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Note on wiggler field "squareness".

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Abstract

The effects of squareness of a wiggler field along the beam line on vertical tune shift, octupole moment and on field integral associated with radiation damping are considered.

1 Analytical model

Consider a wiggler of length L_w consisting of n periods with period length of $\lambda = L_w/n$, with infinity wide poles. Vertical and longitudinal magnetic field, B_y and B_z, can be described by well known equations:

$$B_y(y,z) = \sum_m B_m \cosh(mky)\sin(mkz) \tag{1}$$

$$B_z(y,z) = \sum_m B_m \sinh(mky)\cos(mkz) \tag{2}$$

Here $k = 2\pi/\lambda$, z is the coordinate along magnet, B_m are Fourier harmonics of the vertical field as a function of z. In middle plane, $y = 0$, the equation for B_y gives:

$$B_y(z) = \sum_m B_m \sin(mkz) \tag{3}$$

Integrating the vertical field,B_y, along z one can find the horizontal angle of the particle trajectory, x', as a function of the position along magnet:

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$$x'(y,z) = \frac{1}{B\rho}\int_0^z B_y(y,\bar{z})d\bar{z} = \frac{1}{kB\rho}\sum_m \frac{B_m}{m}\cosh(mky)[1-\cos(mkz)] \quad (4)$$

Here $B\rho$ is the beam stiffness. Due to x', the longitudinal magnetic field $B_z(y,z)$ will create an effective horizontal field component which will deflect the trajectory vertically. In a short lens approximation, $\beta_y \gg L_w$, the change in vertical angle caused by this effect will be:

$$y'(y) = -\frac{1}{B\rho}\int_0^{L_w} x'(y,z)B_z(y,z)dz = \frac{\lambda L_w}{8\pi(B\rho)^2}\sum_m \frac{B_m^2}{m}\sinh(\frac{4\pi m}{\lambda}y) \quad (5)$$

From this equation one can find the vertical tune shift for "zero" vertical betatron amplitude:

$$\delta\nu_y = \frac{\overline{\beta}_y L_w}{8\pi(B\rho)^2}\sum_m B_m^2 \quad (6)$$

and vertical cubic nonlinearity introduced by the wiggler:

$$C = \beta_y \frac{d\nu_y}{da_y^2} = \frac{\overline{\beta}_y^2 L_w}{8\pi(B\rho)^2} k^2 \sum_m m^2 B_m^2 \quad (7)$$

Here $\overline{\beta}_y$ is the vertical beta function at the wiggler location.

The contribution of the wiggler field to radiation damping rate will be proportional to B_y^2 integrated over the wiggler length, i.e.:

$$\delta\left(\frac{1}{\tau_{rad}}\right) \sim \int_0^{L_w} B_y^2(z)dz = \frac{L_w}{2}\sum_m B_m^2 \quad (8)$$

2 CESR-c wiggler

On figure 1 is shown the vertical magnetic field along the magnet calculated for 20 and 40cm period wigglers. Here is only 1/2 of the wiggler period. The field dependence was fitted with a function consisting of two harmonics, $B_y(z) = B_1\sin(kz) + B_3\sin(3kz)$. Using the maximum wiggler field B_{max} and a "squareness" parameter α:

$$B_{max} = B_1 - B_3 \quad (9)$$

$$\alpha = B_3/B_{max} \quad (10)$$

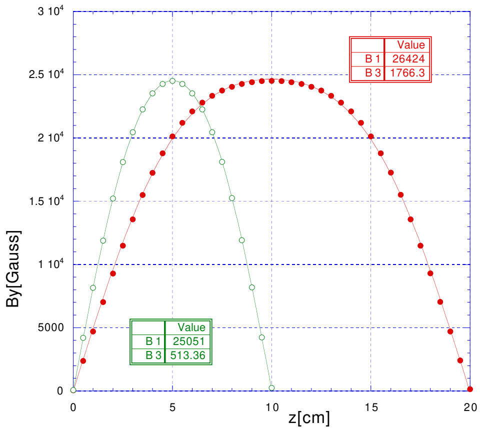

Figure 1: Vertical magnetic field as a function of coordinate along the magnet calculated for 20 (open points) and 40cm (solid points) period wigglers. Fitting with two harmonics, see text, gives $B_{max} = 2.4 \times 10^3$ Gauss, $\alpha = 0.02$ for 20cm period wiggler and $B_{max} = 2.4 \times 10^3$ Gauss, $\alpha = 0.07$ for 40cm period.

one can rewrite formulas for the vertical tune shift $\delta\nu_y$, vertical cubic nonlinearity C and $\delta\left(\frac{1}{\tau_{rad}}\right)$ as:

$$\delta\nu_y = \frac{\overline{\beta}_y L_w}{8\pi(B\rho)^2} B_{max}^2(1 + 2\alpha + 2\alpha^2) \tag{11}$$

$$C = \frac{\overline{\beta}_y^2 L_w}{8\pi(B\rho)^2} k^2\, B_{max}^2(1 + 2\alpha + 10\alpha^2) \tag{12}$$

$$\delta\left(\frac{1}{\tau_{rad}}\right) \sim \frac{L_w}{2} B_{max}^2(1 + 2\alpha + 2\alpha^2) \tag{13}$$

From these expressions one can see that the relative variation of vertical tune and radiation integral with "squareness" parameter $\sim 2\alpha + 2\alpha^2$. Variation of cubic nonlinearity $\sim 2\alpha + 10\alpha^2$ is bigger. This is depicted on figure 2.

3 Conclusion

Using harmonics representation of wiggler magnetic field, the formulas for field integral associated with damping time as well as for vertical tune shift

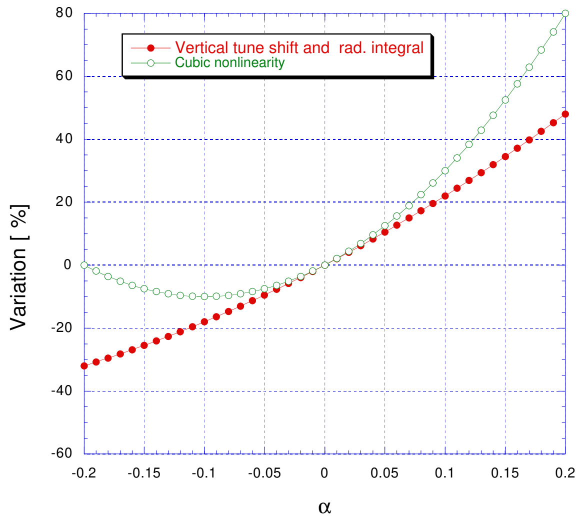

Figure 2: Variation of vertical tune shift, radiation integral (solid points) and variation of cubic nonlinearity (open points) as a function of "squareness" parameter α.

and vertical cubic nonlinearity have been derived.

Comparing vertical tune shift and radiation integral for the 20cm ($\alpha = 0.02$) and 40cm ($\alpha = 0.07$) period wigglers of equal length and maximum magnetic field, one can see that due to larger α, the 40cm period wiggler gives 11% more radiation damping and introduces 11% bigger vertical tune shift. The vertical cubic nonlinearity scaled as $1/\lambda^2$ for the 40cm period wiggler will be 4 times smaller, but larger α will cause 15% increase of the nonlinearity.

Acknowledgment

I would like to thank David Rice for his attention to this work.