Transverse Tune Shift Dependence on Beam-Pipe Wall-Conductivity *

Z. Greenwald †, D. L. Rubin, Cornell University, Ithaca, NY

INTRODUCTION

In $CBN\ 04-01$[1] we compared the horizontal tune shift of a counter rotating beam calculated in an Aluminum beam pipe, with the tune shift calculated in free space. To better understand the fact that the obtained tuned shifts were of the same order, we look at the effect of the wall resistance on the tune shift. We first expressed the tune shift as function of the transverse kick and then calculate the tune shift for three pipes with different wall resistance.

TUNE SHIFT CALCULATION: GENERAL EXPERSSION

The tune shift, $\triangle Q_x$, can be written [2] as:

$$\triangle Q_x \simeq \frac{1}{4\pi} \cdot < \beta_x > \int k_x \cdot dl \tag{1}$$

Where k_x is the quadrupole strength:

$$k_x = \frac{\frac{\triangle E_t}{\triangle x}}{p \cdot c} \tag{2}$$

E_t is the total transverse kick in $\frac{V}{m}$, p, is the bunch momentum, and c, is the speed of light. The dependence of the tune on the transverse kick is obtained by substituting Equation 2 into Equation 1:

$$\triangle Q_x \simeq \frac{\frac{1}{4\pi} \cdot < \beta_x >}{p \cdot c} \int \frac{\triangle E_t}{\triangle x} \cdot dl \tag{3}$$

The horizontal kick, E_t, experienced by an off center witness bunch traveling in the $-z$ direction, is calculated from the electro magnetic field created by another bunch, the source bunch, which travels along $+z$ with horizontal displacement from the witness bunch. $E_t = E_x - c \times B_y$. Where, E_x, is the horizontal component of the electric field, B_y, is the vertical component of the magnetic field.

An example of transverse kick experienced by a counter rotating witness bunch at $x_w = -1.5\ m$ generated by a $1\ cm$ long source bunch traveling at $x = 1.5\ cm$ off center, in a pipe with radious of $2.5\ cm$ can be seen in Figure 1 . The kick in Figure 1 was simulated for three different wall materials: Stainless Steel, Aluminum, and Copper. Note, when the wall resistance is increased, the transverse kick peak pulse increases but the pulse duration is shorter. The induced electric and magnetic fields generated for the Aluminum wall pipe can be seen in $CBN\ 04-01$[1].

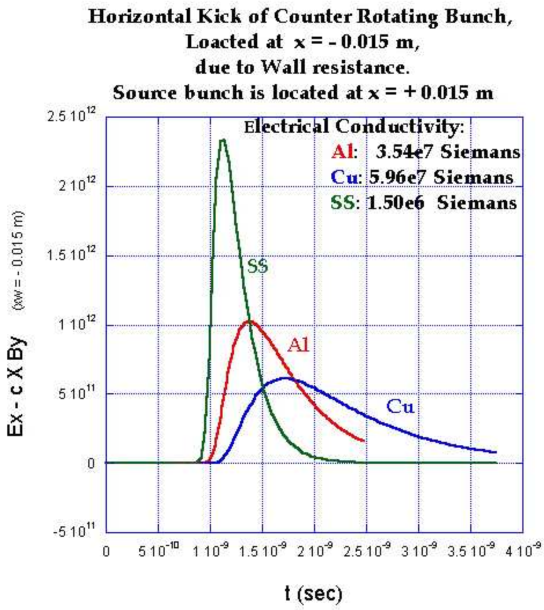

Figure 1: Transverse kick of SS, Al, Cu, at $x = -0.015\ m$ seen by a bunch traveling in $-\ z$ direction due to a 1 $coulomb$, 1 cm Gaussian bunch displaced at $x = +0.015\, m$ traveling in the $+\ z$ direction.

Next we substitute the horizontal kick in Equation 3:

$$\triangle Q_x \simeq \frac{\frac{1}{4\pi} \cdot < \beta_x >}{p \cdot c} \int \frac{\triangle (E_x - c \times B_y)}{\triangle x} \cdot dl \tag{4}$$

$$\triangle Q_x \simeq \frac{\frac{1}{4\pi} \cdot < \beta_x >}{p \cdot c} \frac{\triangle}{\triangle x} \int (E_x - c \times B_y) \cdot dl \tag{5}$$

In the last expression, Equation 5, the integral of the kick gives the total voltage, V_t, induced at x_w due to the transverse kick. This voltage depends on the source bunch charge, speed, and spacing $x - x_w$ from the witness bunch. It does not depend on the wall conductivity, as can be seen in Figure 2 where all three curves of $\frac{V_t}{c} = \int E_t \cdot dt$ approach the same value of 922 $\left[\frac{V}{m} \cdot sec\right]$. Thus, it is expected that the integral of the kick of the counter rotating witness bunch in free space at the same spacing from the source bunch will result with the same calculated tune shift. To support this expectation we performed the following calculation.

* Work supported by the National Science Foundation.
† Email: zipi@cesr10.lns.cornell.edu

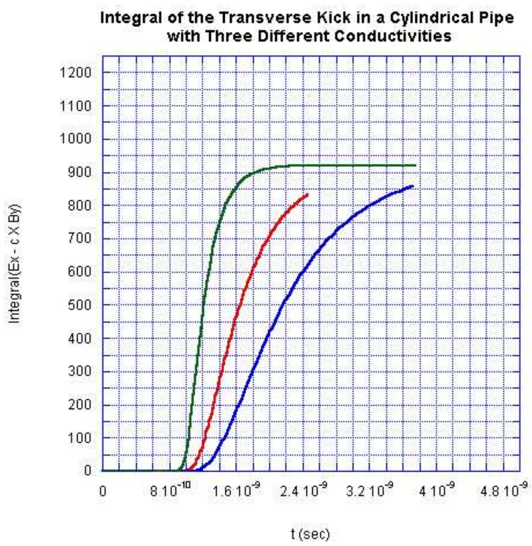

Figure 2: Integral of the Transverse Kick seen at $xw = -0.015\ m$ generated by a charge moving in a $0.02\ m\ radius$ cylindrical pipe along $+z$ at $x = +0.015$, for three different pipe conductivity as seen in $Figure\ 4$

TRANSVERSE KICK IN FREE SPACE OF COUNTER ROTATING BEAMS

In $CBN\ 04-01$ we simulated the bunch in a free space as a line charge which generated only electric field. In this paper the bunch in free space is simulated as a moving charge. Using Jackson [3] derivation for the electro magnetic fields, the electric field E_x (in cgs units) of a point charge traveling in the z direction in free space seen by a particle displaced horizontally by $b\ cm$ is:

$$E_x = \frac{\gamma q b}{(b^2 + \gamma^2 c^2 t^2)^{3/2}} \tag{6}$$

and the magnetic field B_y is:

$$B_y = \beta E_x \simeq E_x \tag{7}$$

Since the induce voltage, $V_t[MKS\ units] = \int E_x \cdot dl - \int c \times B_y \cdot dl$, we calculate:

$$\int E_x \cdot dt = \frac{qb}{\gamma^2 c^3} \int \frac{dt}{\left(\frac{b^2}{\gamma^2 c^2} + t^2\right)^{3/2}} \tag{8}$$

$$\int E_x = \frac{q}{b \cdot c} \left[\frac{statvolt \cdot sec}{cm}\right] = \int B_y \cdot dt \tag{9}$$

Using similar parameters as in the above section, $b = 0.03\ m$, point charge $q = 1\ Coulomb$, the time integral on the transverse kick $\int E_t \cdot dt = 1000\ \left[\frac{V}{m} \cdot sec\right]$ which is comparable to the value of $922\ \left[\frac{V}{m} \cdot sec\right]$ obtained for $1\ cm$ long Gaussian source bunch traveling in a beam pipe.

CONCLUSION

Having different wall resistance in the beam pipe affect the transvers kick as seen in $Figure\ 1$. The total induced voltage, which is the integral of the transverse kick, is independent on the wall resistance. Since the tune shift is proportional to the induced voltage, it will depend only on the bunch charge and the horizontal separation of the counter rotating beams. This also means that the case of counter rotating beams in free space can be calculated as beam traveling in a resistive beam pipe where the resistance approaches infinite.

REFERENCES

[1] Z. Greenwald, D. L. Rubin, ”Tune Shift Due to Transverse Wakefield in an Aluminum Cylindrical Pipe”, CBN 04-01

[2] A. W. Chao, M. Tigner, ”Handbook of Accelerator Physics and Engineering”, World Science 1998

[3] J. D. Jackson, ”Classical Electrodynamics”, Jhon Wiley & Sons 1975