Estimation of Bunch Lengthening for the Designed Low Energy (1.88 GeV) Ring*

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1 INTRODUCTION

2 THE BUNCH LENGTH FORMULA

In general, the bunch length, σ_s, is inversely proportional to the synchrotron frequency Ω_s:

$$\sigma = \frac{\alpha_p}{\Omega_s} \cdot \frac{\sigma_E}{E} \tag{1}$$

α_p, $\frac{\sigma_E}{E}$ are the momentum compaction and energy spread. At low current $\sigma = \sigma_o$, $\Omega_s = \Omega_{so} \propto \sqrt{\dot{V}_{RF}}$. When the current is increased, the slope of the beam induced voltage, $\dot{v}_b$, at the center of the bunch perturbs the slope from the RF system $\dot{V}_{RF}$ causing a change in the synchrotron frequency $\Omega_s \propto \sqrt{\dot{V}_{RF} + \dot{v}_b}$.

Treating this change in slope as a perturbation over the entire bunch, we may estimate the bunch length by:

$$\frac{\sigma}{\sigma_o} = \frac{\Omega_{so}}{\Omega_s} = \frac{1}{\sqrt{1 + \frac{\dot{v}_b}{\dot{V}_{RF}}}} \simeq \left(1 - \frac{\dot{v}_b}{2\dot{V}_{RF}}\right) \tag{2}$$

It can be seen from $Equation$ 2 that there will be lengthening or shortening of the bunch depending on the sign of the slope of the induced voltage v_b.

3 INDUCED VOLTAGE

In places where there are small discontinuieties in the beam pipe, like in sliding joints and bellows, the impedance seen by the bunches [1] [2] is inductive with $\dot{v}_b < 0$.

$$\dot{v}_b \propto L\frac{di}{dt} \tag{3}$$

Which will cause bunch lengthening.

Large discontinuietis like the horizontal separator behave like a resistor, as shown in $Figure$ 1 with $\dot{v}_b = 0$

$$\dot{v}_b \propto R \cdot i \tag{4}$$

Causing no change in the bunch length

The super conducting cavities look slightly capacitive [?] with $\dot{v}_b > 0$

$$\dot{v}_b \propto -\int i \cdot dt \tag{5}$$

Making the bunches shorter.

4 ESTIMATION OF THE EXPECTED SINGLE BUNCH LENGTHENING FOR LOW ENERGY RING (1.88 GEV)

The low energy ring is designed to operate with an equilibrium (low current) bunch length of $0.7\ cm$. The energy spread and bunch current for both the L.E. designed ring and the present $5.4\ Gev$ can be seen in $Table$ 1.

Table 1: Loss parameter and energy loss by $1\ cm$ bunch for 1.06 inch and 1.3 inch opening

	CORNELL STORAGE RING	
	Designed Low-Energy Ring	Present Operation High Tune Lattice
E (GeV)	1.88	5.4
$\frac{\Delta E}{E}$	$8 \cdot 10^{-4}$	$6.699 \cdot 10^{-4}$
α_p	0.01116	0.006786
$I_b\ (ma)$	4	9

4.1 RF Voltage Needed to support 0.7 cm bunch at the L.E. Ring

In order to achieve the shorter bunch of $0.7\ cm$ the RF voltage, V_{RF}, should be increased. An estimation of this voltage can be done by scaling the present equilibrium bunch length, in the High Tune Lattice, of $1\ cm$ with the desired $0.7\ cm$. Recalling that the synchrotron frequency is proportional to $\sqrt{\frac{\dot{V}_{RF}}{E}}$ the equilibrium bunch length is:

$$\sigma_o \propto \frac{1}{\sqrt{\frac{\dot{V}_{RF}}{E}}} \tag{6}$$

The ratio of the bunch lengths at the two energies:

$$\frac{0.7}{1.0} = \sqrt{\frac{E_{(L.E.)}}{\dot{V}_{RF_{(L.E.)}}}}\sqrt{\frac{\dot{V}_{RF}}{E}}\frac{\alpha_p(L.E.)}{\alpha_p}\frac{\left(\frac{\Delta E}{E}\right)_{(L.E.)}}{\left(\frac{\Delta E}{E}\right)} \tag{7}$$

$$\sqrt{\frac{\dot{V}_{RF_{(L.E.)}}}{\dot{V}_{RF}}} = \frac{1.0}{0.7}\sqrt{\frac{E_{(L.E.)}}{E}}\frac{\alpha_p(L.E.)}{\alpha_p}\frac{\left(\frac{\Delta E}{E}\right)_{(L.E.)}}{\left(\frac{\Delta E}{E}\right)} \tag{8}$$

$$\frac{\dot{V}_{RF_{(L.E.)}}}{\dot{V}_{RF}} = 2.74 \tag{9}$$

Since the RF frequency remains the same:

$$V_{RF(L.E.)} = 2.74 V_{RF} \tag{10}$$

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4.2 *Bunch Induced Voltage at the Low-Energy Ring*

The induced voltage v_b is proportional to the wake potential w_b:

$$v_b = -w_b \cdot Q = -w_b \cdot I_b \cdot T \qquad (11)$$

and the slope of the induced voltage $\dot{v}_b$ is:

$$\dot{v}_b = -\dot{w}_b \cdot I_b \cdot T \qquad (12)$$

Where Q is the total charge of the bunch and T is the revolution time around the ring. A plot of the calculated wake field using the MAFIA code with 1.0 cm and 0.7 cm bunches going through a sliding joint with 1.3 $inch$ opening is shown in $Figure$ 2. Note the slope of $\dot{w}_b$ at the center of the bunch for the 0.7 cm bunch length is 2.6 times higher than that of the 1.0 cm bunch.

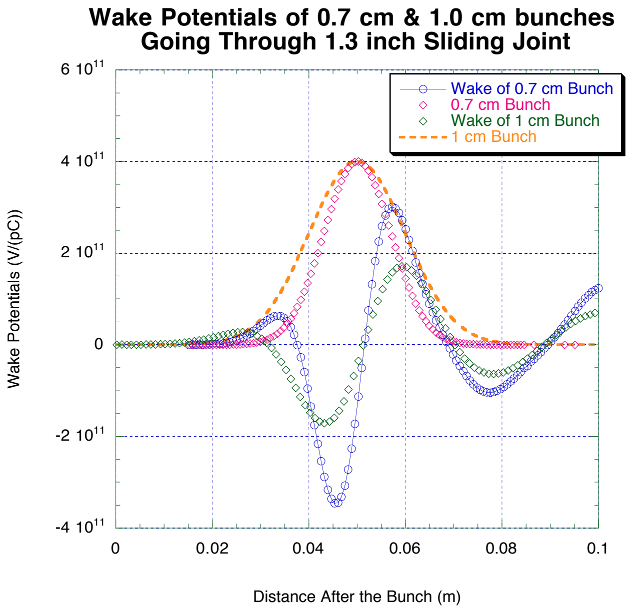

Figure 1: Wake potential of 0.7 cm and 1.0 cm bunches going through a 1.3 inch opening sliding joint.

4.3 *Bunch Lengthening at the Low-Energy*

rewriting the bunch length as function of the wake potential:

$$\sigma_s = \sigma_o \left[1 + \frac{1}{2}\frac{\dot{w}_b \cdot I_b \cdot T}{\dot{V}_{RF}}\right] = \sigma_o + \Delta\sigma \qquad (13)$$

And the bunch lengthening:

$$\Delta\sigma = \frac{1}{2}\frac{\dot{w}_b \cdot I_b \cdot T}{\dot{V}_{RF}} \qquad (14)$$

Since the bunch lengthening is caused mainly by the sliding joints, as shown in $Section$ 2, we will plug in $Equation$ 14 the value of $\dot{w}_b$ extracted from $Figure$ 2 and the L.E. ring parameters summarized in $Table$ 1.

$$\Delta\sigma(L.E.) = \frac{1}{2}\frac{\dot{w}_b 2.6 \cdot \frac{I_b}{2.25} \cdot T}{\dot{V}_{RF} 2.74} \qquad (15)$$

$$\Delta\sigma(L.E.) = \Delta\sigma_{(5.4\ GeV,\ 1.0\ cm)} \cdot 0.21 \qquad (16)$$

Where $\Delta\sigma_{(5.4\ GeV,\ 1.0\ cm)} = 0.07\ cm$ as calculated in $Reference$ [2] for a single 1 cm bunch going through 100 sliding joints with $V_{RF} = 6\ MV$ and $I_b = 9\ ma$. Thus: $\frac{\Delta\sigma(L.E.)}{\sigma_o} = 2.1\%$

5 LONGITUDINAL LENGTHENING OF BUNCHES IN BUNCH TRAINS

Measurements of bunch lengthening in the present 5.4 Gev operation versus current using streak camera [4] and bunch spectrum [3] resulted in lengthening of 15% while the calculation of a single bunch predicted only 7% lengthening. This could be explained by the additional increased in the slope of the wake due to interaction between bunches as simulated in $Reference$ [5].

6 CONCLUSIONS

The bunch lengthening for the Low-Energy ring is expected to be about 2.1% for single bunch. Assuming the same variations between the bunches in a bunch train operation, we should expect 5% lengthening.

7 REFERENCES

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[5] Z. Greenwald, D. L. Rubin, "Variation in Lengthening of Bunches in Bunch Train Operation in CESR", CBN 00-15