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Closed orbit distortion and collision crossing angle induced by short-range wakefields

G. Dugan
Laboratory of Nuclear Studies
Cornell University
Ithaca, NY 14853

Transverse wakefields are responsible many of the dynamic instabilities observed in high-energy accelerators. However, wakefields can also cause “static” effects.[1] For a storage ring with a single bunch, the short-range transverse wakefield produced by the head of the bunch will cause a distortion in the closed orbit of the tail of the bunch. When observed at a fixed location in the storage ring, the bunch will acquire a static “tilt” or distortion. The magnitude of this tilt will depend on the bunch intensity. It will also depend on the transverse displacement of the bunch from the center of the vacuum chamber at each point around the ring. The closed orbit of the bunch centroid will also be affected, and will become a function of the beam intensity. For machines with many bunches, in addition to the single bunch effect, the trailing bunches will be influenced by the wakefields from the leading bunches; the extent of the effect will depend on the range of the wakefields.

For a collider, this static “tilt” can have deleterious consequences for the luminosity. When the beams collide, the “tilt” will amount to a crossing angle; this can cause a luminosity reduction if the difference between the orbit of the bunch head and tail is comparable to the bunch rms size at the interaction point. In this note, I make some simple estimates of this effect and work out the numbers for CESR.

Following Chao[2], the transverse kick received by a test charge e at position z, due to the preceding bunch, is

$$\Delta y'(z) = -\frac{N r_0 y_0}{\gamma}\int_z^{\infty} dz'\rho(z')W_1(z-z') \tag{1}$$

in which we consider only short-range wakes, neglecting multi-turn effects. In this equation, N is the number of particles per bunch, y_0 is the transverse displacement of the bunch, $\rho(z)$ is the bunch longitudinal charge density (normalized), and W_1 is the (short-range) wakefield.

Let us consider the case of a simple wake function

$$W_1(z) = \begin{Bmatrix} -W_0 \ \ if \ \ 0 > z > -2\hat{z} \\ 0 \ \ otherwise \end{Bmatrix} \tag{2}$$

in which $\hat{z}$ is the bunch length for a uniform bunch, for which

$$\rho(z) = \begin{Bmatrix} \dfrac{1}{2\hat{z}} \ \ if \ \ \hat{z} > z > -\hat{z} \\ 0 \ \ otherwise \end{Bmatrix} \tag{3}$$

Then Eq. (1) gives

$$\Delta y'(z) = \frac{Nr_0 y_0}{\gamma} \frac{W_0}{2\hat{z}} (\hat{z} - z) \;\; if \;\; -\hat{z} < z < \hat{z} \tag{4}$$

The maximum value of $\Delta y'$ is

$$\Delta y'_{max} = \Delta y'(-\hat{z}) = \frac{Nr_0 y_0}{\gamma} W_0 \tag{5}$$

Using the following approximate relations,

$$\begin{aligned} Z_1^{\perp} &\approx \frac{b}{c} W_0 \approx \frac{2R}{b^2} \left[\frac{Z_0^{||}}{n} \right] \\ W_0 &\approx \frac{2Rc}{b^3} \left[\frac{Z_0^{||}}{n} \right] \end{aligned} \tag{6}$$

in which b=radius of the vacuum chamber, R=ring radius, and $\frac{Z_0^{||}}{n}$ is the total longitudinal impedance, we have

$$\Delta y'_{max} \approx \frac{Nr_0 y_0}{\gamma} \frac{2Rc}{b^3} \left[\frac{Z_0^{||}}{n} \right] \tag{7}$$

This equation is in cgs units. To convert to SI units, we replace $Z_0^{\|} \Rightarrow 4\pi\varepsilon_0 Z_0^{\|}$, and introduce the impedance of free space $Z_0 = \frac{1}{c\varepsilon_0}$, to get

$$\Delta y'_{max} \approx \frac{2Nr_0 y_0}{\gamma} \frac{R}{b^3} \left[\frac{1}{30} \frac{Z_0^{||}}{n} \right] \tag{8}$$

with $\frac{Z_0^{||}}{n}$ in Ohms. As a result of this angular kick, the tail of the bunch will have a closed orbit which is different from that of the head. As a worst-case rough estimate, let us consider the entire impedance of the machine to be localized at $s=s_Z$. Then the closed orbit difference between the head and tail of the bunch will be given by

$$\Delta y_{max}(s) = \Delta y'_{max} \frac{\sqrt{\beta_y(s)\beta_y(s_z)}}{2\sin\pi\nu_y} cos\left[\psi_y(s) - \psi_y(s_Z) - \pi\nu_y\right] \tag{9}$$

In the spirit of a worst-case estimate, let us assume that the phase difference between the location of the impedance and the IP is such as to make the cos function in Eq. (9) equal to 1. Then we have, for ν_y near a half-integer,

$$\Delta y(IP) \approx \frac{\Delta y'_{max}}{2} \sqrt{\beta_y(IP)\beta_y(s_Z)} \tag{10}$$

This displacement of the bunch tail relative to the head is equivalent to an angle in the y-s plane; the angle is

$$\alpha(IP) \approx \frac{\Delta y(IP)}{2\hat{z}} \approx \frac{2Nr_0 y_0}{\gamma}\frac{R}{b^3}\left[\frac{1}{30}\frac{Z_0^{||}}{n}\right]\frac{\sqrt{\beta_y(IP)\beta_y(s_Z)}}{4\hat{z}} \qquad (11)$$

For two counter-circulating beams which have a common offset (Fig. 1) at the impedance, when the beams collide, the beams will have an effective crossing angle given by Eq. (11). For beams on pretzel orbits (electrostatic deflection), as in Fig. 2, each beam is "tilted" in the same sense, and there is no effective crossing angle when they collide (provided the tilt is perfectly linear, which is only true for a uniform bunch).

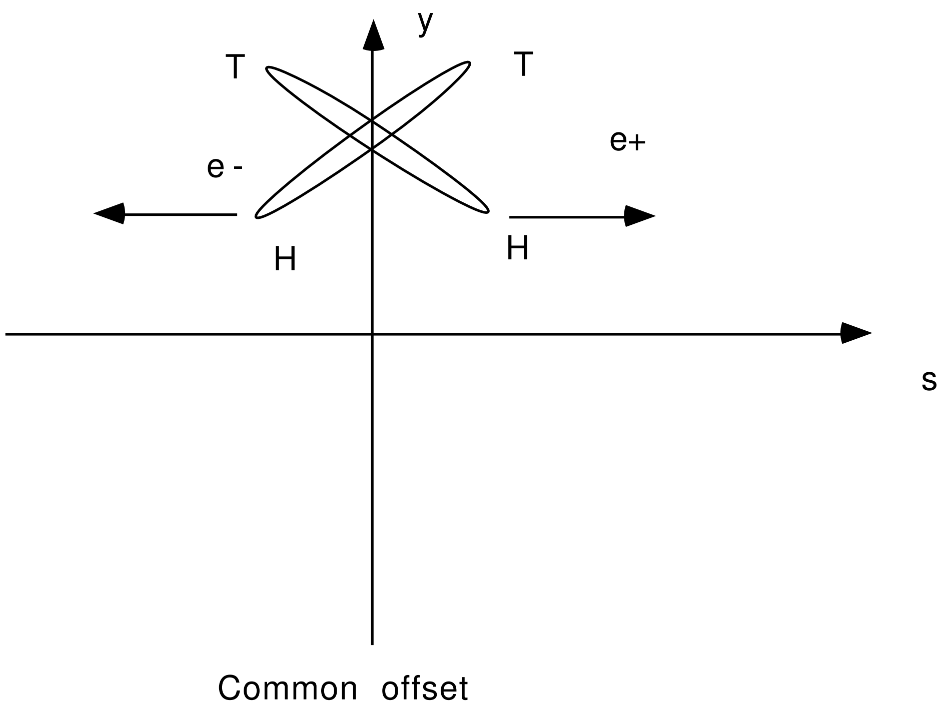

Fig. 1

Bunch tilt due to common offset of both beams from the center of the vacuum chamber

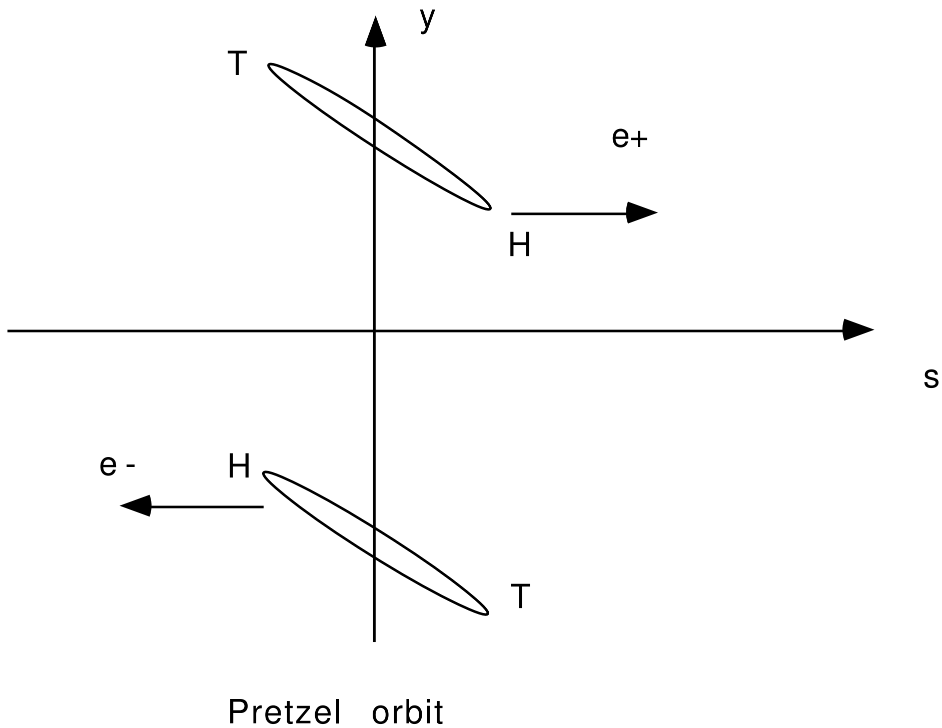

Fig. 2
Bunch tilt due to equal and opposite offsets of each beam from the center of the vacuum chamber

The figure of merit for luminosity reduction due to a crossing angle is

$$\begin{aligned} \nu_0 &= \frac{\alpha(IP)\hat{z}}{\sigma_y} = \frac{Nr_0 y_0}{\gamma}\frac{R}{b^3}\left[\frac{1}{30}\frac{Z_0^{||}}{n}\right]\frac{\sqrt{\beta_y(IP)\beta_y(s_Z)}}{2\hat{z}}\frac{\hat{z}}{\sqrt{\beta_y(IP)\varepsilon_y}} \\ &= \frac{Nr_0 y_0}{2\gamma}\frac{R}{b^3}\left[\frac{1}{30}\frac{Z_0^{||}}{n}\right]\sqrt{\frac{\beta_y(s_Z)}{\varepsilon_y}} \end{aligned} \tag{12}$$

The luminosity is reduced roughly by the factor $\sqrt{1+\nu_0^2}$.

Let us evaluate this for some typical parameters of CESR. First consider the horizontal plane: We'll use N=1.3x10^{11} (8 ma), r_0=2.8x10^{-15}m, R=123.8 m, γ=10^4, b=40 mm, $\frac{Z_0^{||}}{n}$=1 Ω, β(s_Z)=15 m, β(IP)=1 m, ε=0.2 mm-mrad, and $\hat{z}$=2 cm. For a 1 mm closed orbit error at the impedance, we have $\Delta y'_{\max}$=4.7 μrad, Δy(IP)= 9μm, α(IP)=0.28 mrad, and ν_0=0.01. This is a negligible effect.

However, the effect on the luminosity is much larger in the vertical plane (for flat beams). We take most of the same parameters as for the horizontal plane, but use ε=0.003 mm-mrad (1.5% coupling), b=25 mm, and β(IP)=2 cm. Then, for a 1 mm closed orbit error at the impedance, we have $\Delta y'_{\max}$=19 μrad, Δy(IP)= 5μm, α(IP)=0.13 mrad, and ν_0=0.34. Such a value of ν_0 would result in a significant (~5%) luminosity reduction, as well as possibly contributing to the excitation of beam-beam driven synchrobetatron resonances.

Of course, the worst case assumptions have been made to obtain this number, and a very simple model for a highly localized impedance has been used. On the other hand, the estimates are made for only one bunch, and could increase with multiple bunches. The estimated centroid offset of the beam due to this effect (half of $\Delta y(IP)$, or about 2.5 μm) is of the same order of magnitude as the vertical trajectory difference seen between cars 2 and 5 with the BBI luminosity monitor[3] (about 1 μm). This effect may be due to the same process described in this note, except that it is the wakefield of car 2, as seen by car 5, which is at work, which may explain the reduced size of the effect.

To investigate this further, one might look for the centroid offset directly. At a location where β=30 m, rather than the IP, the centroid offset in the vertical plane should be about 100 μm for the above parameters. This would appear as an intensity-dependent shift in the closed orbit; the CESR orbit measuring system should be able to observe this. The effect can be enhanced by introducing a local vertical bump at the location of a large transverse impedance. In fact, depending on the sensitivity of the measurement, one might be able to use this technique to map out the short-range transverse impedance of the ring as a function of azimuth.

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[1] A. W. Chao and S. Kheifets, "Beam Shape Distortion Caused by Transverse Wake Fields", *IEEE Transactions on Nuclear Science",* Vol. NS-30, No. 4, p 2571 (1983)

[2] A. W. Chao, *The Physics of Collective Beam Instabilities in High Energy Accelerators",* p. 145

[3] D. Sagan, J. Sikora, S. Henderson, "A Luminosity Monitor using the Coherent Beam-Beam Interaction", CBN 97-13 (1997)