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IDEAL WIGGLER[1]

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Abstract. Described is the wiggler with reduced nonlinear components for usage in the damping ring of the Linear Collider. Zigzag field dependence on longitudinal coordinates made by profiling of poles.

INTRODUCTION

Wigglers are important insertion devices for storage rings for use of SR generation. The search for an ideal wiggler is on the arena today as it appears as an inevitable part of the linear collider in many projects for damping rings.

Vertical emittance plays a more important role than the radial one, as it was mentioned first in [1]. The idea why is that, can be simplified by the following. As at IP one needs to have *flat beams* for reduction of energy spread forced by synchrotron radiation, then the easiest way to reach this is with enlarged horizontal emittance, rather than horizontal envelope function. So linearity in vertical motion plays a key role in this emittance business in the search for luminosity in the linear collider.

In some publications the wiggler is treated as a series of dipoles installed with reversed polarities, one after another. Despite this, it is true formally; some incorrect conclusions were made on the quality improvement for these installations. First, as in some laboratories, the measurements of the wiggler field were carried out with long coils, so the wiggler was treated as a somehow averaged field. So conclusions were made there that the width of the poles were often incorrect. They allowed being narrow in a view of cancellation of the field integral in neighboring poles. This might be especially confusing for the wiggler with even numbers of poles, where integral is zeros by definition, despite significant roll-off the field across the pole. Publication [2] was probably the first one where there was attracted attention to these effects.

In addition, formally, computer codes and presentation of Hamiltonian of transverse motion is based on coordinates which are expansions with respect to the reference (equilibrium) trajectory so for equilibrium particle all transverse coordinates are zeros. In a wiggler the particle wiggles with the amplitude of $\sim \lambdabar_W K/g$ with respect to origin wiggler coordinates, ($\lambdabar_W$ stands for wiggler period divided by $2p$ and deflection parameter K defined as usual $K = \frac{ZeB_0\lambdabar_W}{mc^2}$, where B_0 is amplitude of first harmonic of the magnetic field). The field expansion with respect to this wiggling trajectory is extremely complicated, however.

Other approach associated with presentation of wiggler field with respect to Cartesian coordinates where wiggler field has rather simple presentation, which is not associated with reference equilibrium trajectory. So equilibrium trajectory is not having zero coordinates now as it wiggles through the field described by this way.

In our previous publications we considered some improvements of field quality, which can be easily done practically for any wiggler [3], [4]. The main desire was to make the pole field across aperture as flat as possible. Formally, this can be reached with wide poles. Here we are

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[1] Electronic version is available at http://www.lns.cornell.edu/public/CBN/2005/CBN05-1/cbn05-1.pdf. Work supported by NSF.

continuing the quest for the wiggler with reduced nonlinear action to the particles in vertical direction.

FIELDS IN A WIGGLER

Let us consider a wiggler with finite pole width across transverse coordinate which directed along Cartesian coordinate x, vertical axes y and longitudinal one –s. Particles supposed to move along s –axis. If zero x lies in the medial plane y=0 and transversely coincides with the middle of the pole, then analytical representation for the wiggler fields is the following [3], [5]

$$B_x(x,y,s) = -\frac{xy}{4}B''(s) + 2S(s)xy + \frac{x^3y+xy^3}{48}B^{(IV)} + D(s)xy(x^2-y^2)+\dots$$

$$B_y(x,y,s) = B(s) + S(s)(x^2-y^2) - \frac{x^2+3y^2}{8}B''(s) + D(s)\cdot(x^4+y^4-6x^2y^2)+\dots \qquad (1)$$

$$B_s(y,s) = y\cdot B'(s) - \frac{x^2y+y^3}{8}B'''(s) + \frac{3x^2-y^3}{3}S'(s) + D'(s)\frac{5x^4-10x^2y^2+y^5}{5}+\dots\ .$$

where $B(s)$, $S(s)$, $D(s)$ stand for generation functions for dipole, sextupole, decapole, … field along the wiggler respectively. Here derivatives are taken along s. So these expansions are valid for particular choice of coordinates. We also suggested that plane sy is a plane of symmetry for the wiggler. In this case function $B(s)$ can be defined simply by measurements along s –axis or by calculations, as in this case x=y=0. After that, all necessary derivatives appeared in (1), can be taken numerically with necessary accuracy. So now the difference between calculated (or measured) transverse field variations and found from (1) can be treated as higher harmonics, with sextupole $S(s)$ as a lowest one. Or with other words, if one has transverse field roll-off, say at field maximum, calculated or measured, sextupole component $S(s)$, having dependence~x^2 can be identified *after subtraction* the terms, associated with derivatives. Other harmonics, such as $D(s)$ can be found in the same manner, taking in consideration dependence ~x^4 and so on. This gives a *simple and powerful recipe* for representation of wiggler field for the purposes of calculation of nonlinearities and dynamical aperture associated with this.

In case, when the poles are much wider, than aperture opening, the field can be described by single generating function $B(s)$ only, which stands for the vertical dipole field. All multipole components responsible for transverse dependence must be vanished and, according to formula (1) this will bring association of derivatives with appropriate multipole [3]. For example, in this case $S(s) = \frac{1}{8}B''(s)$, $D(s) = \frac{1}{16}S''(s) - \frac{1}{192}B^{(IV)}(s)$ and so on, and field expansion now can be represented as the following

$$B_y(y,s) = B(s) - \frac{y^2}{2!}B''(s) + \frac{y^4}{4!}B^{(IV)}(s) - \dots$$

$$B_s(y,s) = y\cdot B'(s) - \frac{y^3}{3!}B'''(s) + \frac{y^5}{5!}B^{(V)}(s) - \dots\ , \qquad (2)$$

If the field is time dependent, then the following substitutions need to be done in formulas (1), (2)

$$\frac{\partial^{2k}B(s)}{\partial s^{2k}} \to \left(\frac{\partial^2}{\partial s^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)^k B(s,t)\,, \tag{3}$$

Where c –is speed of light. Odd derivative obtained by taking integral over s in this combination.

One can easily conclude from (2), that if longitudinal field distribution is a (piecewise) linear one, then all derivatives are going to be zero. For the wiggler, of cause linear dependence everywhere is not possible, just piecewise smooth linear function, so these derivatives emerge at points where the field reaches its extremes, see Fig 1.

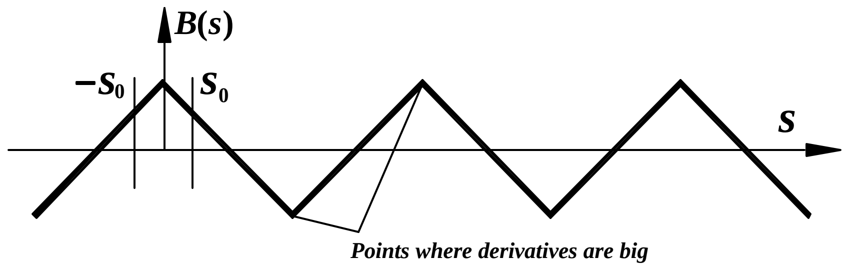

Figure 1: Longitudinal profile of magnetic field with linear dependence between extremes.

At other than these points all derivatives higher than the first one are zeros and the field has no nonlinearity, according to (2). Let us check if these points can give an input to the vertical kick however. Formally as the second derivative is proportional to the local curvature, it can be very big at these points. Fortunately there might be two factors helping in kick reduction. First one is that the vertical forces acting to the particle, is proportional to the transverse angle, $\alpha \cong x'$, which is zero at points of extreme[2]. Other is that the second derivative is limited by physical means, going to be $B''(s) \cong B_{max}/a^2$, where *stands* for characteristic dimension, which might be an aperture opening at this place.

Vertical field B_y defines local amplitude of horizontal wiggling angle[3],

$$\alpha(s) = \frac{1}{(HR)}\int_0^s \frac{B_y(s')ds}{Cos\,\alpha(s)} \cong \frac{1}{(HR)}\int B_y(s')ds\ , \tag{4}$$

Meanwhile B_s defines vertical kick $y' \cong \alpha \cdot B_s$. The regions around maximal or minimal values of field, the particle passing with zero angle with respect to longitudinal axis. So we can suggest, that around maximal field value the angle depends like odd function of longitudinal coordinate s, $\alpha \cong k_1 \cdot s + k_3 \cdot s^3 + \ldots$, where coefficients $k_1, k_3, \ldots$ defined from (2) and (4). While particle passes from $-s_0$ to $+s_0$, where s_0 *is* arbitrary coordinate, not far from maximum, so resulting nonlinear vertical kick is proportional to the following

$$y' \cong \frac{1}{(HR)}\int_{-s_0}^{s_0}(k_1 s + k_3 s^3 + \ldots)B_s(s)ds \cong -\frac{y^3}{3!(HR)}\int_{-s_0}^{s_0}(k_1 s + k_3 s^3 + \ldots)B'''(s)ds \tag{5}$$

[2] For steady trajectory without a systematic angle.
[3] Cosine in denominator is due to the fact that integration is going along straight line.

Function $B(s)$ is an even function around its maximum so $B'''(s)$ is odd function, the same as the angle. So in a view of this, the integral (5) is not a zero by symmetry. It can be evaluated by making substitution

$$(k_1 s + k_3 s^3) B''' = \frac{d}{ds}\{(k_1 s + k_3 s^3) \cdot B''(s) ds\} - (k_1 + 3k_3 s^2) \cdot B''(s)\,. \tag{6}$$

Substitute (6) into (5) one can obtain, taking into account, that second term is an even function

$$y' \cong \frac{y^3}{6(HR)}\{(k_1 s + k_3 s^3 + ...) \cdot B''_s(s)\}\Big|_{-s_0}^{s_0} \cong \frac{s_0 y^3}{3(HR)}(k_1 + \tfrac{1}{4} k_3 s_0^2 + ...) \cdot B''_s(s_0) \tag{7}$$

So one can see if second derivative of field distribution at this point $s = s_0$ is zero, then the integral (7) goes to be a zero. Although assumption was about symmetry of field distribution around maximum/minimum of magnetic field this result still valid under broader conditions, as soon as second derivation is zero at the ends of integration interval.

Other look at dipole wiggler which supports the idea about preference of linear dependence can be seen from Fig.2, where the wiggler is represented as series of partial *transversely oriented quadrupoles*, so the particle passes them *across* longitudinal coordinate s through central region and developing wiggles along x, [5].

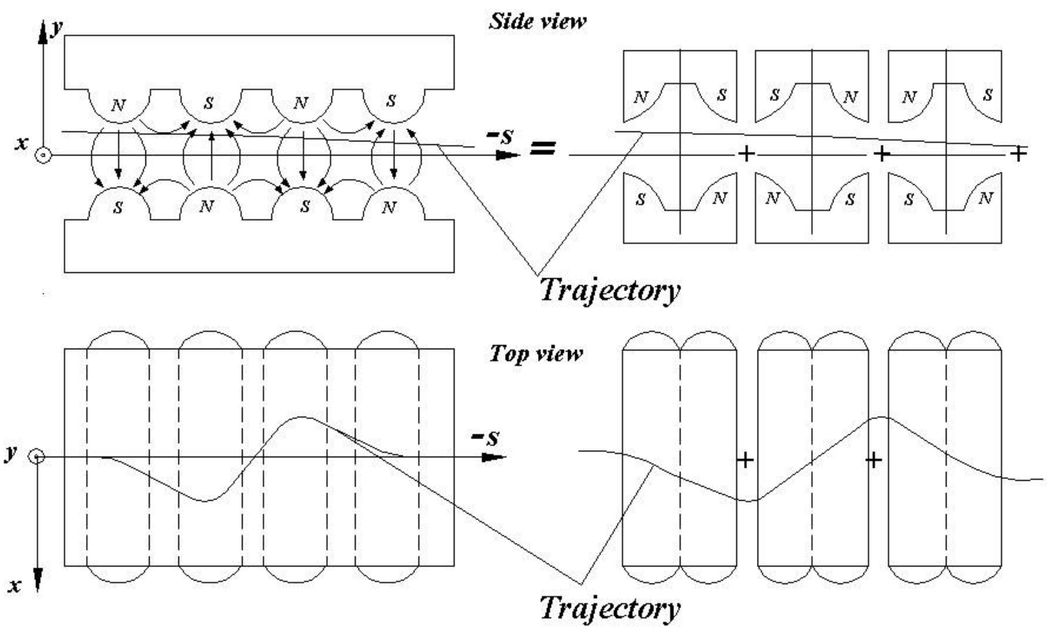

Figure 2: Representation of wiggler as series of quadrupoles with transverse orientation.

This presentation exactly corresponds to the one in (2) where effective gradient of longitudinal field in lowest order is proportional to its longitudinal derivative, so action is proportional $\propto ayB'(s)$. One can see that nonlinear terms associated with higher derivatives. So as particle is moving along x, towards and backwards form viewer in Fig.2, one can easily see, why the wiggler always focusing particle in vertical direction: with reverse of longitudinal field direction in neighboring quads, the angle is reversed too.

Again, now one can see that requirements of nonlinearity for linear field dependence all derivatives are disappeared except in points around extremes. But namely here the angle is zero, so there is no nonlinear action at all. Of cause this idealized jump of curve slope, represented in Fig.1 is not possible in practice, but it was interesting to investigate on how close to this distribution it is possible to approach.

Coming back to Fig.2 let say again that if the field is represented as series of quadrupoles, one wants, naturally, to have the field in these quadrupoles as linear as possible.

WIGGLER's HARDWARE

To see how our considerations can be applied to the real case, we have modeled real wiggler. 3D model was erected and field calculations with MERMAID have been carried for the hardware represented in Fig.3. Basically there is an attempt to install quadrupoles close one to another. This model does not associated with any specific project, however.

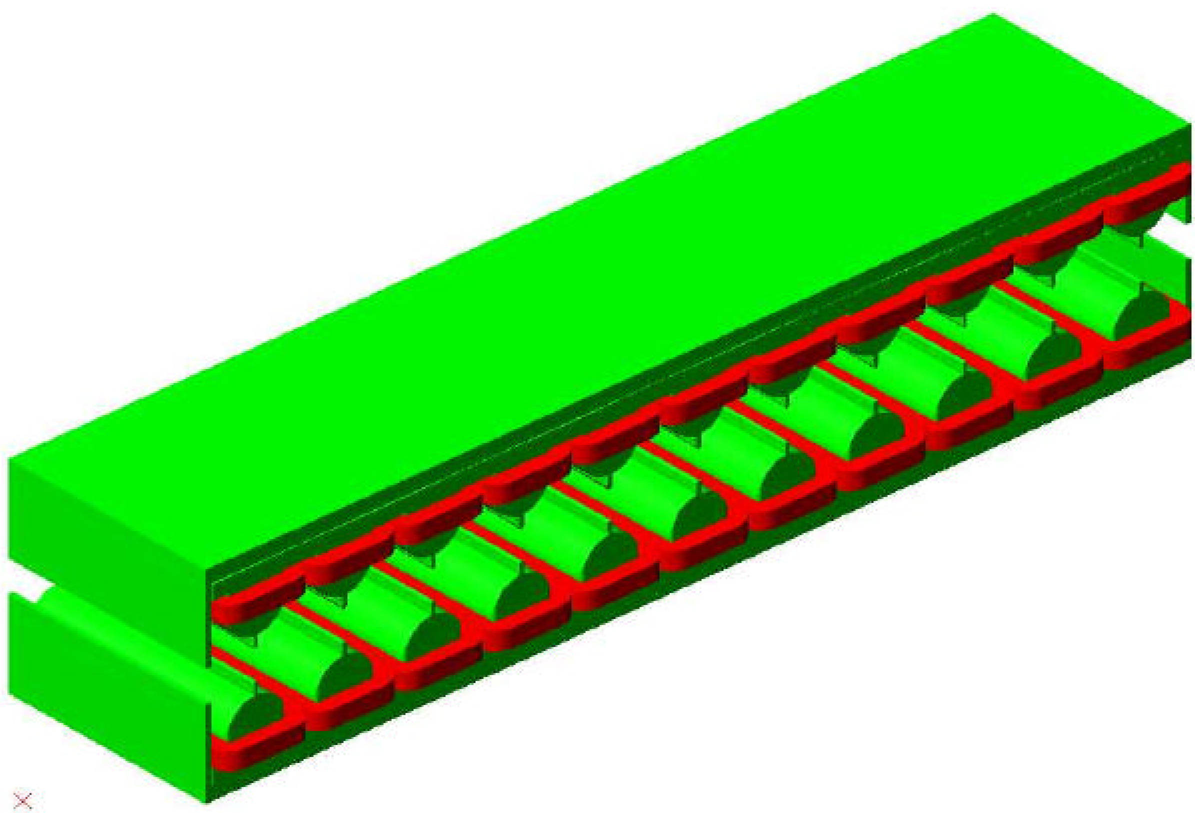

Figure 3: Wiggler with profiled poles.

Eleven pole wiggler in Fig.3 has period 25 *cm* and the pole width 30 *cm*. Poles approximated by cylinders with radius 5.5 *cm* with ribs having height 0.83 *cm*, width 0.8 *cm* with fillet radius 0.4*cm*. So the pole in lowest point runs 2.7 *cm* above medial plane. All iron in this model is Steel 1010. Current running in central pole coil is 60*kA*. In the end coils the current is ~14 and ~43 *kA* respectively. Coil itself has dimensions 2.3´ 2 cm^2. Supposedly this is a coil with SC wires. Details of cryostat are not shown, just cold mass only.

We did not optimize the profile as the iron is saturated; cylindrical surface rather than hyperbolic is adequate here.

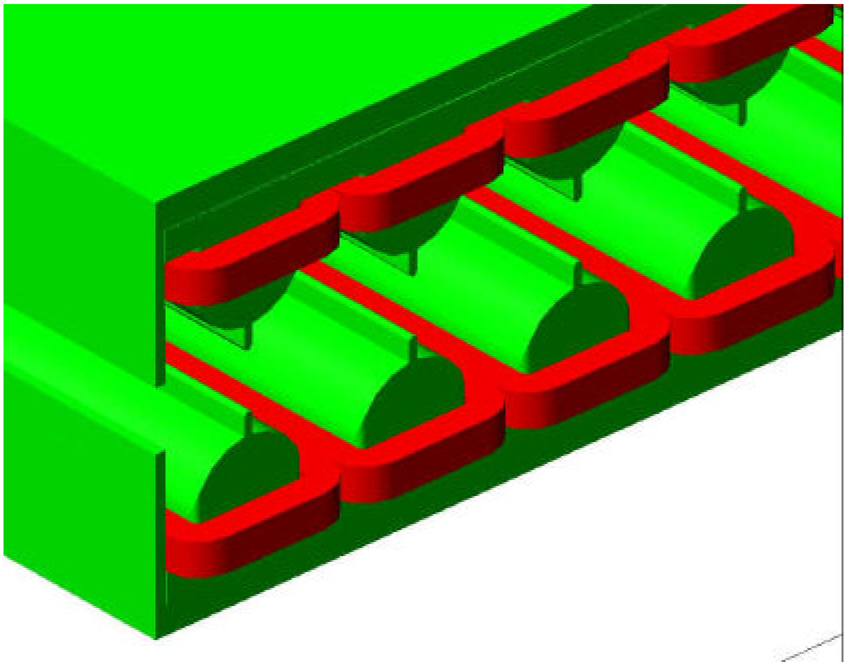

Figure 4: Scaled view from Fig.2.

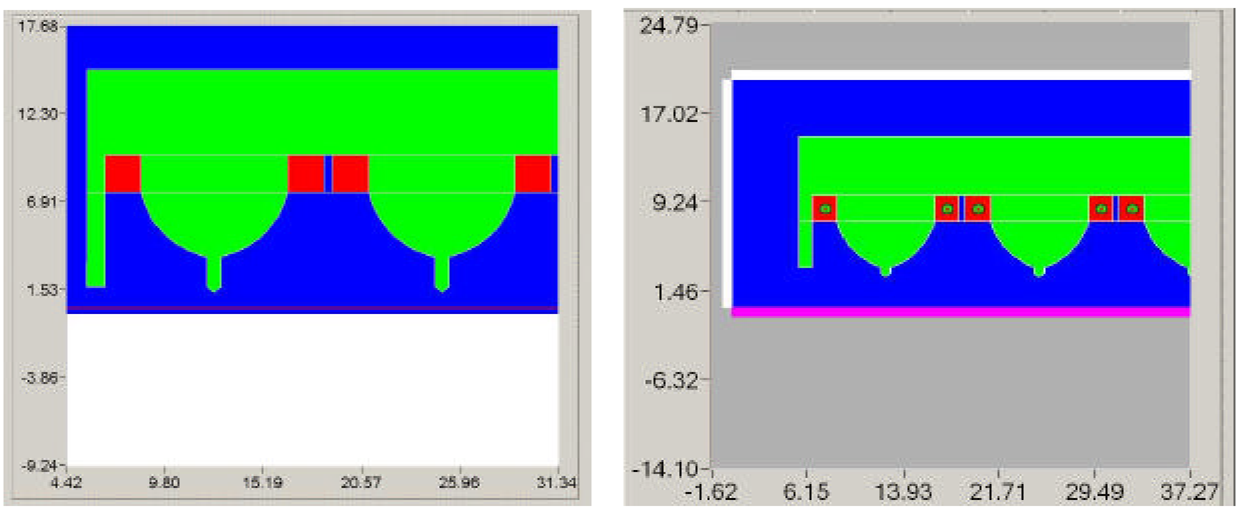

Figure 5: Two different poles profile around end region like they appear in MERMAID. Left one is an extreme case made for investigation of influence of the rib heights.

Finally we finished with profile represented in Fig. 5, right. Here the ribs have reduced height.

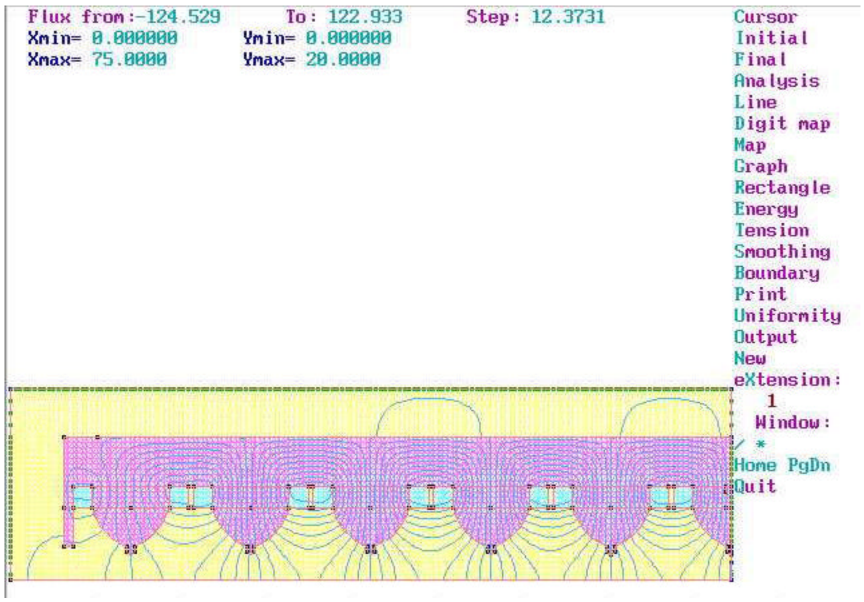

Figure 6: Magnetic lines for ~¼, ½, 1 tapering.

One can see from Fig.6 that the quadrupole field is disturbed between second and third poles, as the current running in these neighboring coils are different. To eliminate this one can consider local change in period and different profile. We did not exercise in this however.

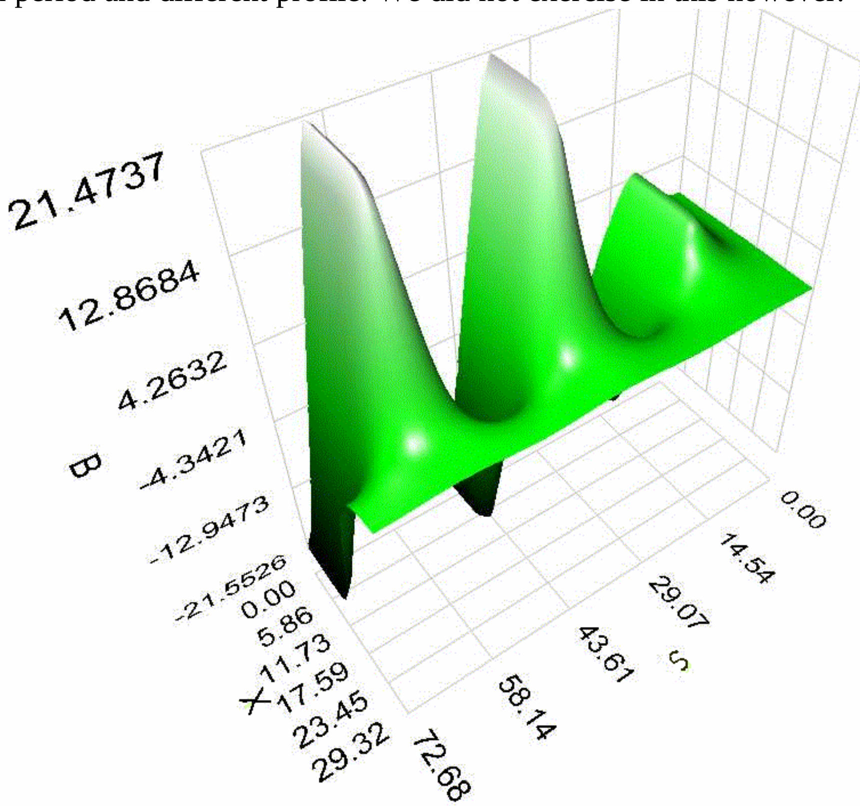

Figure 7: 3D presentation of magnetic field in ¼ of wiggler. Transverse coordinate with x=0 corresponding to the center of wiggler runs at far side of this picture. Coordinate s=75 *cm* corresponds to the middle of wiggler.

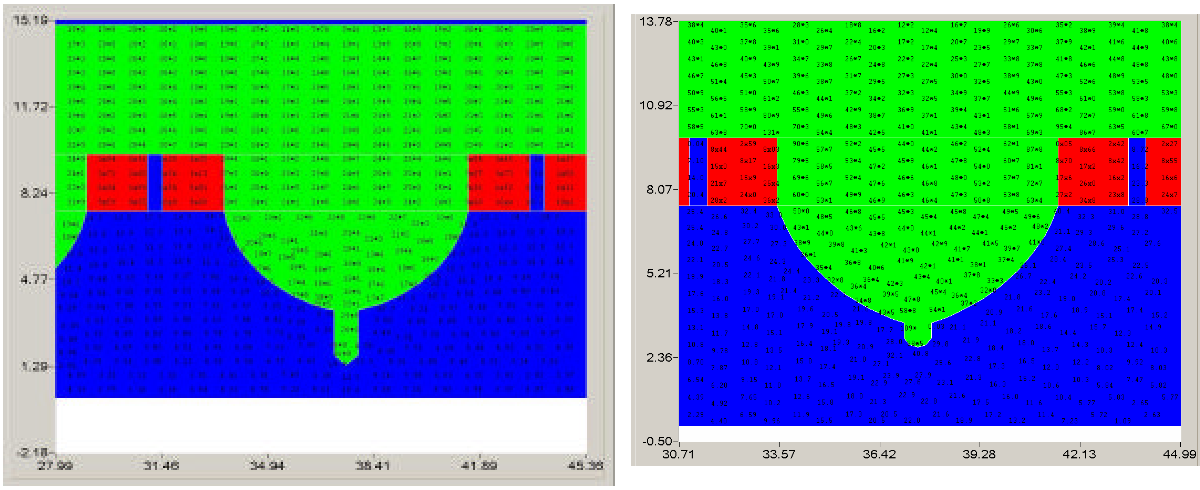

Figure 8: Numerical map used for control of saturation. Slice made around central part.

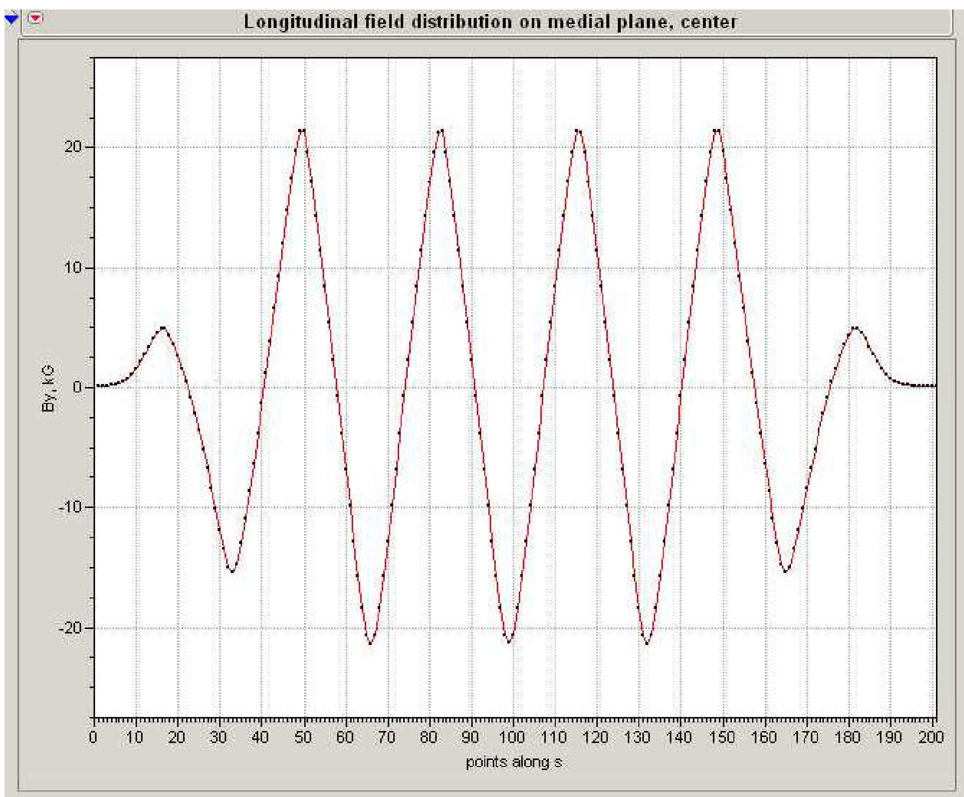

Figure 9: Longitudinal field dependence (as function of s) at $x=0$ calculated for wiggler which is represented in Fig. 3 with tapering ~ ¼, ¾, 1. Right point on this picture corresponds to s=150 *cm*.

Tracking in this field done with help of numerical code [7]. Result is represented in Fig. 10 below. Vertical focusing could not be eliminated, of cause. Once again, the purpose of our investigation is to make this focusing as linear as possible

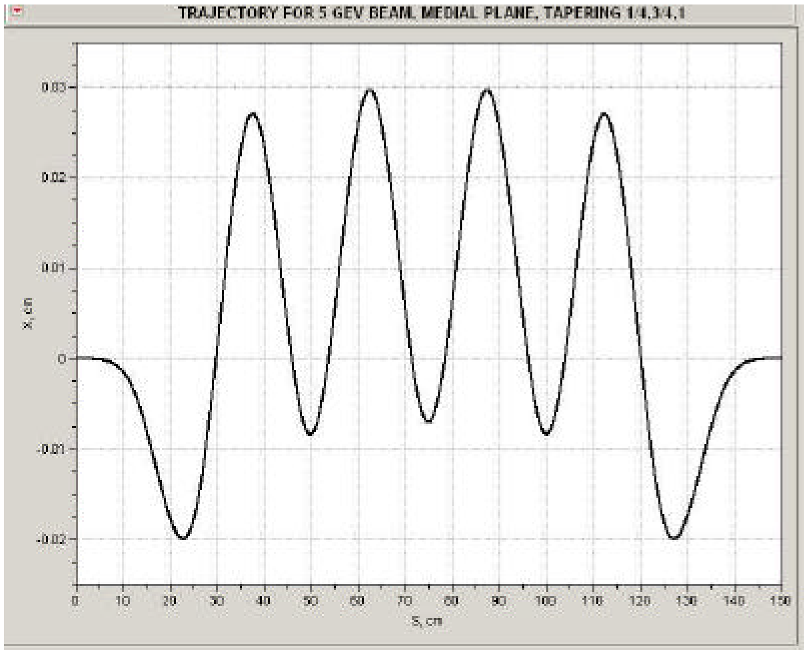

Figure 10: Trajectory of electron with energy 5 GeV entering at x=y=0.

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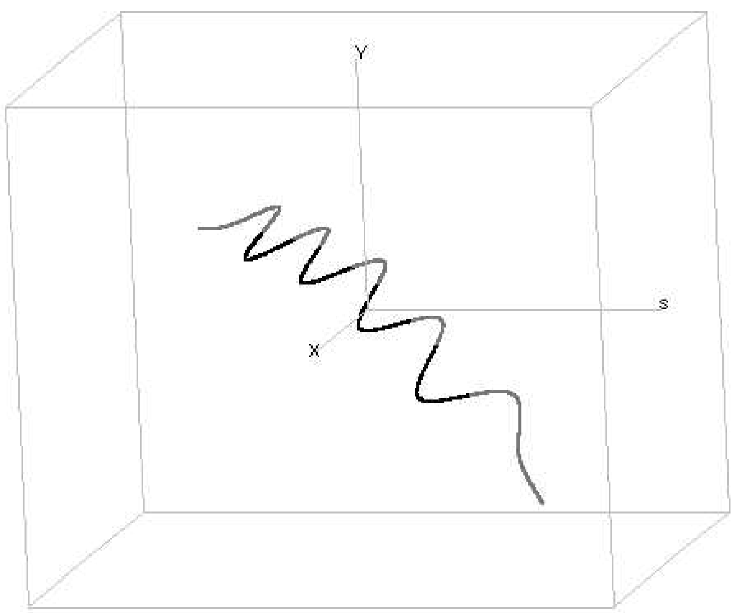

Figure 11: 3D representation of trajectory started at y=0.7*cm* above the medial plane. Vertical scale enlarged; in reality out–coordinate is lowered by ~40 micrometers only with respect to the one at the entrance.

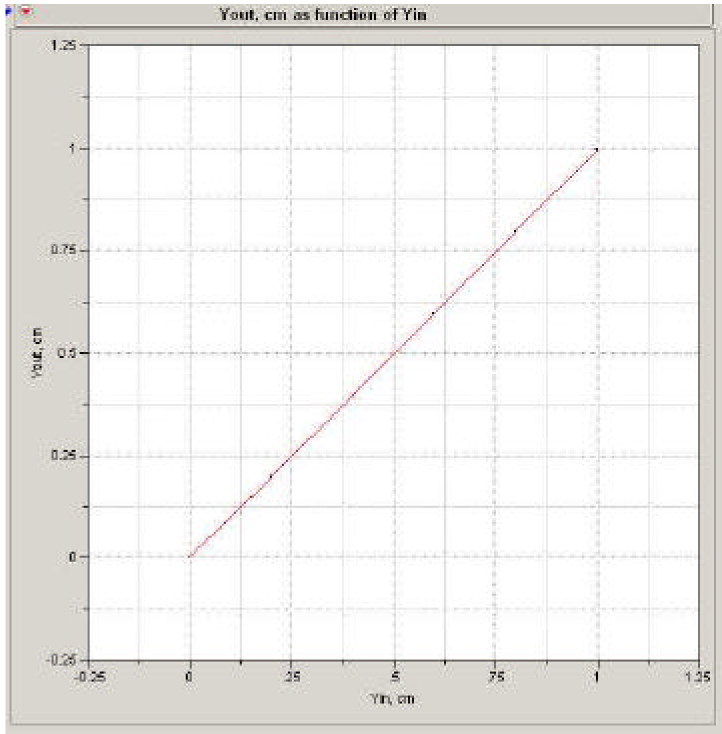

Figure 12: Vertical coordinate at the exit of wiggler as a function of vertical position at the entrance.

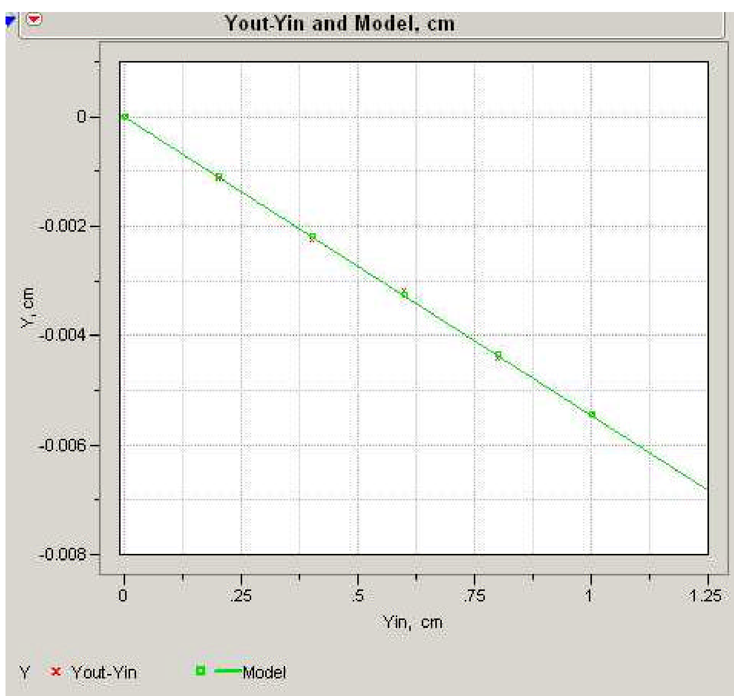

Figure 13: Difference between exit and entrance y –coordinates and the linear model fit.

One can see from Fig. 13 that at least for 1-*cm* incoming coordinate there is no nonlinear action to the particle, just pure focusing.

Other field distribution can be thought as more simplified and is just tapering ½, 1 at the end. The field distribution and particle's trajectory for this type of tapering are represented in Fig.14 and Fig. 15 respectively. The reason for this type of field distribution might be thought for reduction of nonlinear field slope visible at transition region between first and second pole.

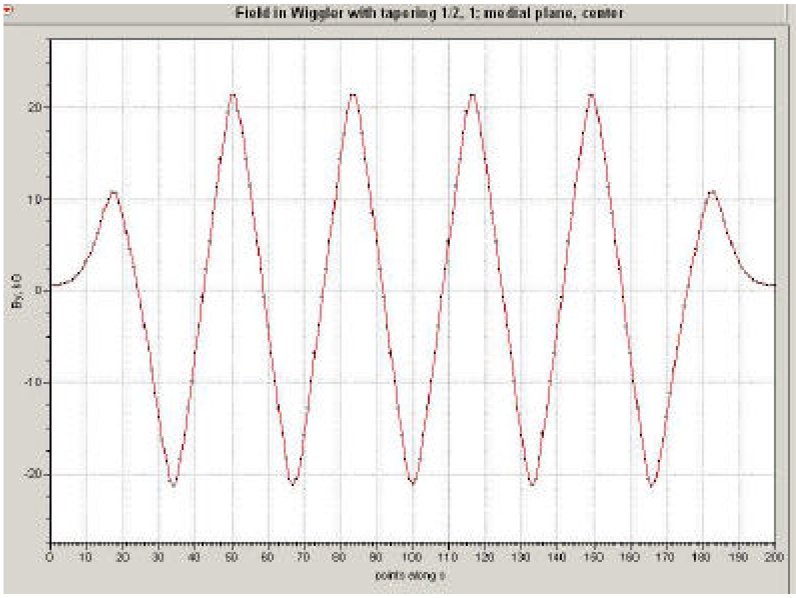

Figure 14: Longitudinal field dependence (as function of s) at $x=0$ calculated for wiggler with tapering ~ ½, 1. Right point on this picture corresponds to the s=150 *cm*.

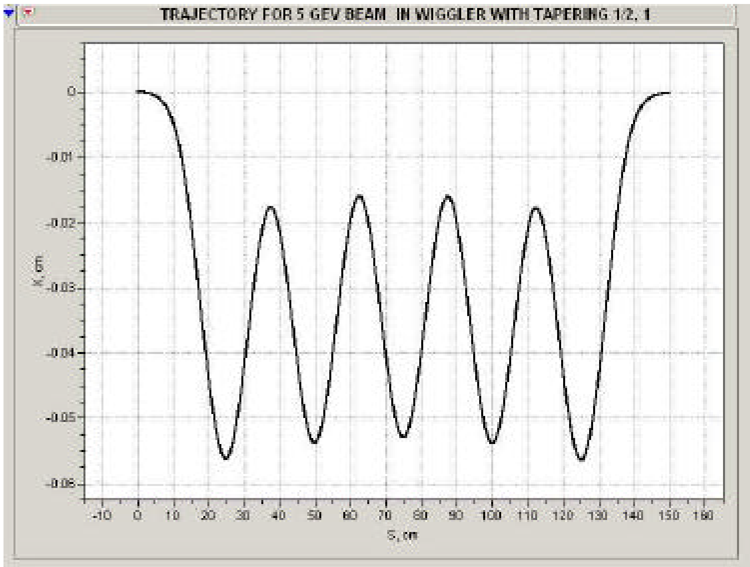

Figure 15: Trajectory in field from Fig.14.

We did not investigate trajectories for this field however, leaving this to another occasion.

CONCLUSIONS

We originated here a new approach to the wiggler design. Basic idea behind is very simple: if the wiggler can be considered as series of (transversely oriented) quadrupoles, then let make these quadrupoles having an ideal field. The region with linear dependence is a piecewise smooth linear one which can be generated by poles, having a profile of quadrupole (hyperbolas). One can consider formation of linear dependence with distributed currents as well. Other recommendation can be given is that the pole must be as wide as possible.

Period required for minimization of emittance must be as small as possible. This contradiction with requirement for minimization of non-linearity is eliminated by linear zigzag dependence like just described above.

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