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Some aspects of the use of Vibrating Wire Technique for a wiggler magnetic field measurement.[1]

Alexander B. Temnykh

Laboratory of Nuclear Studies, Ithaca NY 14853, USA

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Abstract

Nonlinear perturbation in beam dynamics caused by wiggler's magnetic field may significantly degrade storage ring performance. The commonly used magnetic measurement techniques such as Hall probe, long flipping coil measurement, scan with a small searching coil can not revile all causes of the perturbation. Since the beam trajectory is wiggling, the field seen by particles moving through the wiggler is different from the field measured along strait lines with above techniques.

In contrast, the modified vibrating wire (VW) technique described here may provide a simple way to measure magnetic field along the path imitating wiggling beam trajectory. The VW technique uses the taut wire stretched through the testing field region as a magnetic field probe. Application of DC current to the wire will cause its displacement imitating the beam trajectory[2], then exciting the wire vibrating modes with AC current one can measure magnetic field along the wire, i.e., along the path imitating beam trajectory.

The paper presents results of the wiggler magnetic field measurements using "wiggling" and strait vibrating wire. Comparison reveals effects caused by the trajectory wiggling. Good agreement with theoretical model was observed.

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[2]This effect is well known and sometimes is used for magnetic field measurement

1 Introduction and Theory

Equation for the displacement of the taut wire with DC current stretched through the magnetic field and equation for a particle motion in the magnetic field are very similar:

$$T\frac{\partial^2 X_w}{\partial z^2} = -I_{dc}B_y(z) \tag{1}$$

$$\frac{\partial^2 X_p}{\partial z^2} = \frac{q}{P}B_y(z) \tag{2}$$

Here (1) describes the wire displacement and (2) is for particles motion. In these equations z is a longitudinal coordinate, $B_y(z)$ is vertical magnetic field as function of z, X_w and X_p are the wire and particle horizontal displacements resprectively, T is the wire tension, I_{dc} is the DC current applied to the wire, q and P is a particle charge and momentum. The equations become identical if

$$\frac{I_{dc}}{T} = -\frac{q}{P} \tag{3}$$

In many cases the vertical magnetic field in wigglers can be approximated as

$$B_y \simeq B_w \sin(2\pi z/d) \tag{4}$$

where d is a wiggler period, B_w maximum wiggler field. For this type of field the wire horizontal displacement (and beam trajectory) will be sinus-like function

$$X_w(z) = A_w \sin(2\pi z/d) \tag{5}$$

with the wiggling amplitude

$$A_w = \frac{I_{dc}B_w}{T}(\frac{d}{2\pi})^2 \tag{6}$$

For $B_w = 0.780$T and $d = 12$cm (used in experiments described below) the later gives:

$$A_w[mm] = 0.28\ \frac{I_{dc}[A]}{T[N]} \tag{7}$$

Let's consider effects caused by the wiggling of beam trajectory using theories described in [2] and [3].

One effect is a vertical focusing. Because of the trajectory wiggling in horizontal plane, longitudinal wiggler field component in coordinate system related to the trajectory will produce horizontal field. In linear approximation and under assumption that the wiggler field is a sinus-like function (4), the dependence of the horizontal field integrated along the wiggling path (5) on vertical position at $x = 0$ will be:

$$dI_x(x=0,y) = \int_0^L B_z X'_w(z)dz = -\frac{B_w A_w L}{2}(\frac{2\pi}{d})^2 y \tag{8}$$

Here L is wiggler length. This field causes a beam deflection in vertical plane which results in vertical focusing. [1] For $B_w = 0.780$T, $d = 12$cm, $L = 3m$ and $A_w = 0.060$ mm (parameters used in our experiments) the above expression gives:

$$dI_x(x=0,y)[Gm] = -1.92 \cdot y[mm] \tag{9}$$

Note that at $x = 0$ the wiggling does not effect horizontal field integral, i.e.,

$$dI_y(x=0,y) = 0 \tag{10}$$

Another effect caused by the trajectory wiggling relates to the single poles field roll-off. Let $B_p(x)$ be the dependence of vertical field on horizontal position across a single pole. Consider trajectories at $y = 0$. Because of the different horizontal position in "positive" and "negative" wiggler poles the vertical field integral over the wiggling trajectory will depend on wiggling amplitude, $B_p(x)$ and horizontal position x as:

$$dI_y(x,y=0) = -\frac{A_w L_w}{2}\frac{\partial B_p(x)}{\partial x} \tag{11}$$

If wiggler pole design is symmetric in x, i.e., $B_p(x)$ is even function of x, then $\partial B_p(x)/\partial x$ and $dI_y(x, y = 0)$ should be odd function and will produce terms like quadrupole, octupole and etc. multipoles. The Hall probe measurement of the vertical magnetic field across of single pole of the wiggler used in our experiments indicated dependence:

$$B_p(x)[G] = 7.80 \times 10^3 - 6.53 \times 10^{-8} \cdot x^6[mm] \tag{12}$$

[1] It should be mentioned that while particles moving trough the wiggler are always experiencing focusing effect, the taut wire with DC current, independent of the current direction, is always pushed off the magnet center indicating defocusing effect.

Length [m]	3.0
Number of poles	50
Period [cm]	12.0
Peak magnetic field [T]	0.780
Gap [cm]	4.0
Pole width [cm]	11.0

Table 1: General parameters the wiggler used in measurements.

For $A_w = 0.060$ mm and $L = 3m$ (experimental condition) equation (11) gives the dependence of vertical field integral on x :

$$dI_y(x, y = 0)[Gm] = 3.56 \times 10^{-8} \cdot x^5[mm] \quad (13)$$

Note that at $y = 0$ horizontal field integral should not depend on the trajectory wiggling:

$$dI_x(x, y = 0) = 0 \quad (14)$$

2 Measurement Setup

In the measurements the wiggler recently built for Cornell High Energy Synchrotron Source was used. The wiggler's general parameters are given in Table 1.

The measurement setup schematically shown on Figure 1. It consisted of copper-beryllium 100 micron diameter wire 383.2cm in length (1), stretched through the wiggler(2). The wire mid point has been placed exactly in the middle of wiggler. The wire ends were fixed on stages movable in horizontal and vertical directions. At one end of the wire was a tension mechanism(3), while optical wire motion detectors(4) were placed close to the opposite end. A "LAB-NB" board installed in the Macintosh and programs based on "LabView" software provided all needed signal analysis. Power supply driving DC current used to "wiggle" the wire was connected to the wire ends in series with transformer. AC current in wire was excited by wave form generator "HP33120A" connected to the other transformer input. The wire sag estimated from fundamental mode frequency ($\sim$ 18Hz) was 0.9mm.

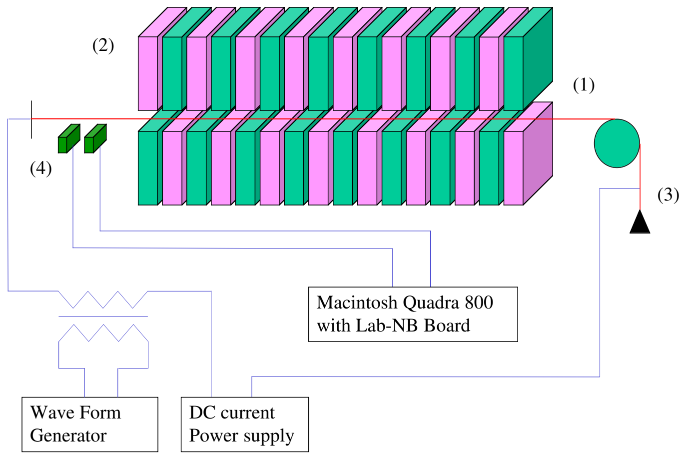

Figure 1: Measurement setup. (1) - 100 micron copper-beryllium wire, (2) - wiggler, (3) - tension mechanism, (4) - horizontal and vertical wire position detectors.

3 Results

3.1 Wiggling amplitude calibration

The first experiment was to calibrate wire wiggling amplitude. The wire x position in the 7 adjacent poles was measured by optical means as function of DC current flowing though the wire and tension. The results of the measurement shown in Figure 2 indicated the linear dependence of wiggling amplitude A_w on ratio I_{dc}/T. Fit with linear function of the data yields:

$$A_w[mm] = (0.255 \pm 0.006) \cdot \frac{I_{dc}[A]}{T[N]} \tag{15}$$

This is in good agreement with calculation in Section 1.

3.2 The horizontal field appearence due to wiggling.

In this experiment, horizontal magnetic field along strait and wiggling wire was measured using VW technique at three vertical positions (y = 5mm, $y = 0$ and $y = -5$mm) at $x = 0$. The 1.68N tension and 0.8A DC current applied to the wire resulted, according to calibration (15), in $A_w = 0.124$mm

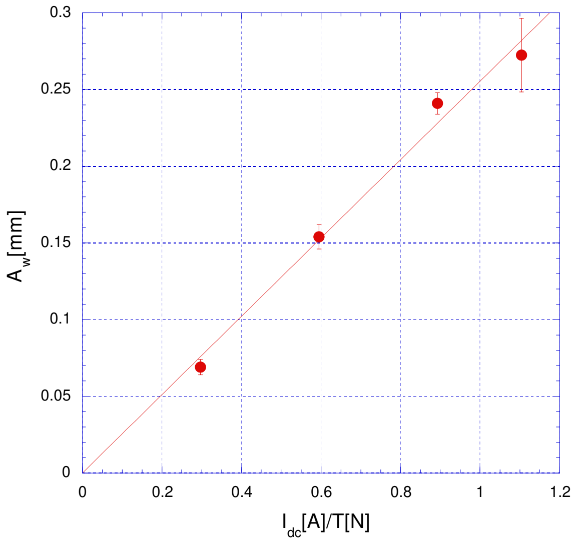

Figure 2: Measured dependence of the wire wiggling amplitude on ratio I_{dc}/T. I_{dc} is a current flowing through the wire, T - the wire tension. Fit with linear function gives $A_w[mm] = (0.255 \pm 0.006) \cdot I_{dc}[A]/T[N]$

of the wire wiggling amplitude. Horizontal field distribution along the wire was measured using 9 odd (1^{th},3^{d} ... 17^{th}) modes of vertical wire vibration. Only odd order modes were used because modes of even orders were strongly affected by systematic field perturbation at the wiggler ends. Results are shown in Figure 3. The plots depict the difference between data obtained with strait and wiggling wire, i.e., the horizontal field appeared due to the wire wiggling. At $y = 5$mm and $y = -5$mm the field has approximately equal amplitude but opposite sign. At $y = 0$, where longitudinal field is zero, the horizontal field did not appear. This is consisted with a wiggler model. It should be mentioned that for 12cm wiggler period, 5mm vertical offset and 0.125mm amplitude wiggling, the model calculation gives the horizontal field averaged over wiggler period of 0.8×10^{-3} of maximum wiggler field. This suggests that 1[r.u.] of the vertical scale corresponds to 4×10^{-4} of maximum wiggler field or 3.3G.

Another remark is regarding field distribution along z. Because the length of the wire standing waves used in this measurement was longer than the wiggler period, the horizontal component of magnetic field was averaged

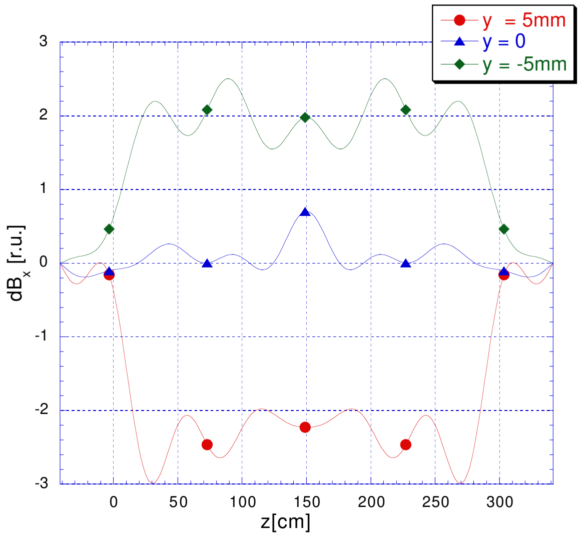

Figure 3: Horizontal field at different vertical position caused by wiggling. 1[r.u] in vertical scale $\simeq 4 \times 10^{-4}$ of maximum wiggler field or 3.3G, see in text. Magnetic structure (wiggler) located between $z = 0$ and $z = 300$cm.

over few the wiggler periods. The result was that the measured distribution appeared to be uniform along the magnet.

3.3 Effect of the wiggling on vertical and horizontal field integrals.

If the function describing distribution of the field along magnet is known, one can use only one wire vibrating mode to measure the field integral. Let's prove this statement for a wiggler field. Suppose the distribution of the field distortion caused by the wiggling along integrating path (wire) is:

$$\delta B(s) = \delta B_0 \sin^2(2\pi s/d);\ \ s_1 \le s \le (s_1 + L) \tag{16}$$

Here s_1 is the position of wiggler's end, L is the wiggler's length, $L = Nd$. B_0 is the distortion amplitude, which is not known. The field integral over the path along the wire, $0 \le s \le l_w$, l_w is the wire length, will be:

$$dI = \int_0^{l_w} \delta B(s) ds = \frac{\delta B_0 L}{2} \tag{17}$$

From other side, the first longitudinal field harmonic, H_1, which can be directly measured with fundamental wire vibrating mode can be written as:

$$H_1 = \frac{2}{l_w}\int_0^{l_w} \delta B(s)\sin(\frac{\pi s}{l_w})ds = \delta B_0 \frac{2}{\pi}\sin(\frac{\pi L}{2l_w}) \tag{18}$$

It was assumed that the wire middle point is placed exactly in the middle of wiggler. Comparing expressions (17) and (19) one can find

$$dI = H_1 \frac{l_w}{2}\frac{\pi L}{2l_w}\sin^{-1}(\frac{\pi L}{2l_w}) \tag{19}$$

i.e., the field integral is equal to the first longitudinal harmonic multiplied by well defined coefficient related to the geometry of setup. Note that if the wire is much longer than the magnetic structure, $l_w \gg L$, the coefficient does not depend on the length of the structure and the expression for field integral becomes:

$$dI = H_1 \frac{l_w}{2} \tag{20}$$

In experiments described below the vibrating wire measurement was calibrated with small shim placed in the middle of magnet. Previously, the field integral change due to the shim was measured with a long flipping coil. Geometric factor was calculated using formula (19).

In the first experiment, there was measured the dependence of the change in horizontal and vertical field integrals due to wiggling versus vertical position at $x = 0$, i.e., $dI_{x,y}(x = 0, z)$. To create the wire wiggling 0.42A DC current was applied. At 1.68N tension, according to calibration (15), it resulted in 0.060mm of wire wiggling amplitude. The experimental data plotted on Figure 4. One can see evident variation of dI_x with vertical position and much smaller variation of the dI_y. Fit with liner function of dI_x gives:

$$dI_x(x = 0, y)[Gm] = (-1.99 \pm 0.17)\cdot y[mm] \tag{21}$$

This is in a perfect agreement with a model calculation made in Section 1. Note that according to model vertical field integral should not depend on wiggling,i.e., $dI_y(x = 0, y) = 0$. The observed dependence of dI_y on y can be explained by a small tilt of horizontal wire position detector around z. In result the sensor may parasitically detect signal from vertical wire vibration.

In second experiment dI_x and dI_y were measured versus horizontal wire position at $y = 0$. The wire wiggling amplitude was 0.060mm, the same as

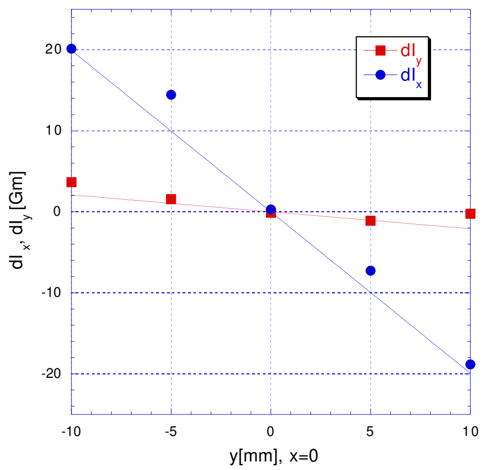

Figure 4: Difference in horizontal and vertical field integrals measured with strait and wiggling wire as function of vertical wire position at x=0. Fit with linear function gives: $dI_x[Gm] = (-1.99 \pm 0.17) \cdot y[mm]$, $dI_y[Gm] = (-0.21 \pm 0.08) \cdot y[mm]$

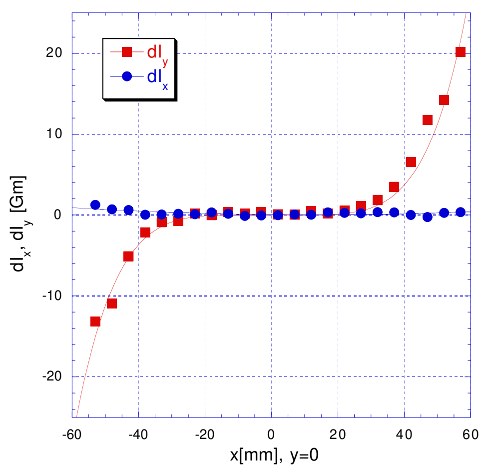

Figure 5: The difference in the field integrals measured with strait and wiggling wire as a function of horizonal wire position at y=0. Dependence of vertical field integral on x was fitted with function $dI_y = C \cdot x^5$. The fit gives $C = 3.60 \pm 0.12$ [Gm/mm^5]

in previous experiment. Results are showed in Figure 5. Fit of the measured dependence of dI_y on x yields

$$dI_y(x, y = 0)[Gm] = (3.60 \pm 0.12) \times 10^{-8} \cdot x^5[mm] \tag{22}$$

This is also in a very good agreement with the calculations made in Section [1]. Variation of dI_x with x is negligible as it should be according to the model.

4 Conclusion

Vibrating Wire technique was used to measure magnetic field distribution and field integrals along a wiggling path imitating the beam trajectory in a wiggler's magnetic field. To wiggle the wire, which ws used as a field probe, DC current was applied to it. Obtained data are in a good agreement with model calculation.

The Vibrating Wire technique has unique features which can be effectively exploited for the measuring of wiggler-like magnetic structures.

References

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[3] J. Safranek et, Nonliner Dynamics in SPEAR Wigglers, Proceedings of the 1999 Particle Accelerator Conference, New York, 1999, pp. 157-161.