TO THE CALCULATION OF KICK BY INCOMING BUNCH IN VACUUM CHAMBER[1]

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Abstract

We are attracting attention to the fact that the vacuum chamber can manifest a significant screening effect for calculation of cross-kick from an oppositely moving bunch. The field, reduced by the chamber, demonstrates about the same *gradient* as the field in free space, however. This might induce a mistake in calculation of kick strength, if confirmation of kick formula is done by measurements of betatron tune shift in real machine, which is sensitive to the gradient only.

INTRODUCTION

For calculation of kick from counter-rotating bunch in damping ring, many authors use the field values, calculated for free space. If the size of the vacuum chamber is much bigger, than the distance between the beams–this approximation might be valid. Check of validity of this assumption needs to be done every time, however, taking into account specifics in shape of chamber and beams displacement.

Due to relativistic compression of magnetic field spread$\sim 1/g$ in longitudinal direction, its value can be calculated just as the static 2D field generated by instant current value as

$$|E_{\perp}| \sim H_{j} = I/2p\, r\,, \tag{1}$$

where $I(s\text{-}ct)$ stands for the local current value, r is the radial distance. Really, the field of the charge moving with constant velocity could be expressed by the following [1]

$$\vec{E} = \frac{e\vec{R}_t}{R_t^3}\frac{1}{g^2(1-b^2 Sin^2 J)^{3/2}}\,, \quad \vec{H} = \vec{b}\times\vec{E}\,, \tag{2}$$

where R_t –is the distance between the observation point and the present instant position of the charge (moment t), J –is an azimuthal angle to the observation point, b –is a speed of charge normalized to the speed of light. From the above formula one can obtain that

$$\vec{E}_{\perp} = \vec{e}_{\perp} g\frac{e}{R_t^2}, \quad \vec{E}_{\|} = \vec{e}_{\|}\frac{e}{g^2 R_t^2}, \quad H_j = g\frac{eb}{R_t^2}, \quad H_{\|} = 0, \tag{3}$$

where the first expression corresponds to $J \cong p/2$ and the second one to $J \cong 0$, $\vec{e}_{\perp}, \vec{e}_{\|}$ –are unit vectors in rectangular to the trajectory and parallel directions respectively.

Let us calculate the flux of rectangular component $\vec{E}_{\perp}$ of electrical field over a strip having the width $2R_t/g$ (see Fig.1 below). As the area of the strip is $2pR_t\cdot(2R_t/g)$, from (3) one can obtain [2]

$$Flux \cong \oint \vec{E}_{\perp} dS \cong \vec{E}_{\perp}\cdot 2p\,R_t\frac{2R_t}{g} = g\frac{e}{R_t^2}\cdot 2p\,R_t\frac{2R_t}{g} \cong 4p\,e\,. \tag{4}$$

This is exactly as it must be from the equation $div\vec{E} = 4p\,r$, where r –is a macroscopic charge density as $Flux \cong E_{\perp}2pr\cdot l_b = 4peN$ with formal substitution $l_b = 2R_t/g$. So in the laboratory

[1] Electronic version is available at http://www.lns.cornell.edu/public/CBN/2005/CBN05-14/cbn05-14.pdf.
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frame at the distances $r \geq l_b g / N$ one can use macroscopic definition of electric 2D field. The field at smaller transverse distances is much more complicated, however.

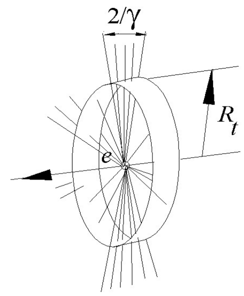

FIGURE 1: The strip around the relativistic charge used for integration of flux.

So that is why the 2D static formula (1) is valid here.

MODELING

We begin with test run made for round geometry. Well known solution is that the field in not dependent on surrounding tube radius, if the current is running through the center.

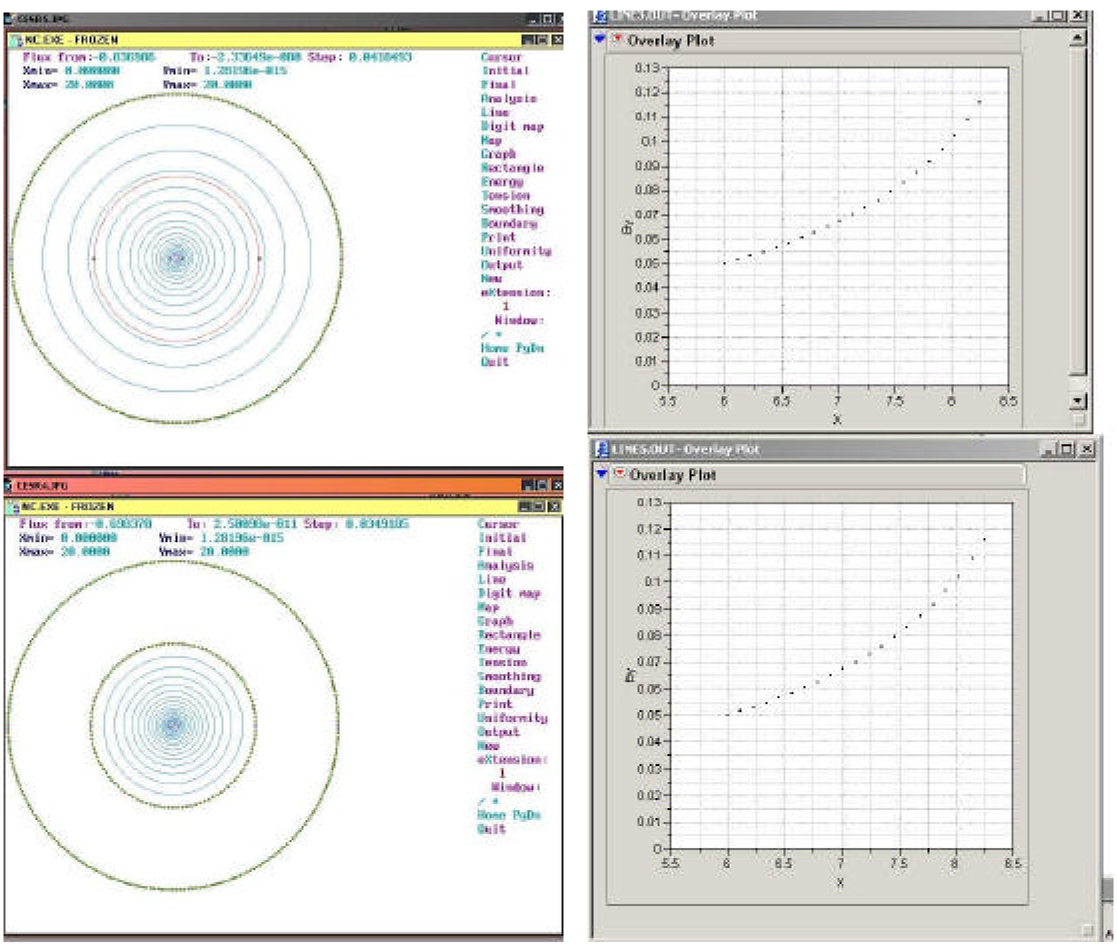

FIGURE 2: Magnetic lines for two cases, left. Field values across the region, specified by x–coordinate, –right. Center located at x=10 cm. Calculations done with MERMAID.

In this cylindrical geometry the outer diameter is 20 cm and the smaller outer diameter is 10 cm, Fig 2, left. The current is running through the axis of these cylinders, which is located at x=10 cm. So the field for x=7 cm corresponds to the horizontal shift equal to 3 cm. The vertical field around this point represented in Fig. 2, right, for two cases:20 and 10 cm diameter. As expected, the field is the same does not matter what is the diameter of external cylinder.

Now the real cross section inserted into program MERMAID as shown in Fig 3. This time the boundary is limited by cylinder having 30 cm in diameter. For modeling fields in vacuum, the CESR's chamber contour is just transparent.

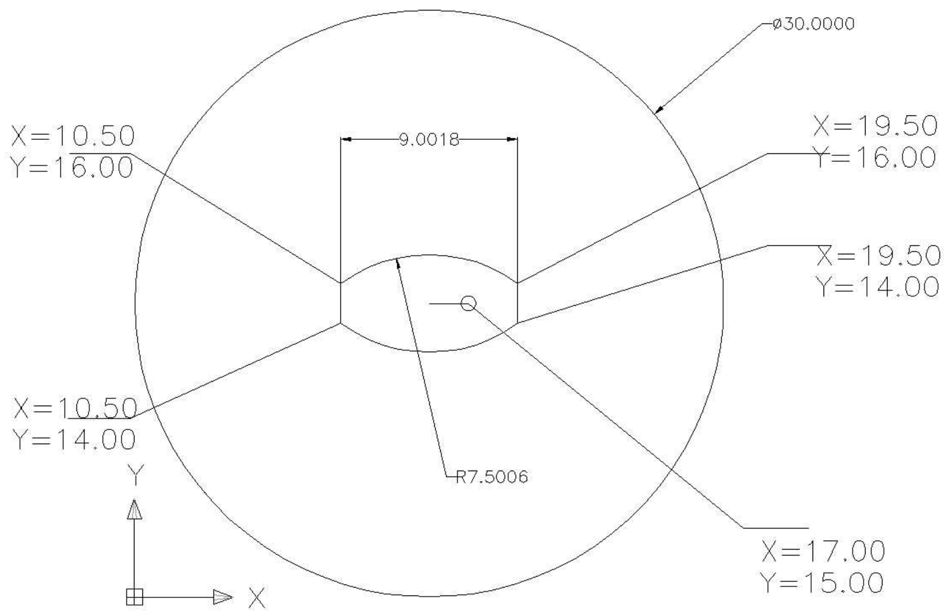

FIGURE 3: Geometry. All dimensions are given in cm. Current is running at x=17cm, y=15cm, i.e. +2cm from the center of chamber, which is located at x=15, y=15cm.

We are interesting in fields around the opposing beam location, which center lies at x=13, y=15 *cm*. So this corresponds basically to the pretzel amplitude ± 2 cm.

The current was taken as big as I=1 kA. This value is arbitrary. For estimations one can take CESR's 10mA average current. As the perimeter of CESR~768*m* so for 1cm bunch length the pick current corresponding to this average value will be $\hat{I} \cong \frac{76800cm}{1cm} 10 \cdot 10^{-3} A = 0.768kA$. One can easily scale this value to any other value and other longitudinal distribution.

This model is valid for the chamber having infinitely high conductivity. For the finite conductivity situation becomes more complicated as magnetic field penetrates inside material of the walls what yields misbalance of electric and magnetic forces. In this case residual magnetic field demonstrates shape dependence and, hence gradient in residual field, see [3] and references there.

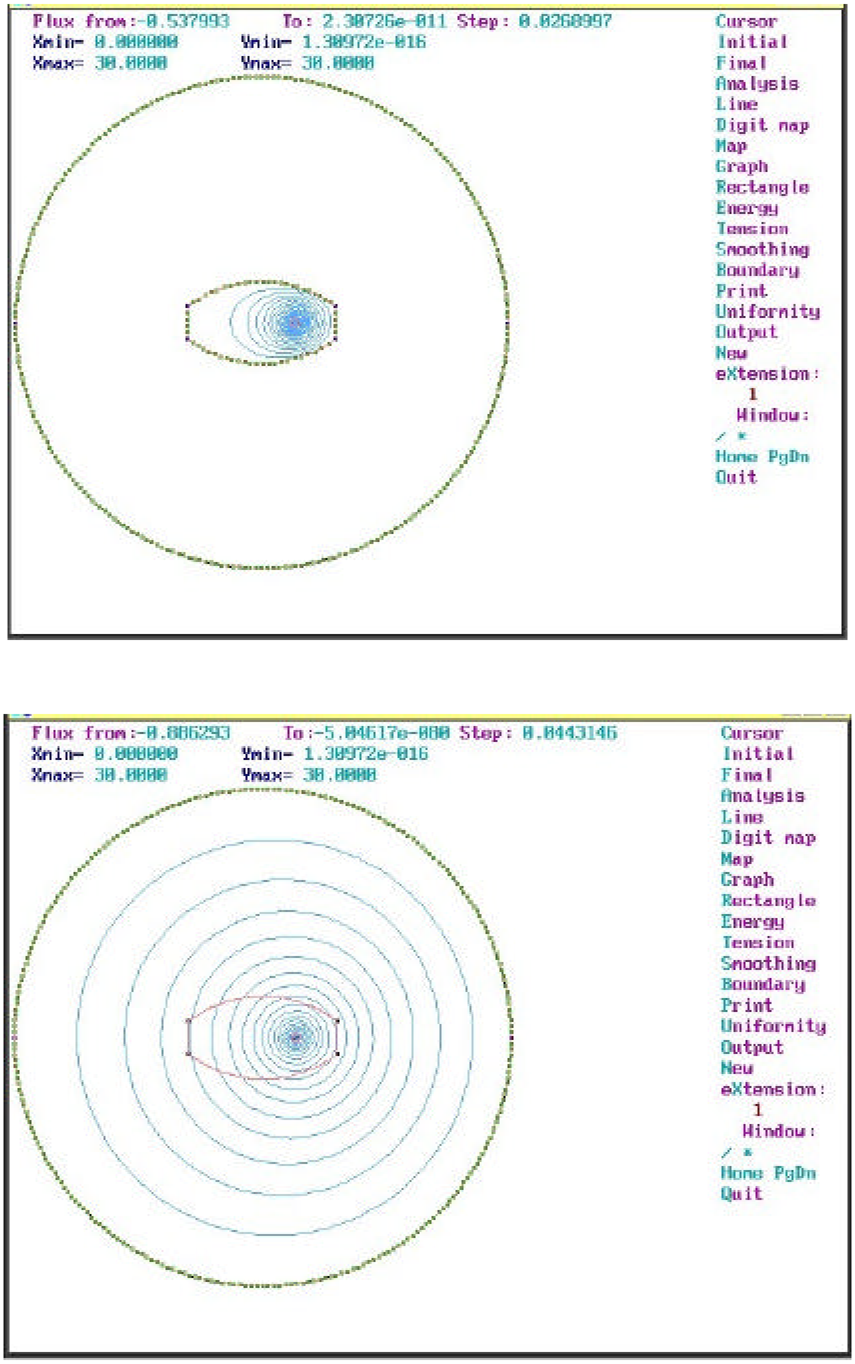

FIGURE 4: Magnetic lines in CESR's chamber represented in upper picture. Boundary conditions correspond to infinite conductivity. Lower picture shows the lines in surrounding round chamber having diameter 30 cm. This approximates free space well enough.

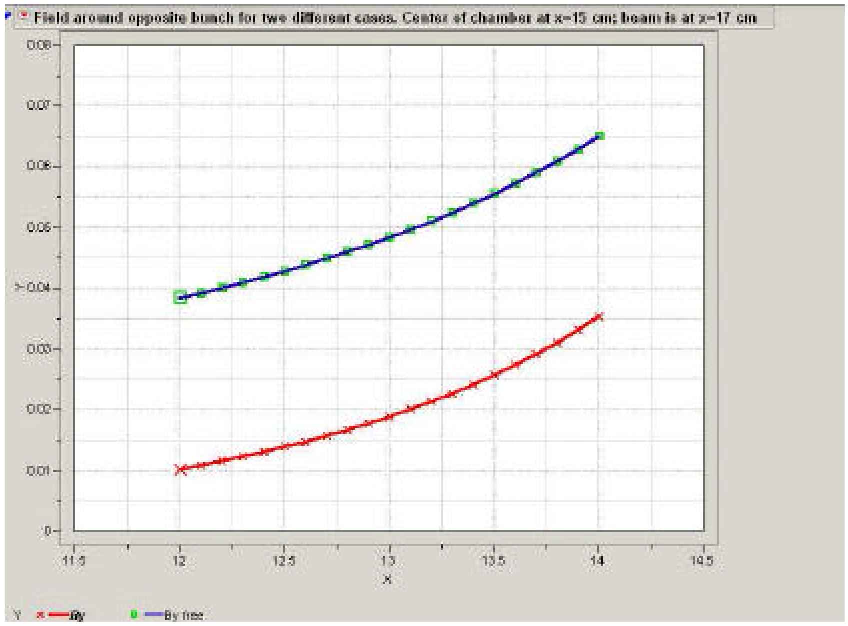

FIGURE 5: Vertical magnetic field ($H_j \equiv H_y$), kG in comparison. Field in CESR's chamber is lower. Center of opposing beam located at x=13*cm* (-2*cm* from the middle of chamber). Beam generating this fields located at x=17 cm, see Fig. 3.

So one can see, that despite the magnetic field becomes reduced to the~ 25% of its initial value, the *gradient* remains about the same. So one might be inspired by formal coincidence of calculated betatron tune shift

$$\Delta Q \cong \frac{1}{4p\,(HR)}\int b\,(s)\frac{\partial H_j\,(r,s)}{\partial r}ds \cong \frac{b\,(s_0)}{4p\,(HR)}\int \left.\frac{\partial H_j\,(r,s)}{\partial r}\right|_{y=0} ds\,,$$

and the betatron tune measured in real machine and extend the kick formula to all cases. Numerical calculations require knowledge of real kick, however, so here the source of mistake might be hidden.

These numbers are valid for the chamber of CESR's shape and pretzel conditions. For other rings this effect is different, naturally. Change in strength of this effect remains important for multi-turn tracking.

CONCLUSION

Screening by a vacuum chamber is important and needs to be taken into account. But basically this is working in a favor of kick reduction. Effective kick is ~2-4 times lower with the chamber. So the calculations with formulas for free space majorate the effect. The screening definitely needs to be taken into account when numerical simulations carried for multi-turn tracking.

REFERENCES

[1] Landau L., Lifshits E., "*The Field Theory*", Pergamon Press, 1983.

[2] A. Mikhailichenko, "*Space Charge Effects in Diluted Beam*", PAC1999, FRA 106, Proceedings, Ed. A. Luccio, W. MacKay, Vol. 5, pp. 3636-3638.

[3] N. Dikansky, D. Pestrikov, "*The Physics of Intense Beams and Storage Rings*", New York, USA: AIP (1994) 483 p.