OCTUPOLE/QUADRUPOLE/… ACTING IN ONE DIRECTION

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We propose to use elements of beam optics (quads, sextupoles, octupoles, …) so that they can act in one transverse direction only. For these purposes particles go *across* the elements, entering them and coming out far from the edges. This one-directional acting can be useful for fine-tuning of storage-rings with wigglers.

It is well known, that wiggler with horizontally wide poles acts to the particle only in vertical direction. Mostly important, however, is that wiggler also introduces *dynamically* the octupole-type nonlinearity, which also acts in vertical direction only. This brings problems with tuning, as usual elements of beam optics act in both directions simultaneously, $\partial_x B_y = \partial_y B_x$. A quadrupole, if it is focusing particles in one transverse direction, introduces theirs defocusing in other direction. Octupole focuses/defocuses particles in both directions simultaneously. So compensation of focusing introduced by wiggler sometimes is not a trivial problem.

Here we are suggesting new way of utilization of well-known elements, so they can act in one transverse direction.

Let us remind, first, how the ordinary wiggler woks. Transverse cross-section and top view is represented in Fig.1. At the right side of this figure a schematic presentation of the wiggler field done with the help of partial *quadrupoles*, which installed so that particle transfers them across in central part.

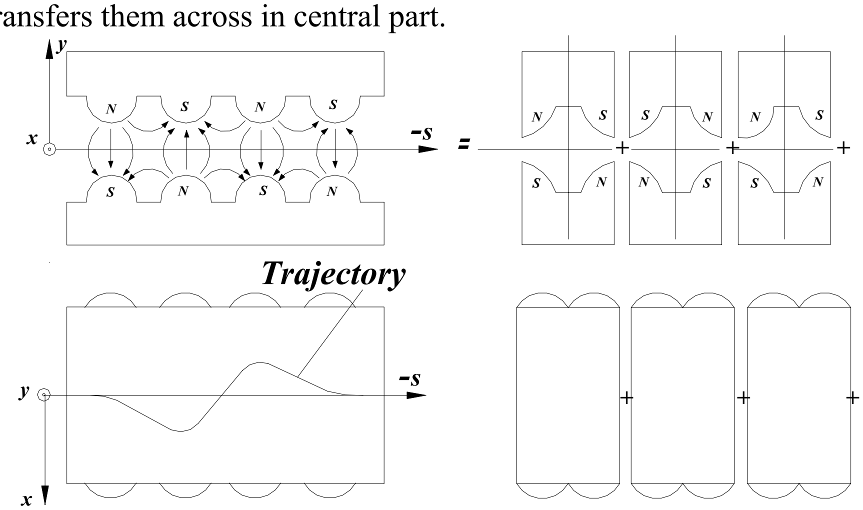

Figure 1: Ordinary dipole wiggler can be represented as series of *quadrupoles* oriented transversely. Horizontal plane of this dipole wiggler can be oriented vertically as well. Particles are moving along –*s*.

In this type of field, the wiggling angle defined by magnetic field value $\alpha \cong K/\gamma$, where

$$K = \frac{eH_{0w}\lambda\!\!\!^{-}_w}{mc^2} \cong 93.4 \cdot H_{0w}[T] \cdot 2\pi\lambda\!\!\!^{-}_w[m]$$

is so called wiggler parameter, $\lambda_w = 2\pi\lambda\!\!\!^{-}_w$–stands for the wiggler period, and H_{0w} is peak wiggler field.

One can conclude, first, that the wiggler does not act to the particle in *x*- direction (if the poles are wide enough). Second, the wiggler is focusing particles in vertical direction due to *transverse* wiggling with effective gradient (per single gap) as

$$G_{eff} \cong \alpha \frac{H_{0w}}{\lambda_w}, \tag{1}$$

which is not a function of wiggler's period. Typically gradient is strong, but the angle α is small, however.

So, usual dipole wiggler is nothing else but series of quadrupoles, oriented transversely.

Next term acting to the particle is an octupole one arising from the wiggling with angle in effective sextupole field

$$S(s) = \tfrac{1}{8} H_w''(s), \tag{2}$$

so the effective octupole acts as

$$\int O(s)ds \cong \tfrac{1}{3}\alpha_{max} S_{max} \cong \tfrac{1}{24}\alpha_{max} \frac{H_{0w}}{\lambda_w^2}. \tag{3}$$

This octupole term is a second order one with respect to the linear focusing one, so this nonliearity is weak. Namely this term responsible for dynamic aperture limitations, however.

Now let us consider the system, which generates nonlinear field in one direction in first order. Namely let us install octupoles (for example) with theirs natural axes of symmetry rectangular to the beam direction, similar as it was in previous case.

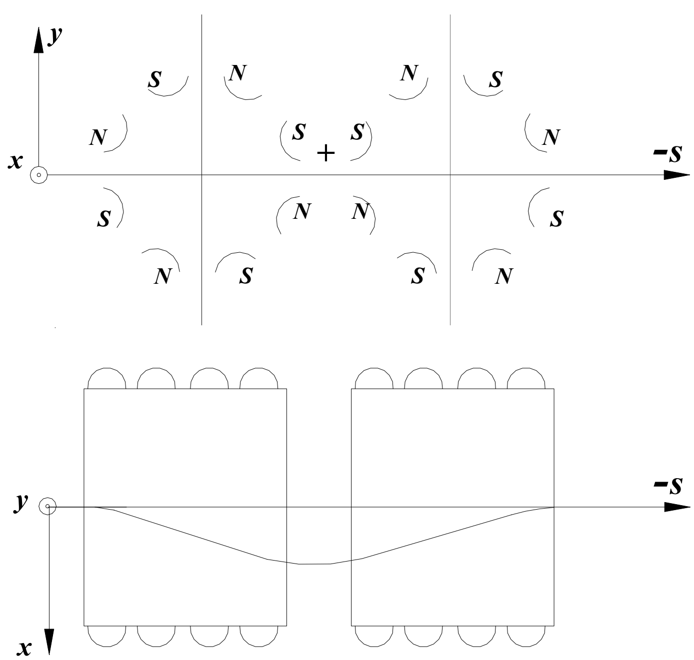

Figure 2: Octupoles installed rectangular to the beam trajectory. Trajectory is shown also. It runs ~in central part of each octupole ~along axis -*s*. For these purposes the yoke must have side openings there.

First, one can agree, that this system does not act to the particle in *x*-direction: any other trajectory, which starts parallel to the first one will come out also parallel. Second, one can conclude, that this system does not have linear component in vertical direction as the vertical focusing fields varies cubically with vertical deflection so

$$O_{eff} \cong \alpha(s)O\,, \tag{4}$$

where again α stands for a wiggling angle, and O stands for octupole value.
Octupole field $O(s)$ in normal (natural) coordinates can be expressed as

$$\begin{aligned} B_x &= O(s)(3x^2y - y^3) - \frac{5x^4y - y^5}{20}O''(s) + \cdots \\ B_y &= O(s)(x^3 - 3xy^2) - \frac{x^5 - 5xy^4}{20}O''(s) + \cdots \\ B_s &= \left(O'(s) - \frac{x^2 + y^2}{20}O'''(s) + \ldots\right)(x^3y - xy^3) \end{aligned} \tag{5}$$

These formulas allow finding the current distribution along yoke, as the current density running at the boundary must compensate tangent component of magnetic field. For example it the yoke shape is rectangular, the current density running along the walls must be parabolic.

In our case as the system of coordinates is rotated by 90 deg, $x \rightarrow s$ and $s \rightarrow x$. For wide poles field component in x direction must be zero, so in rotated coordinates $B_x \equiv 0$. The last corresponds to absence of s dependence in formulas for octupole expressed in natural coordinates (above). In new coordinates the field looks like

$$\begin{aligned} B_x &= 0 \\ B_y &= O\cdot(s^3 - 3sy^2) + \cdots \\ B_s &= O\cdot(3s^2y - y^3) - \cdots \end{aligned} \tag{6}$$

Now the octupole value itself is a free parameter, as octupoles can be made strong enough. In simplest case these might be usual octupoles. They can be made strong, for example, by making aperture (period) small. One can see, that this effective octupole, acting in one direction can be more powerful, than the one, generated in a wiggler.

Let us make some comment on possible utilization of tilted magnets. For single octupole tilted with respect to the trajectory, as it is shown in Fig.3, the acting power defined *as the difference* of the kicks obtained at the entrance of the octupole and inside. So the acting power will be determined *not* by the angle of tilt φ.

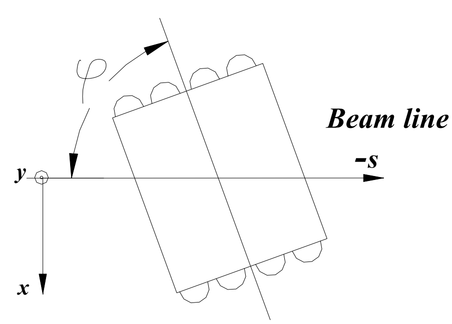

Figure 3: Single octupole/quadrupole/… tilted by angle φ in horizontal plane. The tilt can be arranged in vertical plane as well.

Acting power defined by the difference in laws between internal kick (octupole) and by the field drop outside the magnet (exponential) as well as kick angle given by the poles at entrance. Fringe field effects at the outer sides on magnet act in phase. So again, the acting power in first place defined by internal kick angle generated by the poles at the entrance. Of cause the *laws* of kick at outer parts of lens and inside are different. The law inside is an octupole one, the law outside *defined by details of magnet yoke design*. If the yoke is not closed, the field drop looks like the one from infinitely wide poles. The last one is more likely exponential. It the yoke is closed, having the holes in iron for the beam pass, the drop is much stiffer. So the net effect *for multipole desired* might be bigger, than simple difference. This single octupole can provide both focusing and defocusing effect depending on polarity of octupole and the tilt angle value. One can see that this single lens in Fig.3 generates linear term and displacement.

For practical realization one can consider octupoles with current distribution as well as octupole with poles. Examples of octupole and other multipoles with pole/current distribution one can find, for example in CBN 95-16. In this case device can be rather compact.

It is clear that instead of octupole *any other multipole* magnet can be used in this multipole wiggler as well. So this gives a possibility to make quadrupoles, sextupoles, … acting in one direction.

One can consider the multipole wiggler, Fig.2 rotated along the beam axis vertically, as well as in any other direction. Multipole tilted vertically gives multipole acting only in radial direction, while tilt with arbitrary angle generates skew component.

In scheme shown in Fig.4 three-magnet scheme generates crossing angle and closed bump. For arranging closed bump the field integrals must be in proportion 1,-2,1.
As it was mentioned the angle in wiggler is

$$\alpha \cong \frac{K}{\gamma} \cong \frac{70}{4000} = 0.0175 \text{ rad, i.e.}\sim 1\text{degree.}$$

It is necessary to remember, that in a wiggler all gaps are acting synchronously. In a typical case, as the number of gaps is ~6-7, that can be easily overwhelmed by typical octupole, however.

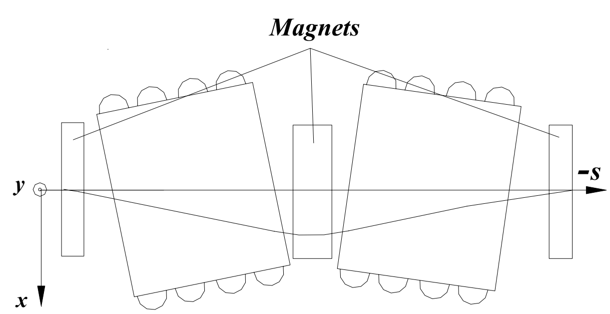

Figure 4: The attempt to increase angle α . Magnets generate closed bump.

In Fig.4 multipoles installed with additional angle, what increases the action of these multipoles, meanwhile closing trajectory done with three-magnet scheme. Again the effective power defined by *difference* in the law of the field drop inside/outside the octupole and by angle generated internally.

Let us compare the strength of wiggler octupole/quadrupole with the ones suggested above. Period can be shorter, of the order of aperture of vacuum chamber, what in case of CESR is ~2x5 =10cm compared with 40 cm, which gives ~16 times in a favor of octupole wiggler. Additional factor 24 also acts in a favor of octupole wiggler. So for single gap the power of octupole wiggler can be bigger in 16x24 times. AS the magnetic field for room temperature magnet can be generated ~10 kG, this brings the numbers to gives advantage~100. For whole wiggler this number is lower, coming to ~13 times, so two octupole wiggler system, Fig. 2, can be equivalent to ~5-10 wigglers, depending on β -function values.

One can see from Fig. 2 that if octupole is focusing in radial direction it is focusing in vertical direction too (for central region; that is why octupole is not present in field expansion of a quadrupole). So combination of octupole and appropriately oriented octupole wiggler can be used for generation of arbitrary focusing/defocusing octupole. For this multipole wiggler must be installed in series with usual octupole, and oriented so that it cancels vertical defocusing. For compensation of CESR wigglers octupoles in a wiggler must be oriented vertically. In that case the wiggler will focus in radial direction, so if additional octupole is defocusing, this will give effective defocusing only in vertical direction.

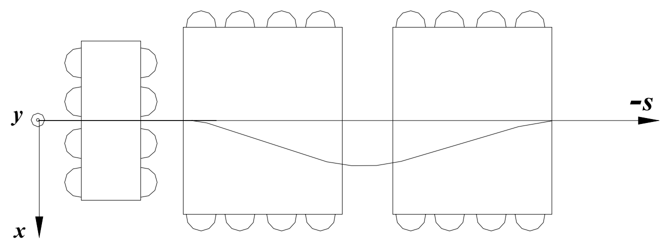

Figure 5: Combination of defocusing octupole and octupole wiggler (pair of octupoles installed normally to the beam's trajectory) with appropriate orientation (*x/y*) can be focusing/defocusing in any direction.

In this tandem of lenses, octupole acts according to formulas (5), i.e. vertical kick defined by coordinates at the entrance

$$\Delta y'(x_0, y_0) \cong \frac{\left(3x_0^2 y_0 - y_0^3\right) \cdot \int O(s) \cdot ds}{(HR)} \tag{7}$$

Meanwhile vertical kick given by octupole wiggler by pass from one pole to another is

$$\Delta y'(x_0, y_0) \equiv \frac{O_w \int \alpha(s) \cdot \left(3s^2 y_0 - y_0^3\right) ds}{(HR)} \cong \frac{O_w^2 \int\limits_{-\lambda_W/4}^{\lambda_W/4} \left(\int\limits_{-\lambda_W/4}^{s} \left(\sigma^3 - 3\sigma y_0^2\right) d\sigma \right) \cdot \left(3s^2 y_0 - y_0^3\right) ds}{(HR)^2} . \tag{8}$$

It is clear from here, that characteristic cubic dependence on vertical coordinate is present here. One can see also that total bending angle for particle from pole to pole is zero, as it must be

$$\Delta\alpha \cong \frac{O_w \int\limits_{-\lambda_W/4}^{\lambda_W/4} \left(s^3 - 3sy_0^2\right) ds}{(HR)} \equiv 0\,, \qquad (9)$$

as this is symmetric integration of odd function of longitudinal coordinate s.

This material has the level of estimations, and more careful calculations required; however the principle of operation is clear. These octupole wigglers can be made compact and removable, so they can be installed in place of CHESS permanent magnet wigglers, when they are open. We hope that such installation will help in handling nonlinearities in CESR machine.

CONCLUSION

Let us summarize the idea of proposal. In usual dipole wiggler, octupole component is a second order value and defined by deviation of longitudinal dependence from linear (next to linear components in expansion of sin-like field dependence).

Octupole (and so on) wiggler allows making this nonliearity acting in one direction much more strong (first order value) and realize it as a compact device. This device also can be easily oriented for horizontal/vertical focusing only. Together with *defocusing* octupole this allows cancellation of focusing effect introduced by usual wiggler.

In some sense one can install two usual dipole wigglers, first with magnetic field oriented vertically (usual orientation) and second one with bending magnetic field oriented horizontally. Besides the technical difficulty, much higher radiation and excitation of vertical emittance, this is the similar idea.

All other methods of cancellation of nonlinearities such as manipulation by betatron phase shift between nonlinear elements are working here too.

This publication also opens a new approach to design of a wiggler with reduced octupole nonlinearity. For this purposes, the longitudinal field dependence must be between the poles as linear as possible, *not a sin-like* (linear zigzag). This can be done either by poles profile or by distribution of currents. Unavoidable change of sign of derivative now becomes local, where transverse speed is small, so nonlinearities do not influence much.

In conclusion author thanks Maury Tigner, who attracted attention to this problem.