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TO THE PECULIARITY OF RADIATION IN A SOLENOIDAL MAGNETIC FIELD

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We attract attention that the law of radiation during motion in solenoid, allowing transverse momentum loss independently from its longitudinal one, can be used effectively for cooling of charged particles beams practically not dependently of theirs energy.

INTRODUCTION

It is well known that the force due to radiation reaction to the particle directed along the instant particle's velocity and can be expressed as the following [1]

$$\vec{F}_{rad} = \frac{2}{3}\frac{e^4}{m^2c^4}\left(F_{kl}u^l\right)\cdot\left(F^{km}u_m\right)\cdot\vec{n}\,, \tag{1}$$

where e and m are the electron charge and mass respectively, c is a speed of light, $\vec{n}$ is unit vector directed along the instant particle's velocity $\vec{n} = \vec{v}/|\vec{v}|$, $F_{kl}(x^i) = \partial_k A_l - \partial_l A_k$ (with A_k as components of 4–vector of potential) are components of 4–tensor of electromagnetic field, $u^i = \{c, \vec{v}\}$ is 4–vector of particle's velocity. This expression (1) can be represented in 3–components as the following

$$\vec{F}_{rad} = -\frac{2}{3}\frac{e^4\gamma^2}{m^2c^5}\left\{\left(\vec{E} + \frac{\vec{v}\times\vec{H}}{c}\right)^2 - \frac{\left(\vec{E}\cdot\vec{v}\right)}{c^2}\right\}\cdot\vec{v}\,, \tag{2}$$

where $\vec{E}, \vec{H}$ are electric and magnetic fields respectively. One can see that in absence of electric field the force $\vec{F} \neq 0$ only if $\vec{v}\times\vec{H} \neq 0$, or only for transverse to the speed components.

The circumstance that the particle's friction force opposes the instant velocity yields the requirements of radiation particle's energy in full to reduce the transverse momentum while longitudinal one is compensated by RF cavity. By this way the temperature of electron beam associated with transverse moment is lowering in a damping ring at the cost of radiation the full energy of particle. Even in extreme case of energy losses in a single act, the transverse emittance is reduced on expense of full energy loss [2].

One other important circumstance associated with the fact that devices supposed to force the radiation process installed in the damping ring are likely with transverse dipole fields, such as wigglers or bending magnets. They still force the radiation with the same rate even for the beams having zero transverse temperature.

Meanwhile the energy associated with transverse momentum itself is much less, than the full one. So desire to operate somehow only with transverse component of full momentum was a subject of interest since the synchrotron radiation limit in synchrotron was predicted in [3].

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In [4] an attempt was made to consider radiation in focusing channel of linear collider. In [5] more fundamental consideration of the radiation in focusing channel was made. As the focusing channel is a system of focusing/defocusing quadrupoles having gradient $\pm G(z)$ installed with period $2\pi\lambda\!\!\!^{-}_u$, the radiation here have all peculiarities of undulator radiation with undulatority factor $K = eH_\perp \lambda\!\!\!^{-}_u / mc^2$ where local magnetic field $H_\perp \cong G \cdot x$ defined by instant amplitude $x(z)$ of oscillations in the channel. Radiation is going on harmonics defined by period of focusing/defocusing lenses shrunken by Doppler effect in factor $1/(1-\vec{n}\vec{\beta})$ [6]. The length of formation of radiation with wavelength λ as always $l_f \cong 2\lambda\gamma^2 \cong 2\pi\lambda\!\!\!^{-}_u$. In this focusing channel radiation is absent for particles running at the axis (having low transverse temperature). The particle here must radiate however it's all energy as the focusing field is transverse. As the amplitude is decreasing, the power of radiation drops significantly.

All these considerations are important in attempt to reach extreme temperatures, in particular temperatures when the beam trapped in focusing channel can be considered as degenerated Fermi-gas [7].

Meanwhile in [8,9] there were considered radiation in a solenoidal field as example in each case. In both cases was indicated, that particle can lose it's transverse moment without affecting its longitudinal one. In this sense the solenoidal field in any circumstances can not be considered as a limit of any kind of focusing system with quadrupoles.

In this publication we are investigating briefly this peculiarity from the point of possible utilization for the cooling beams for lowering temperature. We found that the rates of cooling might be interesting.

LOW OF MOTION IN MOVING SYSTEM OF REFERENCE

Let us remind first the radiation law for relativistic particle in solenoidal field.

In the coordinate system moving with average velocity of an electron, particles move along helical trajectory with instant radius $R = \beta'\varepsilon' / eH$ (Fig.1). The intensity of radiation by electron[2] is

$$I = \frac{2}{3}\frac{e^4 H^4 v'^2 \gamma'^2}{m^2 c^5} = \frac{2}{3} c r_e^2 H^2 \beta'^2 \gamma'^2, \tag{3}$$

where primes denote quantities calculated in the moving system of reference: v' is the electron's velocity, $\beta' = v'/c$, $\gamma' = \varepsilon'/mc^2 = 1/\sqrt{1-\beta'^2}$, ε' is the electron's instant energy. It was substituted here also $r_e = e^2/mc^2$ –classical electron radius.

The time dependence of the electron energy is described by the equation $d\varepsilon/dt = -I$, which can be represented in the form [1]

$$\frac{d\gamma'}{dt} = -\frac{2}{3}\frac{r_e^2 H^2 \beta'^2 \gamma'^2}{mc} = -\frac{2}{3}\frac{r_e^2 H^2}{mc}(\gamma'^2 - 1), \tag{4}$$

This equation has the solution

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[2] Lower we will talk about electron; for ions an evident substitution need to be done: $e \to Ze, m \to AM$, where Z -is the charge of ion and A- it's atomic number.

$$\gamma' = \frac{sinh[\alpha\cdot(t-t_0)] + \gamma_0' \, cosh[\alpha\cdot(t-t_0)]}{cosh[\alpha\cdot(t-t_0)] + \gamma_0' \, sinh[\alpha\cdot(t-t_0)]} \tag{5}$$

where $\alpha = \frac{2}{3} r_e^2 H^2 / mc \cong 1.94\times10^{-8} H^2[G]$. One can see that at the initial moment $t = t_0$ the energy $\gamma = \gamma_0$. In the limit $t - t_0 \to \infty$ the value $\gamma' \to 1$.

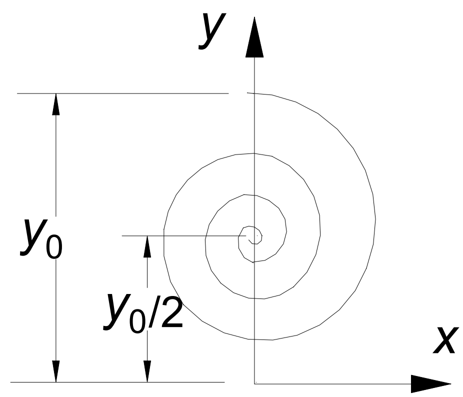

FIGURE 1: Projection of the particle's trajectory to the transverse plane.

Time dependence of the gamma for two different initial values is represented in Fig.2. The damping of the energy and amplitudes of electron oscillations in the magnetic field in the non-relativistic case described by exponential function. In the ultrarelativistic case $\gamma_0 >> 1$, according to (3) they decay independently on theirs initial energy as

$$\gamma'\big|_{\gamma_0 >> 1,\; t-t_0 >> 0} \cong ctgh[\alpha\cdot(t-t_0)] \; .$$

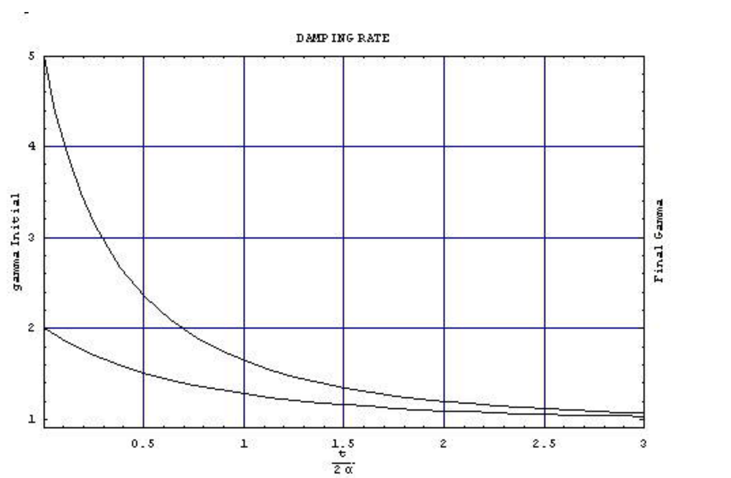

FIGURE 2: γ' as a function of normalized time $t/(2\alpha)$ for two different initial values $\gamma_0' = 2$ and $\gamma_0' = 5$.

One can see, that equilibrium value reaches practically independently of its initial values. In the non-relativistic case kinetic energy $T = mc^2(\gamma' - 1)$ changes according to (5) as

$$T \cong T_0 e^{-2\alpha(t-t_0)} . \tag{6}$$

So the damping time according to (6) goes to

$$\tau' = \frac{1}{2\alpha} = \frac{3mc}{4r_e^2 H^2} \cong \frac{2.58 \cdot 10^8}{H^2[G]} . \tag{7}$$

We have considered the process of the energy loss in the moving system. In the laboratory system the damping time will be higher $\tau = \tau' \gamma_{||}$, where $\gamma_{||} = 1/\sqrt{1-\beta_{||}^2}$, $\beta_{||}$, $\gamma_{||}$ are the relative longitudinal velocity and relative longitudinal relativistic factor in the laboratory coordinate system.
Usually the relative energy γ and the transverse relative velocity $\beta_\perp$ are given in the laboratory coordinate system. That is why the relative longitudinal velocity $\beta_{||}$ and relativistic factor $\gamma_{||}$ can be expressed through these values. For example, $\gamma_{||} = \gamma / \sqrt{1-\beta_\perp^2 \gamma^2}$. The velocity $\beta_\perp = \beta' / \gamma_{||}$.
This yields the damping time in the laboratory coordinate system to be

$$\tau = \frac{3mc}{4r_e^2 H} \frac{\gamma}{\sqrt{1+\beta_\perp^2}} . \tag{8}$$

In the laboratory coordinate system the electron moves along special drill spiral of the radius

$$R = R' = \frac{mc^2\sqrt{\gamma'^2 - 1}}{eH}\Big|_{\gamma'-1<<1} \cong \frac{\sqrt{2}mc^2}{eH} e^{-\alpha(t-t_0)} . \tag{9}$$

Let us estimate the damping times however. For longitudinal field in solenoid as high as 100 kG, (7) yields $\tau' \cong 2.58 \cdot 10^{-2}$ s in the rest frame. For a 50 MeV beam the time constant goes according to (8) to $\tau \cong 2.6$ s. If we suggest that the solenoid has the length L_{sol} ~$100m$ and the rest length of the ring $L_{rest} \cong L_{sol}$ is about the same length, one can obtain that the number of turns for cooling will need to be $N_{turns} \cong c\tau/(L_{sol} + L_{rest}) \cong 2c\tau / L_{sol} \cong 2.3 \cdot 10^6$ turns.

ENTERING CONDITIONS

Let us say a little about possible adjustment of the injection into solenoidal field. The trajectory of the electron drawn at the Fig.1 corresponds to the case when the electron enters the solenoid with zero initial polar angle with some vertical deviation from the axis. In more general axisymmetrical case let us first mention that equation $div\vec{B} = 0$ yields

$$H_r(z) = -\frac{1}{r}\int_0^r r' \frac{\partial H_z(r',z)}{\partial z} dr' \tag{10}$$

So the angular momentum defined simply by Bush's theorem

$$\Delta p_\vartheta(z) \cong \frac{e}{c}\int_{-\infty}^{z} \dot{z} \cdot H_r dt = -\frac{e}{c}\int_{-\infty}^{z} H_r dz = \frac{e}{cr}\int_0^{y_0} r' \cdot H_z(r',z) dr' \equiv \frac{e}{c}\frac{\Phi(y_0)}{2\pi r} \tag{11}$$

The same formula describes the change of the transverse momentum at the out end of solenoid, but with y_{out} in formula for flux. Without radiation the total change of transverse momentum is zero. As the dipole radiation carries polarization the particle will decrease some transverse momentum. This is a sequence of the fact that the center of helix is shifted from radial position required by $<\rho>=\rho/2$ for exit without angular moment. The coil of solenoid will accept the angular momentum arising at the entrance and the exit of solenoid.
The final position of the electron corresponds to the case when its deviation from the axis of solenoid is equal to the half of the initial one. If we will focus the electron beam on the axis of the solenoid then the initial conditions of all electrons will be such that they will rotate relative the solenoid axis and will tend to its axis.

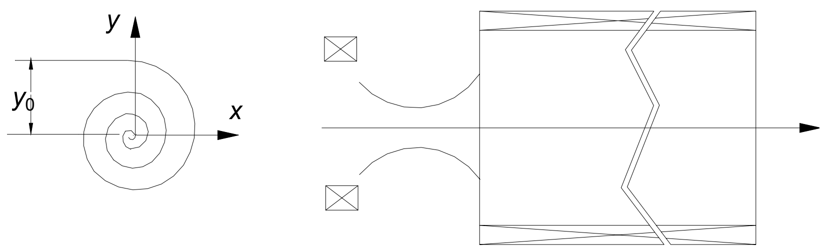

FIGURE 3: Entrance into solenoid. Short solenoid has focal point in front of the main one. Manipulating by polarity, location and strength of short trim pre-solenoid one can adjust the entering conditions.

For the initial values as the entrance of the solenoid in Lab system as $x_0, \dot{x}_0, y_0, \dot{y}_0$ one can obtain the trajectory averaged values as [10]

$$<x>=\frac{x_0}{2}+\frac{\dot{y}_0}{\omega_H}, \qquad <y>=-\frac{\dot{x}_0}{\omega_H}+\frac{y_0}{2}\ , \tag{12}$$

where $\omega_H = eH/\gamma mc$. So for shrinkage to the central axis the right parts of these expressions must be zero. So with arrangement of crossover in the focal point of solenoidal edge one can expect that the beam will shrink to the axis line. This condition is not important in the terms of damping rate and principle described, however.
From the technical point of view the transferring envelope function from the part with focusing quads the symmetrical values required by solenoid can be easily arranged by shifting the last quad to the half of period, for example.
In publication [11] described a ring for electron cooling of positrons. This ring has long solenoids and particle dynamics considered there. However SR is not included into consideration.

CONCLUSION

In general the problem under discussion is a problem of thermal relaxation near temperature of degeneration for relativistic beam with radiation and collisions in solenoidal field. We considered here only a small fraction of the problem. Even so the utilization of long solenoids in damping rings has visible potential.
High magnetic field required can be generated with the help of SC coil. Low emittance of the beam allows smaller diameter of the clearance, so the energy stored in solenoid can be made not high.

One can cool the beams having low energy proportionally decreasing the damping time in Lab system.
Cooling in solenoidal field allows natural solution to the cooling of **polarized** particles. This is in contrast to usual scheme with dipole wigglers, which destroys polarization if adequate measures are not applied[3]. Natural orientation of spin along axes of solenoid preserves it during cooling.
Damping times remain in second range, so the few bunches must circulate in the ring. The lowest temperature in the cooler with solenoid can reach extremes required for degeneration however.

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[3] Say by arrangements a transverse orientation of the spin in the region of the wiggler, or using asymmetric wigglers.