Measuring Higher Order Mode Loss Factors and Wake Voltages Using the Change in Phase of Two Bunches

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The measurement of the Higher Order Mode Loss (HOML) factor is one of the important longitudinal impedance measurements for a ring. There are several techniques to accomplish this measurement. This paper will discuss the analysis of a technique first developed at DESY in the late 1970's[1]. The method is called the phase shift method of measurement and one implementation of it uses two bunches in adjacent RF buckets. Both initially start out having very small currents with the current in the second bunch raised and the change in arrival time between the bunches measured. As the HOML and beam loading of the second bunch changes with current, the relative arrival time will change. Measuring this change gives us the value of the HOML parameter k. A second experiment increases the current in the first bunch. In this case the change in relative arrival times also includes the effect of the longitudinal wakefield from the first bunch as seen at the second bunch. Usually the analysis of these measurements makes the assumption that the very small current bunch is at the RF phase of a single particle. The present analysis does not make this assumption.

Energy Loss

For a circulating bunch the energy loss for the entire bunch is

$$q\,V(\phi) = q\,V_{SR} + q\,W\,q_0 + (\,k_{HOM} + k_{RF}\,)\,q^2$$

where q is the charge in the bunch, q_0 is the charge in a precursor bunch, $V(\phi)$ is the RF voltage at the bunch's synchronous phase, ϕ, $q\,V_{SR}$ is the synchrotron radiation energy loss, $W\,q_0$ is the wake voltage seen by bunch q originating from the charge q_0 and k_{HOM} and k_{RF} are, respectively, the high order mode loss factor and RF fundamental mode loss factor. This may be rewritten, after dividing by q, in terms of the effective voltages,

$$V(\phi) = V_{SR} + W\,q_0 + k_{HOM}\,q + k_{RF}\,q \equiv V_{SR} + V_H + k_{RF}\,q$$

Beam Loading

The bunch of charge q, as it passes through the RF cavity, also induces a voltage V_b into the cavity fundamental which is exactly oppositely phased with the beam current. This voltage may be computed from the energy loss into the fundamental mode,

$$\Delta U_{RF} = k_{RF}\,q^2$$

where

$$k_{RF} = \frac{\pi}{2} f_{RF} \left(\frac{R}{Q}\right) n_{Cells} \exp\left\{ -\left(\frac{2\pi f_{RF} \sigma_z}{c}\right)^2 \right\}$$

The voltage, which is induced into this mode by $\hat{q}$ the Fourier component of the charge at this frequency, and the energy loss are related by

$$V_b = \frac{d U_{RF}}{d \hat{q}} = \frac{d U_{RF}}{d q} \frac{d q}{d \hat{q}} = 2 k_{RF} q \exp\left\{ + \frac{1}{2} \left(\frac{2\pi f_{RF} \sigma_z}{c}\right)^2 \right\}$$

$$= \pi f_{RF} \left(\frac{R}{Q}\right) n_{Cells} q \exp\left\{ - \frac{1}{2} \left(\frac{2\pi f_{RF} \sigma_z}{c}\right)^2 \right\}$$

This voltage will be the total voltage induced into the cavity's fundamental mode by the passage of the beam. The average voltage, seen by the bunch, is the voltage that occurs at the centroid of the bunch and is $\frac{1}{2}V_b$. This voltage V_b adds to the RF voltage in the cavity causing generally a shift of the phase and a slight reduction of the RF cavity's voltage. To treat this effect we must examine a phasor diagram at the excitation frequency for the cavity.

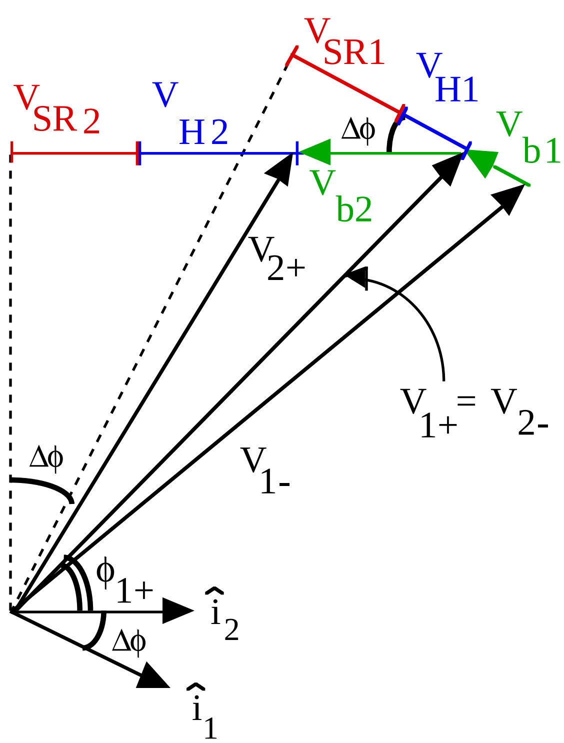

Figure 1: Phasor diagram of RF cavity voltage and beam voltages for two nearby bunches.

Phasor Analysis of Beam Loading

To make this analysis slightly more general we will study the case where there are two closely spaced bunches circulating which we will label as bunch 1 and bunch 2. In Figure 1 we see this phasor diagram which shows the cavity's voltage $V_{n\,+/-}$ after/before the passage of bunch n. Also shown are the components, $V_{SR\,n}$, $V_{H\;n}$ and $V_{b\,n}$, for bunch n. In this diagram $\hat{i}_n$ represents the phasor for the Fourier component of the beam current for each bunch. As shown above, the centroid for each bunch will operate with a synchronous phase ϕ_n at a voltage of $V_n(\phi_n)$ as needed to make up for the energy loss per turn. The phasor which represents the average voltage $V_n(\phi_n)$ seen by the bunch is not shown explicitly in Figure 1 since it is the average of $V_{n\,+}$ and $V_{n\,-}$. In general $\hat{i}_n$ for each bunch will be at different phases with the phase angle between the two bunches being $\Delta\phi$. This $\Delta\phi$ will be proportional to the difference in arrival times of the bunches at the cavity modulo the period of the RF frequency,

$$\Delta\phi = 2\,\pi\,f_{RF}\,\Delta t$$

First of all from Figure 1 we find

$$V_{1+}\cos\phi_{1+} = V_{SR\,2} + V_{H\;2} + V_{b\,2}$$

We can also see by examining the triangle, which has two sides that are made up of dashed lines in Figure 1, that we can write tan $\Delta\phi$ as

$$\tan\Delta\phi = \frac{V_{SR\,2} + V_{H\;2} + V_{b\,2} - \dfrac{V_{SR\,1} + V_{H\;1}}{\cos\Delta\phi}}{V_{1+}\sin\phi_{1+}}$$

or rewriting this in terms of sin $\Delta\phi$, we have

$$\sin\Delta\phi = \frac{(V_{SR\,2} + V_{H\,2} + V_{b\,2})\cos\Delta\phi - V_{SR\,1} - V_{H\;1}}{V_{1+}\sin\phi_{1+}}$$

Now we will assume that the RF system regulates the cavity voltage to be <V> on the average and assume the cavity filling time is longer than the circulation time. Since the bunches are close together, in equilibrium the cavity will start at an initial voltage and then see a voltage transient due to the beam. In the time remaining until the next bunch passage, the RF generator will restore the voltage to the initial value. If the cavity fills more slowly than the circulation time of the ring, the voltage phasor will progress linearly between its post bunch passage and initial values, so the average voltage phasor will be given by

$$\langle \vec{V} \rangle = \frac{1}{2} (\overrightarrow{V_{2+}} + \overrightarrow{V_{1-}}) = \overrightarrow{V_{1+}} + \frac{1}{2} (\overrightarrow{V_{b\,2}} - \overrightarrow{V_{b\,1}})$$

A phasor diagram showing this relationship is found in Figure 2 where the phasor for the average cavity voltage is at a phase $\langle\phi\rangle$ relative to bunch 2's beam current direction. From this diagram we have phasor components which are

$$\langle V \rangle \sin \langle\phi\rangle = V_{1+} \sin \phi_{1+} - \frac{1}{2} V_{b\,1} \sin \Delta\phi$$

and

$$\langle V \rangle \cos \langle\phi\rangle = V_{1+} \cos \phi_{1+} + \frac{1}{2} V_{b\,1} \cos \Delta\phi - \frac{1}{2} V_{b\,2}$$

$$= V_{SR\,2} + V_{H\,2} + \frac{1}{2} V_{b\,1} \cos \Delta\phi + \frac{1}{2} V_{b\,2}$$

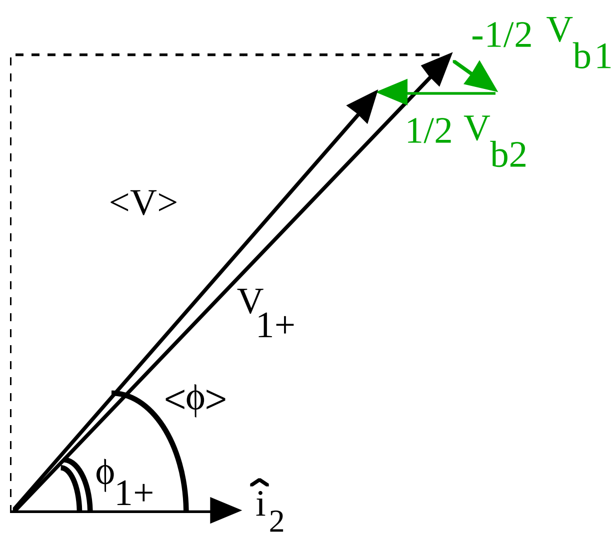

Figure 2. Phasor diagram for the average cavity voltage.

Using

$$V_{1+} \sin \phi_{1+} \sin \Delta\phi = (V_{SR\,2} + V_{H\,2} + V_{b\,2}) \cos \Delta\phi - V_{SR\,1} - V_{H\,1}$$

which has been rearranged from above and the two preceding equations it is possible to write an exact polynomial expression for $\sin \Delta\phi$. However, since we are looking for solutions for $\Delta\phi$ which are near zero, we can approximate $\sin \Delta\phi$ by $\Delta\phi$ which is taken to be much less than 1. Therefore, keeping terms up to $\Delta\phi^2$, the three previous equations become

$$\langle V\rangle \sin \langle\phi\rangle \cong V_{1+} \sin \phi_{1+} - \frac{1}{2} V_{b\,1} \Delta\phi$$

$$\langle V\rangle \cos \langle\phi\rangle \cong V_{SR\,2} + V_{H\,2} + \frac{1}{2} V_{b\,1}\left(1 - \frac{1}{2} \Delta\phi^2\right) + \frac{1}{2} V_{b\,2}$$

$$V_{1+} \sin \phi_{1+} \Delta\phi \cong (V_{SR\,2} + V_{H\,2} + V_{b\,2})\left(1 - \frac{1}{2} \Delta\phi^2\right) - V_{SR\,1} - V_{H\,1}$$

Combining the first two equations gives

$$\langle V\rangle^2 \cong \left\{V_{1+} \sin \phi_{1+} - \frac{1}{2} V_{b\,1} \Delta\phi\right\}^2 + \left\{V_{SR\,2} + V_{H\,2} + \frac{1}{2} V_{b\,1}\left(1 - \frac{1}{2} \Delta\phi^2\right) + \frac{1}{2} V_{b\,2}\right\}^2$$

Multiplying this equation by $\Delta\phi^2$ and combining with the preceding equation produces

$$\langle V\rangle^2 \Delta\phi^2 \cong \left\{(V_{SR\,2} + V_{H\,2} + V_{b\,2})\left(1 - \frac{1}{2} \Delta\phi^2\right) - V_{SR\,1} - V_{H\,1} - \frac{1}{2} V_{b\,1} \Delta\phi^2 \right\}^2$$

$$+ \left\{V_{SR\,2} + V_{H\,2} + \frac{1}{2} V_{b\,1}\left(1 - \frac{1}{2} \Delta\phi^2\right) + \frac{1}{2} V_{b\,2}\right\}^2 \Delta\phi^2$$

If we now recall that the synchrotron radiation loss is the same for all bunches ($V_{SR\,1}=V_{SR\,2}=V_{SR}$) and then expand the previous equation, we obtain the following, after keeping terms no higher than $\Delta\phi^2$,

$$\{V_{H\,2} + V_{b\,2} - V_{H\,1}\}^2 + \{V_{H\,2} + V_{b\,2} - V_{H\,1}\}\{V_{SR} + V_{H\,2} + V_{b\,2} - V_{b\,1}\} \Delta\phi^2$$

$$+ \left\{V_{SR} + V_{H\,2} + \frac{1}{2} V_{b\,1} + \frac{1}{2} V_{b\,2}\right\}^2 \Delta\phi^2 - \langle V\rangle^2 \Delta\phi^2 \cong 0$$

Solving for $\Delta\phi$ and keeping the positive root (as determined from examining the first equations in which we have the small $\Delta\phi$ approximation),

$$\Delta\phi \cong \frac{V_{H\,2} - V_{H\,1} + V_{b\,2}}{\sqrt{\langle V\rangle^2 - \left\{V_{SR}+V_{H\,2}+\frac{1}{2}V_{b\,1}+\frac{1}{2} V_{b\,2}\right\}^2 - \{V_{H\,2}+ V_{b\,2}-V_{H\,1}\}\{V_{SR}+V_{H\,2}+V_{b\,2}-V_{b\,1}\}}}$$

$$\cong \frac{V_{H\,2} - V_{H\,1} + V_{b\,2}}{\sqrt{\langle V\rangle^2 - V_{SR}^2}} \equiv \frac{V_{H\,2} - V_{H\,1} + V_{b\,2}}{\langle V\rangle \sin \phi_{SR}}$$

with the second approximation occuring when there is a large over–voltage for the RF system. This is equivalent to saying that $\sin \langle\phi\rangle \cong \sin \phi_{SR}$, where ϕ_{SR} is the synchronous phase for a single particle (which includes the effect of synchrotron radiation only.) From this result we will be interested in the rate of change in $\Delta\phi$ with charge in the bunch in two separate cases.

Case 1. Precursor Bunch and Main Bunch

For the first case the charge in the precursor (first) bunch is held fixed while the charge in the main (second) bunch is changed. We are interested in the change of $\Delta\phi$ (and, hence, Δt) vs q_2 or

$$\frac{d\,\Delta t}{d\,q_2} = \frac{1}{2\,\pi\,f_{RF}}\,\frac{d\,\Delta\phi}{d\,q_2} = \frac{k_{HOM} + \pi\,f_{RF}\left(\frac{R}{Q}\right) n_{Cells}\,\exp\left\{-\frac{1}{2}\left(\frac{2\,\pi\,f_{RF}\,\sigma_z}{c}\right)^2\right\}}{2\,\pi\,f_{RF}\,\langle V\rangle\,\sin\phi_{SR}}$$

Assuming all the RF system parameters are known, the loss parameter k_{HOM} can then be determined, as

$$k_{HOM} = 2\,\pi\,f_{RF}\,\langle V\rangle\,\sin\phi_{SR}\left(\frac{d\,\Delta t}{d\,q_2}\right) - \pi\,f_{RF}\left(\frac{R}{Q}\right) n_{Cells}\,\exp\left\{-\frac{1}{2}\left(\frac{2\,\pi\,f_{RF}\,\sigma_z}{c}\right)^2\right\}$$

In the circumstance where the $\langle V\rangle \sin \phi_{SR}$ is not known accurately, it may be determined by measuring the synchrotron oscillation frequency, f_S, at low current from

$$\langle V\rangle\,\sin\phi_{SR} = \frac{2\,\pi\,f_s^{\,2}\,T_0\,E_0}{f_{RF}\,\alpha_p\,e}$$

where T_0 is the circulation time for the ring, E_0 is the beam energy, α_p is the momentum compaction factor and e is the charge of an electron.

Case 2. Main Bunch and Trailing Bunch

In this case the charge in the trailing (second) bunch is constant while the charge in the main (first) bunch is changed. This time the change of Δt vs q_1 is

$$\frac{d\,\Delta t}{d\,q_1} = \frac{1}{2\,\pi\,f_{RF}}\,\frac{d\,\Delta\phi}{d\,q_1} = \frac{W_2 - k_{HOM}}{2\,\pi\,f_{RF}\,\langle V\rangle\,\sin\phi_{SR}}$$

Again assuming all the RF system parameters are known and the loss parameter k_{HOM} has been determined, we can then find W_2 the wake function present at the trailing bunch due to the main bunch by

$$W_2 = 2\,\pi\, f_{RF} <V> \sin \phi_{SR} \left(\frac{d\,\Delta t}{d\,q_1} \right) + k_{HOM}$$

Since it is possible to position the second bunch in any (nearby) trailing RF bucket, we can determine the value of the wake function at each of the corresponding times.

Discussion

The two cases calculated above present a technique for determining the loss parameter and the longitudinal wakefield for a storage ring. The measurement of the rate of change of the relative time delay vs bunch charge may be performed in several different ways, two of which are mentioned here. The first method uses a streak camera and fits the bunch shape to determine the time delay between the bunches. A second method measures the bunch signal from a beam position monitor (stripline or button type) with a sampling scope to find the time delay. With today's technology and bunch lengths in the range of 1 to a few centimeters both methods give comparable accuracy. At CESR both types of methods for determining the change in phase vs current have been employed.

Acknowlegement

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References

[1] M. Bassetti, et al , "First Measurement of Parasitic Mode Loss in PETRA", DESY–Report 79/07