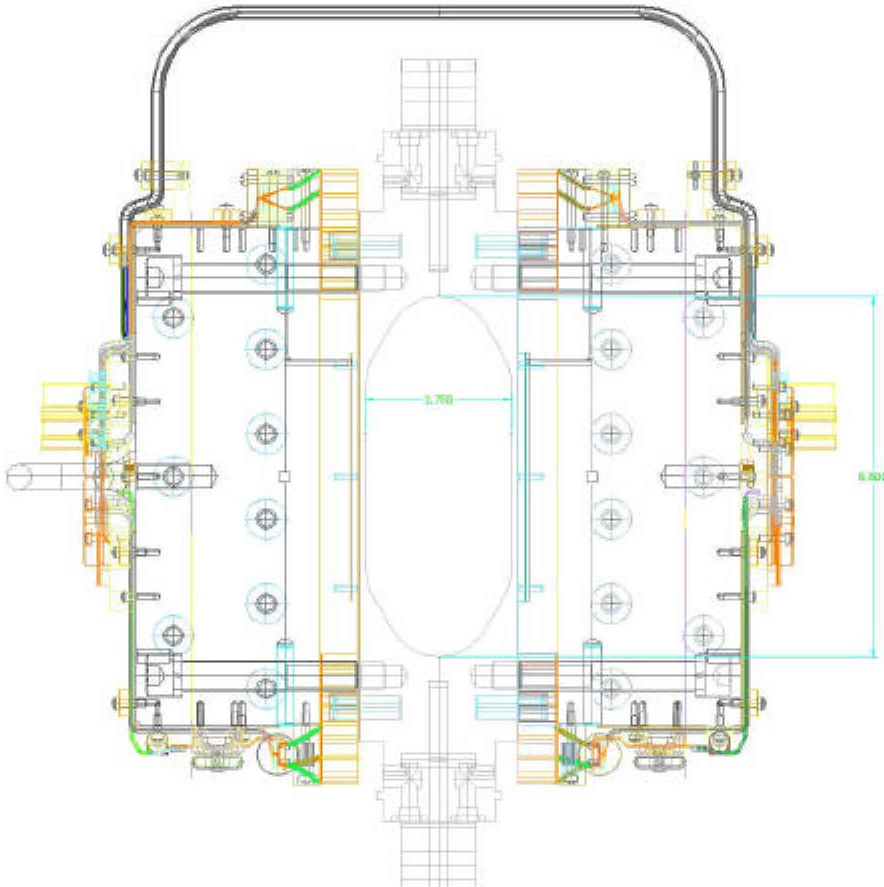


Figure 16: CESR's wiggler cold mass cross section. Two of such wigglers could be donated by Cornell to the project. The wiggler cold mass is shown rotated to 90^o along longitudinal axis, so it could be feed as *gradient wiggler* sensitive to radial displacement (radial emittance cooling). The gap 2¾ is big enough for accommodation the vacuum chamber of the ring.

The wiggler available from Cornell has 5 regular poles plus two ~half strong, brining the number of periods to~2.5. Magnetic field value for I=140A is about 2T. For K=1 and ?$_u$=0.4 m, the field value required $B = K/93.4/l_u \cong 1/93.4/0.4 \cong 0.027$ T i.e. ~270 Gauss. This requires $I = 0.027/2.1 \times 140 \approx 1.8A$ i.e. this wiggler can work without helium cooling, i.e. as usual room temperature magnet. As the wire has diameter ~0.7 mm, then the current density comes to $j \cong I \cdot 4/\pi d^2 \cong 1.8 \cdot 4/3.14/0.5 \cong 4.5 A/mm^2$. In reality the field will be better, as is this region of feeding currents the iron is not saturated. Only utilization of this wiggler in proton or ion machine will require liquid Helium. Although it might be required in gradient wiggler mode.

Upper and lower pole plates feed in series, so changing mode of operation from usual into gradient is just by flipping commutation of two wires in one plate. With minor modification the wiggler can run with Helium even when it is rotated on 90^o.

One backup possibility one can keep in mind is that fabrication of wiggler could be done from scratch. As this wiggler will be a room-temperature one it will be not expensive, compared with the rest equipment. In this case the number of periods could be increased, and additional freedom in choice of period appears (for number of periods also).

OPTICAL AMPLIFIER

Formulas (13), (14), (16) give an idea about amplification required and the power contained in the laser flash. Two examples considered below use these formulas.

Example 1.

For $N \cong 10^{10}$, $l_B \cong 15cm$, $M = 2.5$, $(\Delta E/E) \cong 10^{-3}$, $\gamma \cong 500$ (250 *MeV*), $\lambda \cong 1\mu m$, optical amplifier must be able to have amplification about 300, peak power about 5 *kW*, average power about 25 *W* with repetition rate f of the order of 10 *MHz*. Number of the particles in the bandwidth $N_S \cong 3\cdot 10^5$ defines the number of the turns and the damping time $\tau_C \cong N_S / f \cong 30ms$. Emittance reduction $\varepsilon_f / \varepsilon_0 \cong 1/\alpha N_S \cong 10^{-3}$.

Example 2.

For $N \cong 10^8$, $l_B \cong 5cm$, $M = 2.5$, $(\Delta E/E) \cong 10^{-4}$, $\gamma \cong 500$ (250 *MeV*), $\lambda \cong 1\mu m$, optical amplifier must be able to have amplification about 100, peak power about 225 *W*, average power about 0.075 *W* with repetition rate of 2 *MHz*. Number of the particles in the bandwidth $N_S \cong 2\cdot 10^3$ defines the number of the turns and the damping time $\tau_C \cong N_S / f \cong 1ms$. Emittance reduction $\varepsilon_f / \varepsilon_0 \cong 1/\alpha N_S \cong 7\cdot 10^{-2}$.

Optical amplifier needs to be done with lowest phase distortion and have minimal time delay.

Dye amplifier

For Dye amplifier with Rh6J, the operating wavelength remains within $\lambda \approx 340 \div 540 nm$. Life time of the states excited is $\tau_L \cong 5ns$, absorption cross section $\sigma_{01} \cong 2\cdot 10^{-16} \div 4\cdot 10^{-16}\, cm^2$, density of Dye molecules $n_0 \approx 10^{17}\, cm^{-3}$. The last numbers give for absorption length value $l_{ab} \cong \dfrac{1}{n_0 \sigma_{01}} \cong 0.05 \div 0.1 cm$.

So the pumping area could be estimated as $S_{pump} \cong \lambda \cdot l_{ab} \cong 5\cdot 10^{-6}\, cm^2$.

Saturation power density could be calculated as $P_{sat} \cong \dfrac{\hbar\omega\, n_0 l_{ab}}{\tau_L} \le 100\, kW/cm^2$ and the power of radiation required to pump the dye becomes $I = P_{sat}\cdot S_{pump} \cong 0.5\,W$/stage only. The pumping time required $\tau_{ump} \cong \dfrac{\hbar\omega\, S_{pump} n_0 l_{ab}}{I} \cong 0.15\, ns$. For pumping the Nitrogen laser with $\lambda = 337nm$ could be used here as well as Xe laser with $\lambda = 308nm$ or $\lambda = 248nm$. Amplification can be calculated as $k \cong \exp[\sigma_{10}(\lambda)\cdot n_1 \cdot l] \approx 7$ for the length $l \cong l_{ab}$ and $n_1 \approx n_0$. So *three* stages can give $7^3 = 343$ in amplification. One can increase the length of amplification with transverse pumping, instead of longitudinal, supposed above. For this purpose a cylindrical lens can be used. This arrangement can reduce power density to $10\, kW/cm^2$, coming to amplification per stage about $k \cong 2\cdot 10^4$ [9].

Ti:Al$_2$O$_3$ amplifier

For Titanium Sapphire $\tau_L \cong 3.5\mu s$, $\sigma_{01}(490nm) \cong 10^{-19}\, cm^2$, $\sigma_{10}(790nm) \cong 3\cdot 10^{-19}\, cm^2$, $n_0 \approx 10^{20}\, cm^{-3}$. Absorption length value becomes $l_{ab} \cong 1/(n_0\sigma_{01}) \cong 0.1 cm$, the pumping area $S_{pump} \cong \lambda \cdot l_{ab} \cong 7\cdot 10^{-6}\, cm^2$. Saturation power density becomes $P_{sat} \cong \dfrac{\hbar\omega\, n_0 l_{ab}}{\tau_L} \le 1\, MW/cm^2$ and the power of radiation required to pump the Ti:Al$_2$O$_3$ becomes $I = P_{sat}\cdot S_{pump} \cong 7W$. Amplification can be calculated as $k \cong \exp[\sigma_{10}(\lambda)\cdot n_1 \cdot l] \approx \exp(\sigma_{10}/\sigma_{01}) \approx 20$ for the length $l \cong l_{ab}$. So *two* stages can give a resulting amplification up to $20^2 = 400$. For pumping the Argonne laser can be used here.

In conclusion of this section we can say, optical amplifier must be able to give amplification about 1000 a peak power about 1 *kW*, average power about 10*W*. Repetition rate must be of the order of few *MHz*. Some estimation made for Dye, Titanium Sapphire.

One interesting development of optical amplifier for OSC one can find in [15]. Here Optical Parametric Amplifier suggested to operate at ?=2*µm* brining energy of damping ring to $275/\sqrt{2} \cong 192\,MeV$, what is absolutely acceptable.

CONCLUSION

We conclude that there are no apparent limitations in testing OSC with relatively low investment in compact size ring. The profits might be significant, however, if applied to the proton machines or multi-charged ions. We suggested usage of wigglers with horizontal polarization of magnetic field. This allows easy switch to gradient wiggler mode. As the lengthening is not a function of bending radius (31), some revision of dimensions can be made towards making the bends more compact.

REFERENCES

[1] S. Van der Meer, Report CERN/ISR PO/73-31 (unpublished); D. Mohl, G. Petrucci, L. Thorndahl, S. Van der Meer, Phys. Rep. **58**, 73-119(1980).

[2] A.A. Mikhailichenko, M.S. Zolotorev, "*Optical Stochastic Cooling*", SLAC-PUB-6272, Jul. 1993. 11 pp., Published in Phys. Rev. Lett.71:4146-4149, 1993.

[3] A. Mikhailichenko, "*Damping ring for Testing the Optical Stochastic Cooling*", Published in EPAC 94, London, England, 27 Jun - 1 Jul 1994: Proceedings. Edited by V. Suller and Ch. Petit-Jean-Genaz. River Edge, N.J., World Scientific, 1994. vol. 2*, p. 1214-1216.

[4] E.G. Bessonov, A.A. Mikhailichenko, "*Fast Cooling of Heavy Ions in a Storage Ring*" , CLNS 01/1745, Cornell LEPP, 2001.

[5] A. Mikhailichenko, "*Damping Ring for Generation of Positroniums and Investigation of Fermi Degeneration in Moving Beams*", PAC 2003, Proceedings, TPPG007.

[6] A.N. Matvienko, O.A. Shevchenko, N.A.Vinokurov, "*Isochronous Bend for High Gain Ring FEL*", Proceedings of 2004 FEL Conference, pp. 629-632.

[7] M. Zolotorev, A. Zholents, Phys. Rev. E, **50**, 3087(1994).

[8] A. Mikhailichenko, "*On the physical limitation to the lowest emittance (toward colliding electron-positron crystalline beams*)", 7th Advanced Accelerator Concepts Workshop, 12-18 October 1996, Granlibakken Conference Center, Lake Tahoe, CA, AIP Conference Proceedings 398, p.294.

[9] K.-J. Kim, "*Analysis of OSC including Transverse Effects*", Particle Accelerator Conference, PAC95, Dallas, Texas, May 1-5, 1995.

[10] S.M. Kopylov *ea.*, "*Tunable Dye lasers and it's applications* ", in Russian, Moscow, Radio and Communications publishing, ISBN 5-256-00858-7, 1991.

[11] K. Steffen, "High Energy Beam optics", Interscience Pub., 1964.

[12] A.A. Mikhailichenko, "*Optical Stochastic Cooling and Requirements for a Laser Amplifier*". (Cornell U., LNS). CLNS-98-1539, Dec 1997. 14pp., Talk given at 20th International Conference on Lasers and Applications (Lasers '97), New Orleans, LA, 15-19 Dec 1997.

[13] A.Mikhailichenko, "*Characteristics of Gradient Undulator*", CBN 03-1, Cornell LEPP, February 18, 2003. See also a paper, presented at PAC2003, Portland, OR, May 12-16, as TPPG 005.

[14] E.G.Bessonov, M.V.Gorbunkov, A.A.Mikhailichenko, "*Enhanced Optical Cooling of Ion Beams for LHC*", EPAC 2006 Proceedings, TUPLS001, Edinburgh, UK, 2006.

[15] F.X.Kärtner, "*Amplifier Development for Optical Stochastic Cooling*", Research Laboratory for Electronics, LARP Meeting, Jefferson Port, October 25, 2006.