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Tune Shift Due to Transverse Wakefield in an Aluminum Cylindrical Pipe *

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INTRODUCTION

In this paper we are comparing the tune shift of two counter rotating beams, displaced horizontally by $0.03\ m$, due to two different effects. In the first case, we calculate the tune shift created by the transverse wake field due to the finite wall resistance of Aluminum cylindrical pipe with $2.5\ cm$ radius. In the second case we calculate the tune shift created by the beam-beam kick in a single parasitic crossing in free space.

TUNE SHIFT CALCULATION DUE TO TRANSVERSE WAKE

When particle **1** travels off center at x_1 in a resistive pipe, it induces transverse and longitudinal electromagnetic fields. A witness, particle **2**, representing the counter rotating bunch is displaced by x_w from the center. See $figure$ 1. Particle **2** will have a tune shift due to the transverse wake field created by particle **1**. We were using the $3D\ MAFIA$ code which can calculate transverse wake fields of structures with finite wall resistance. The wake MAFIA calculates uses the standard definition where the witness particle stays at a constant distance behind the traveling source particle. In our case the witness particle travels in an opposite direction to the source, and the distance between the particles changes as they travel. These differences make the standard wake calculation inapplicable and require a separate calculation of the transverse kick experienced by the witness particle while the source particle travels along the pipe. Since the tune shift is proportional to the gradient of the electric field at the location of the witness particle, it is necessary to calculate the total field at two locations spaced by $\triangle X$. The total field is calculated from the transverse kick. In our case, the source particle travels in the speed of light, c, along $+z$ and is displaced by $x1$. The transverse kick, Et, experienced by the counter rotating particle traveling in c in the $-z$ direction and located at $-x_w$ is:

$$Et\left[\frac{V}{m}\right] = E_x - c \times B_y \tag{1}$$

Where E_x is is the electric field in the x direction, and B_y is the magnetic field in the y direction.

In doing the $3D$ field calculation of the cylindrical pipe, using MAFIA, the electro-magnetic field reflections from the discontinuity at the ends of the pipe distorted the calculated wake fields. To solve this problem a 44 cm long pipe was simulated with Aluminum wall conductivity of $3.54 \cdot 10^7\ mhos$. The pipe conductivity was gradually tapered down in 7 steps of 1 cm long, where the conductivity was decreased by a factor of 10 in each step.

The transverse electric field, E_x, and the magnetic field, B_y, created by the particle traveling along the pipe at $x_1 = 0.015\ m$, were calculated at two witness coordinates: ($x_{w,1} = -0.015\ m$, $z = 0.22\ m$) and ($x_{w,2} = -0.017\ m$, $z = 0.22\ m$). The fields calculated for $x_{w,2} = -0.017\ m$ being further from the source particle are lower than those calculated for $x_{w,1} = -0.015\ m$ as expected. See $Figure$ 2. Next we calculated the transverse kick, $E_t(x_w)$ experienced by the counter rotating particle (Equation 1). The change of the kick as function of time as the bunches travel can be seen in $Figure$ 3 for the two locations of the witness particle. The peak values of $E_t(x_{w,i})$, calculated at two locations $i = 1, 2$, for $x_1 = 0.015\ m$ are summarized in Table 1.

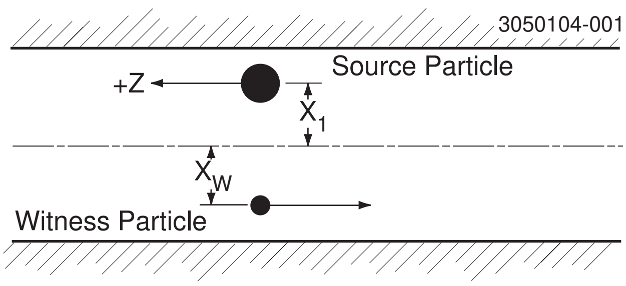

Figure 1: Transverse Electric and Magnetic fields Created in a resistive cylindrical pipe by a 1.5 cm horizontally displaced bunch

Table 1: Transverse kick around $x_w = -0.015\ m$

$x_{w,i}\ [m]$	$E_t(x_w, i)\ [\frac{V}{m}]$
-0.015	$1.15 \cdot 10^{12}$
-0.017	$0.85 \cdot 10^{12}$

The corresponding field gradient $\frac{\triangle E}{\triangle x}$ is:

$$\frac{\triangle E}{\triangle x} = \frac{(E_t(x_{w,1}) - E_t(x_{w,2}))}{|x_{w,1} - x_{w,2}|} \tag{2}$$

$$\frac{\triangle E}{\triangle x} = \frac{3. \cdot 10^{11}}{(1.7 - 1.5) \cdot 10^{-3}} = 1.5 \cdot 10^{14}\ \frac{V}{m^2} \tag{3}$$

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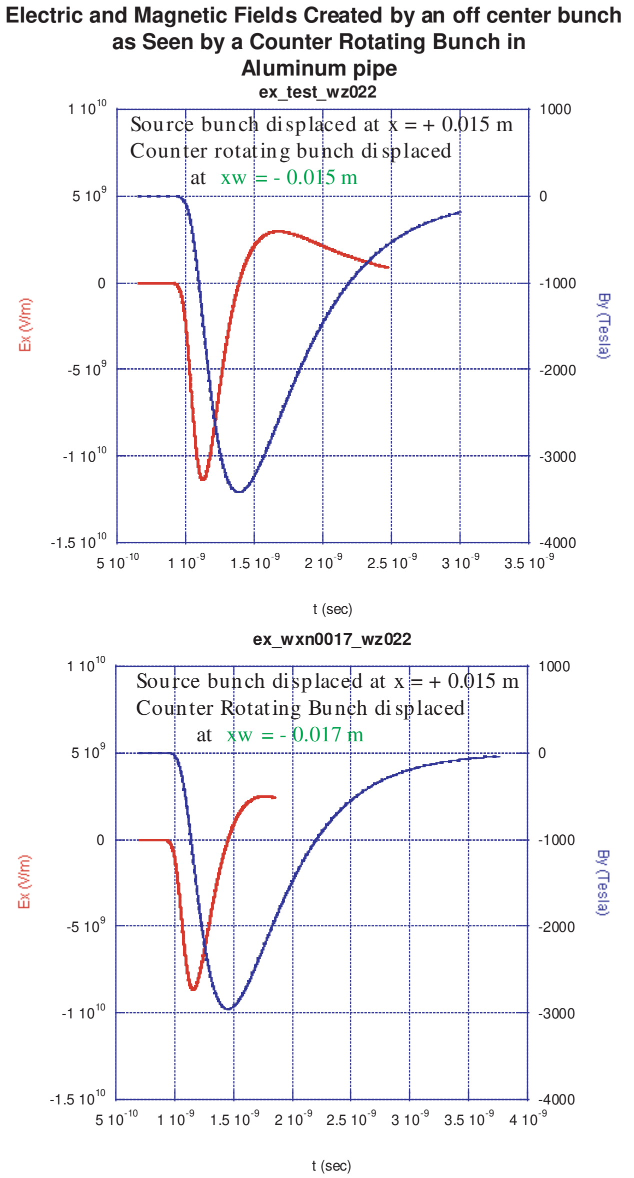

Figure 2: Transverse Electric and Magnetic fields Created in a resistive cylindrical pipe by a 1.5 cm horizontally displaced bunch

The corresponding quadrupole field strength for 1 $Coulomb$, k_x (Sagan[1]), calculated at energy of 1.8 Gev is:

$$k_x\;[m{-}2] = \pm\frac{\frac{\triangle E}{\triangle x}\left[\frac{V}{m^2}\right]}{p\cdot c\,[eV]} = 8.3\cdot 10^4\; m^{-2} \tag{4}$$

And the tune shift is:

$$\triangle Q_x \simeq \frac{1}{4\pi}\cdot <\beta_x> \cdot k_x \cdot l \tag{5}$$

l, is taken as 0.24 m which is the average distance along which the counter rotating particle experience the transverse kick, as seen in Figure 2. For $<\beta_x>= 20\ m$, the tune shift is:

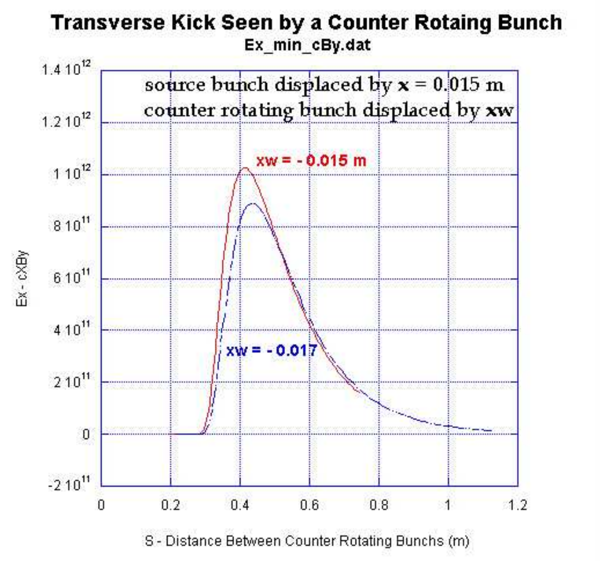

Figure 3: Transverse kick at $x = -0.015\ m$, $x = -0.017\ m$ seen by a bunch traveling in $-\,z$ direction due to a 1 $coulomb$ charge displaced at $x = +0.015\ m$ traveling in the $+\,z$ direction.

$$\triangle Q_x \simeq 3.18\cdot 10^4/Coulomb \tag{6}$$

At 4 ma the charge /bunch is $Q_b = 10.25\cdot 10^{-9}\ Coulomb$, and the tune shift is:

$$\triangle Q_x \simeq 3.18\cdot 10^4\cdot Q_b \simeq 3.26\cdot 10^{-4} \tag{7}$$

or:

$$\triangle f = \Delta Q\cdot 390\cdot 10^3 = 127\ Hz \tag{8}$$

TUNE SHIFT DUE TO BEAM-BEAM KICK AT PARASITIC CROSSING POINTS

The tune shift due to long range beam-beam interaction, in the line charge approximation (Rubin[2]) is:

$$\triangle Q_x = r_o\cdot N\frac{\beta}{2\pi\gamma|x_1 - x_2|^2} \tag{9}$$

Where $r_o = 2.818\cdot 10^{-15}\ m$ is the classical electron radius. N, the number of particles in electron bunch. Using the same condition as in the section above,

$$N = \frac{Q_b}{q} = 6.4\cdot 10^{10}\ particles.\ (\ for\ a\ 4\ ma\ bunch\ ) \tag{10}$$

$$\gamma = 3.5\cdot 10^3\ (\ at\ 1.8\ GeV\ ) \tag{11}$$

$\beta = 20\ m$ and $|x_1 - x_2| = 0.03\ m$. The tune shift is:

$$\triangle Q_x = 1.822\cdot 10^{-4} \tag{12}$$

and

$$\triangle f = 71.07\ Hz \tag{13}$$

CONCLUSION

The tune shift due to the finite Aluminum resistance at the walls is of the same order as the tune shift due to beam-beam interaction at the crossing points.

ACKNOWLEDGMENT

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REFERENCES

[1] D. Sagan and J. Welch, ”The Effect of Leakage Electro static Fields from the Distributed Ion Pumps on the Electron and Positron Beams”, CBN-92-01

[2] D. Rubin, ”Optical Effects of Parasitic Crossing with Nine Trains of Bunches”, CBN 96-02