# MULTIPOLES FOR DUAL APERTURE STORAGE RING ${ }^{\mathbf{1}}$ Alexander Mikhailichenko <br> Cornell University, Wilson Laboratory, Ithaca, NY 14850 

Design of superconducting multipoles and theirs parameters are represented here.

## 1. Introduction.

Multipoles are intristic elements of magneto-optics of the dual aperture storage ring [1-4]. Basically they have the same functions as the similar components in any single aperture ring. As the multipoles have superconducting coils, each of them assembled into separate unit and installed in series in the same cryostat (see Fig. 14, 15 lower). Few elements in such a cryostat represent a focusing unit. Basic focusing unit contains a quadrupole, a sextupole and a dipole steering elements [1]. Some of the units will have an octupole or skew quadrupole instead of vertical steering unit or instead of sextupole. Below we represent the design parameters of each of such element what could be used for the reference. We describe here in brief Sextupole, Skew Quadrupole, Octupole and Steering dipole assembly.
Dual bore multipole magnets have some peculiarities of inter influence of the fields from neighboring element [5]. All these peculiarities, arisen from cross-field interference in a magnet yoke and at the end of the magnet, are taken into account. Fortunately, the field quality for each higher (than quadrupole) multipole is not so high as for the main quadrupole ${ }^{2}$, so some simplifications allowed. These simplifications drastically reduce the mechanical tolerances, and relax mostly of technological problems. The field quality remains at the level $\left[B_{y}(x)-G_{m-1} \cdot x^{m}\right] / G_{m-1} \cdot x^{m} \leq 10^{-3}$ in all aperture required $\Delta x \cong \pm 27 \mathrm{~mm}$ however. We will notify the places where some improvements could be done. The distance between the axes at present design is about 80 mm . This distance could be easily accommodated to other value.
Full 3D description of each multipole will be given in separate publications.

## 2. Sextupole.

### 2.1. The field.

The field distribution in the aperture could be represented as the following (see for example [9])

$$
\begin{equation*}
\bar{B}=B_{x}-i B_{y}=(-i) S z^{2}=(-i) S \cdot\left(x^{2}-y^{2}-2 i x y\right)=2 S x y-i S \cdot\left(x^{2}-y^{2}\right)=\frac{\partial W(z)}{\partial z}=(-i) S r^{2} e^{i 3 \varphi}, \tag{1}
\end{equation*}
$$

where $z=x+i y=r \cdot e^{i \varphi}-$ is a complex variable and $W(z)$-is a complex potential.

### 2.2. Sextupole value.

Sextupole is used for compensation the chromaticity. This chromaticity arises from the dependence of focal distance of the quadrupole lens on the energy of the particle. The focal distance $F$ of the quadrupole-sextupole doublet can be found from expression

$$
\begin{equation*}
\frac{x}{F} \cong \frac{\Delta p_{\perp}}{p} \cong \frac{\int F d t}{p} \cong \frac{e \cdot\left(G l_{Q} x+S l_{S} x^{2}\right)}{p c} \cong \frac{G l_{Q} x+S l_{S} x^{2}}{(H R)} \tag{2}
\end{equation*}
$$

[^0]where $l_{Q}$-is an effective length of the quadrupole, $l_{S}$-is an effective length of the sextupole, $x$-is the actual transverse coordinate of the particle, $p$-is the particle's momenta. For the momenta other, than equilibrium, (2) can be represented as following
\[

$$
\begin{equation*}
\frac{1}{F} \cong \frac{e \cdot\left(G l_{Q}+S l_{S} \eta \frac{\Delta p}{p}\right)}{p_{0} c \cdot\left(1+\frac{\Delta p}{p}\right)} \cong \frac{G l_{Q}}{(H R)_{0}} \cdot \frac{\left(1+\frac{S l_{S} \eta}{G l_{Q}} \frac{\Delta p}{p}\right)}{\left(1+\frac{\Delta p}{p}\right)}=\frac{1}{F_{0}} \cdot \frac{\left(1+\frac{S l_{S} \eta}{G l_{Q}} \frac{\Delta p}{p}\right)}{\left(1+\frac{\Delta p}{p}\right)} \tag{3}
\end{equation*}
$$

\]

where sub indexes 0 numerates equilibrium values, $\eta$ - is the dispersion function at the lens's location. So if one chooses $S l_{S} \eta / G l_{Q} \cong 1$, the doublet will be achromatic. This gives estimation for the sextupole as

$$
\begin{equation*}
S \cong \frac{G}{\eta} \frac{l_{Q}}{l_{S}} . \tag{4}
\end{equation*}
$$

For quadrupole lens with gradient $1 \mathrm{kG} / \mathrm{cm}$, dispersion function value $\eta \approx 200 \mathrm{~cm}$ this yields estimation $S \approx \frac{1[\mathrm{kG} / \mathrm{cm}]}{200[\mathrm{~cm}]} \cdot \frac{60}{27.8} \cong 0.01\left[\mathrm{kG} / \mathrm{cm}^{2}\right]$, where the ratio $60 / 27.8$ represents the ratio in the lengths of quadrupole to sextupole in CESR. This value gives the minimal possible value of sextupole strength.
The real value, however, must be a few times bigger, taking into account, that the sextupole also cancels the chromaticity, generated by final focus lenses and the fact, that cancellation of chromaticity in one degree of freedom generates it in other. So effective cancellation is result of differences of beta function and dispersion functions at location of the sextupoles associated with focusing and defocusing quadrupole lens. As it can be pointed out from (2), the chromaticity arisen from the sextupoles only ${ }^{3}$ is

$$
\begin{equation*}
\gamma \frac{\partial Q_{H}}{\partial \gamma} \approx \frac{1}{2 \pi} \frac{1}{(H R)} \oint S_{x} \beta_{x} \eta_{x} d s \tag{5}
\end{equation*}
$$

The same formula could be written for vertical tune shift with sub-indices $y$ instead $x$. Basically the sextupoles assembled into the families, the minimal number of which is two. For CESR's sextupole the integral value is about $\int S(s) d s \cong 1[k G / \mathrm{cm}]$ for 10 A of a feeding current [6]. Maximal value of feeding sextupole current in working structure is about 7 A , while a typical value is 0.7 A only. So if we fix the integral on the level $0.7 \mathrm{kG} / \mathrm{cm}$, and the length of sextupole about $10 \mathrm{~cm}^{4}$, the sextupole
${ }^{3}$ Full formulas for chromaticity in first order are

$$
\begin{gathered}
\gamma \frac{\partial Q_{x}}{\partial \gamma} \cong \frac{1}{4 \pi} \oint \beta_{x}(s) \cdot\left[-\frac{G(s)}{(H R)}-\frac{2}{\rho^{2}}+\eta_{x}(s) \cdot\left(\frac{2 \cdot S}{(H R)}+\frac{2}{\rho^{3}}+\frac{4 G}{(H R) \cdot \rho}\right)+\frac{\eta_{x}^{\prime}(s) \cdot \alpha_{x}(s)}{\rho}\right] d s \\
\gamma \frac{\partial Q_{y}}{\partial \gamma} \cong \frac{1}{4 \pi} \oint \beta_{y}(s) \cdot\left[\frac{G(s)}{(H R)}-\eta_{x}(s) \cdot\left(\frac{2 \cdot S}{(H R)}+\frac{2 G}{(H R) \cdot \rho}\right)+\frac{\eta_{x}^{\prime}(s) \cdot \alpha_{y}(s)}{\rho}\right] d s,
\end{gathered}
$$

where $\alpha=-\beta^{\prime} / 2$ and $\rho$-is a current bending radius.
${ }^{4}$ Effective length. Geometrical length is something around 8.5 cm .
gradient required will be $0.7[\mathrm{kG} / \mathrm{cm}] / 10[\mathrm{~cm}] \cong 0.07\left[\mathrm{kG} / \mathrm{cm}^{2}\right]$. So the strength of $0.1\left[\mathrm{kG} / \mathrm{cm}^{2}\right]$ could be taken as a major value for the sextupole with 10 cm effective length.
From the other hand, the sextupole value gives an idea of maximal possible dynamic aperture $A$. If we agree, that absolute limit for dynamic stability defined so that the angular kick to the particle arisen from the sextupole is of the order of angular divergence in the beam,

$$
\begin{equation*}
\frac{S x^{2} l_{S}}{(H R)} \approx \sqrt{\frac{\varepsilon_{x}}{\beta_{x}}} \tag{6}
\end{equation*}
$$

where $\varepsilon_{x}$-is an emittance, $\beta_{x}$-is an envelope function. Substitute here estimation from (3), one can obtain

$$
\begin{equation*}
\frac{G A^{2} l_{Q}}{(H R) \eta} \cong \frac{A^{2}}{F_{0} \eta} \approx \sqrt{\frac{\varepsilon_{x}}{\beta_{x}}} \approx \frac{\sqrt{\varepsilon_{x} \beta_{x}}}{\beta_{x}} \cong \frac{A}{\beta_{x}}, \tag{7}
\end{equation*}
$$

or

$$
\begin{equation*}
A \cong \frac{F_{0} \cdot \eta}{\beta_{x}} \tag{8}
\end{equation*}
$$

For betatron tune shift per cell $\mu=\pi / 2$, ratio $\beta_{x} / F_{0} \cong 4.82$ and one can obtain $A \cong \frac{\eta}{5}$. For dispersion function $\eta \approx 100 \mathrm{~cm}$ dynamic aperture could not be bigger, than $A \leq 20 \mathrm{~cm}$. We obtained this estimation supposing, that sextupole cancels the only chromaticity of the neighboring quadrupole. If sextupole is bigger, then this maximal possible dynamic aperture shrinks by the factor, which is the ratio of the real sextupole strength to this minimal one. For CESR, according to previous estimation this is below two-three times in maximum, what still being acceptable. In Fig. 1 the dynamic aperture for CESR is represented. Here sextupoles cancel the only chromaticity arisen from neighboring quadrupoles, not the final focus lenses.


Fig. 1. Dynamic aperture for CESR's regular part of the ring, referred to the entrance of a dipole magnet. Regular part of the ring only.

So as we know now what is the range of the sextupole strength, we can move further.

### 2.3. The field generation.

For generation the field distribution required, there are two major approaches. One uses magnetic poles of necessary shapes, while the influence of the current is practically eliminated. The other one, indeed, uses a current distribution on the walls of the aperture practically without any particular magnetic poles. Saying ahead, we will use an intermediate approach. Let us make, however, some preliminary considerations.
The equation (1) means, that vertical field behavior along the vertical line, say, $x=\mathrm{a}$, is a parabolic one $B_{y}(x=a, y)=S \cdot\left(a^{2}-y^{2}\right)$. If we suggest that at coordinate $x=a$ a vertical magnetic wall is located, the longitudinal current density along the wall in vertical direction will be required for satisfaction the boundary conditions as

$$
\begin{equation*}
j_{s}(y)=\frac{\partial I(y)}{\partial y}=\frac{1}{\mu_{0}} S \cdot\left(a^{2}-y^{2}\right), \tag{9}
\end{equation*}
$$

where $\mu_{0}$ - is a magnetic permeability of the vacuum ${ }^{5}$. One can see that the current distribution must be a parabolic one ${ }^{6}$. One can also consider some other profiles (parabolic) of the wall, to get a uniform current distribution along the surface. From the other hand, the ratio of the current density at the midplane to the current density at height $y=b$ will be $\frac{j_{s}(y=b)}{j_{s}(y=0)}=\frac{a^{2}-b^{2}}{a^{2}}=1-\frac{b^{2}}{a^{2}}$. So for $b / a \approx 1 / 3$, the difference will be on the level of $10 \%$ only. So if we interrupt the current here and continue the wall with a pole of appropriate shape, we can reduce the influence of current density variation. Total current (per one pole) required for generation of the sextupole harmonic can be found from (7) as

$$
\begin{equation*}
\int_{0}^{R} B_{r}(r, \varphi) d r=\int S \cdot r^{2} d r=S \frac{R^{3}}{3}=0.4 \pi I \tag{10}
\end{equation*}
$$

where we used a practical system of units and $R$ - is a radius of the circle, inscribed into the aperture given by a pole shape. For estimation we can suppose, that $S \cong 0.1 \mathrm{kG} / \mathrm{cm}^{2}, R \cong 3.5 \mathrm{~cm}$, then the amount of $A \cdot$ turns will be, according to (10) $I=\frac{S R^{3}}{0.4 \cdot 3 \cdot \pi} \cong 1.13 \mathrm{kA} \cdot$ turns. From (9) one can obtain that

$$
\begin{equation*}
I=\int_{0}^{b} \frac{\partial I(y)}{\partial y} d y=\frac{1}{0.4 \pi} S \cdot\left(a^{2} b-\frac{b^{3}}{3}\right) \tag{11}
\end{equation*}
$$

Compare (10) and (11) one can conclude, $\left(3 a^{2} b-b^{3}\right)=R^{3}$, what basically means, that the point with coordinates $x=a, y=b$ belongs to the pole shape, what is a cubic hyperbola, defined by the sextupole. One can came to this conclusion, considering the pole shape required described by the formula $\operatorname{Re} W(z)=W_{0}=$ Const what is

$$
\begin{equation*}
(-i) \cdot S \cdot \frac{z^{3}}{3}=(-i) \frac{1}{3} \cdot S \cdot\left(x^{3}+3 x y^{2}-i y^{3}+i 3 x^{2} y\right)=\frac{1}{3} S \cdot R^{3} e^{i 3 \varphi} . \tag{12}
\end{equation*}
$$

For angle $\varphi=\pi / 6$, what is a normal location of the pole tip, $e^{i \pi / 2}=i$ and (11) yields a formula for the pole of sextupole as

[^1]\[

$$
\begin{equation*}
-y^{3}+3 x^{2} y=R^{3} \tag{13}
\end{equation*}
$$

\]

In polar coordinates the pole profile can be expressed, according to (11) as

$$
\begin{equation*}
R^{3}=\rho^{3} \sin 3 \varphi, \tag{14}
\end{equation*}
$$

where $\rho, \varphi$ - are polar coordinates of the point at the pole profile. After this preliminary remarks let us go to design aspects.

### 2.4. Sextupole design.

The cold mass of the sextupole is represented in Fig.2. The poles are made removable and manufactured as parts of a cylinder. They are attached to the yoke by screws. At the end they have some cut to improve the harmonics content.


Fig.2. Sextupole cold mass. Dimensions are in mm . The inner boundary of the helium container is the circle with the biggest radius (shown at the right). Outer wall of helium container wraps the yoke.

Six racetrack type single layer coils generate the necessary field profile, Fig. 3. Wire is a superconducting one with 54 filaments of NbTi of 0.4318 mm ( 0.017 in ) in diameter. Each wire wrapped by Kapton insulation and impregnated by Bondal. Coil has 23 turns per pole.
If we fixed this concept, we can move further and calculate the fields is the aperture.

### 2.5. The field quality.

One can see, that similar to quadrupole field [5], the lines in the yoke have the different path lengths. Moreover, the lines in the septum of the yoke, right side on Fig.3, are more condensed and hence reduce the field strength at the right side additionally. To compensate this effect one can make the pole radiuses at the left and right side of the lens slightly different. The field behavior is represented in Fig. 4 for the same pole radiuses. Zero of the field is slightly shifted on 0.042 mm to the right side.


Fig. 3. Magnetic lines in sextupole. Upper half of the left aperture in Fig. 2 is shown.
Neighboring sextupoles have the same value.


Fig.4. Vertical field behavior as a function of transverse coordinate.
Expansion of the field behavior made around mechanical center of the sextupole gives the formula for the feeding current 2.5 kA turns/pole or 108 A of feeding current. This current is about three times the working one. With this current the field dependence is

$$
\begin{align*}
B_{y}(x)[k G]=- & 0.112-4.1 \cdot 10^{-4} \cdot x+0.229 \cdot x^{2}-4.7 \cdot 10^{-4} \cdot x^{3}+1.9 \cdot 10^{-4} \cdot x^{4}+2.8 \cdot 10^{-5} \cdot x^{5}- \\
& -8.66 \cdot 10^{-6} \cdot x^{6}+1.39 \cdot 10^{-6} \cdot x^{7}-3.85 \cdot 10^{-6} \cdot x^{8} \quad[x, \mathrm{~cm}] \tag{15}
\end{align*}
$$

So the field deviation within aperture remains within $2 \cdot 10^{-3}$.

### 2.6. Parameters.

Parameters of the sextupole are summarized in Table 1.

| Parameter | Value |
| :---: | :---: |
| $\int S d s,[k G / \mathrm{cm}]$ | 0.7 |
| $S,\left[k G / \mathrm{cm}^{2}\right]$ | 0.07 |
| Current, $A$ | 35 |
| Turns/pole | 23 |
| Aperture, cm | $5.4($ dia) |

## 3. Skew quadrupole

### 3.1. The field.

The field distribution in the aperture could be represented as the following

$$
\begin{equation*}
\bar{B}=B_{x}-i B_{y}=(-i) e^{i \pi / 4} G z=(-i) e^{i \pi / 4} G \cdot(x+i y) \equiv(-i) G \cdot r \cdot e^{i(\varphi+\pi / 4)}, \tag{16}
\end{equation*}
$$

where $z=x+i y=r \cdot e^{i \varphi}$-is a complex variable and $W(z)$-is a complex potential.

### 3.2. The skew quadrupole value.

Skew quadrupole is used for compensation of coupling in betatron motion arisen from misalignments of installation (and fabrication) elements of beam optics around the ring.

### 3.3. The field generation.

This is basically the same as used for main quadrupole [2,8].

### 3.4. Design.

Skew quadrupole cold mass is represented in Fig. 5. The coil has 36 turns. Four of these coils are used for each of quadrupole. This number of the turns slightly differs from the number of turns in main quadrupole (42) due to reduced area available for the poles. The same mold will be used for winding as it was used for main quad. This mold allows reduction the number of the turns with minimal adjustment. The length of the skew quadrupole's yoke is about 8.5 cm , what yields an effective length about 12 cm .


Fig.5. Skew quadrupole cold mass.

### 3.5. The field quality.

We represent below the graphs for gradient. One can realize the accuracy for the field deviation from linear is higher, as it is defined as

$$
\begin{equation*}
\left[G \cdot x-\int_{0}^{A} g(x) d x\right] / G \cdot x \cong\left[G \cdot x-\int(G-\delta g(x)) d x\right] / G \cdot x=1-\frac{\int \delta g(x) d x}{G \cdot x} \tag{17}
\end{equation*}
$$

where $g(x)$ - is an actual gradient, and $\delta g(x)=g(x)-G \cdot x$ - is a deviation from a linear part.

### 3.5.1. Symmetric case



Fig. 6 . Magnetic lines map and gradient as a function of transverse coordinate. Gradient at both apertures have the same sign. Notice that gradient have a linear slope down to the right. Poles are cylindrical with radius 3.6 cm . Variation of gradient is $\pm 1.5 \cdot 10^{-3}$.

### 3.5.2. Asymmetric case.



Fig. 7. Magnetic lines map and gradient as a function of transverse coordinate. Gradients in neighboring apertures are opposite. Notice that gradient have a linear slope $u p$ to the right. Poles are cylindrical with radius 3.6 cm . Variation of gradient is $\pm 1.5 \cdot 10^{-3}$.

### 3.6. Parameters.

| Parameter | Value |
| :---: | :---: |
| $\int G d s,[k G]$ | 3.0 |
| $G,[\mathrm{kG} / \mathrm{cm}]$ | 0.25 |
| Current, $A$ | 35 |
| Turns/pole | 36 |
| Aperture, cm | $5.4($ dia) |
|  |  |

## 4. Octupole

### 4.1. The field.

The field distribution in the aperture of octupole could be represented as the following

$$
\begin{equation*}
\bar{B}=B_{x}-i B_{y}=(-i) O z^{3}=O \cdot\left(-y^{3}+3 x^{2} y\right)-i O \cdot\left(x^{3}-3 x y^{2}\right)=\frac{\partial W(z)}{\partial z}=(-i) O \cdot r^{3} e^{i 4 \varphi} \tag{18}
\end{equation*}
$$

where $z=x+i y=r \cdot e^{i \varphi}$-is a complex variable and $W(z)$-is a complex potential.

### 4.2. The octupole value.

Octupole used for compensation dependence the betatron tune on the betatron amplitude, generated by different nonlinarities in machine. The horizontal force acting to the particle can be represented as $F_{x}=e O(s) \cdot x^{3}$, where $x$-is an actual coordinate in sextupole. Transverse displacement could be represented as

$$
\begin{equation*}
x(s)=a \sqrt{\frac{\beta(s)}{\beta_{0}}} \cos (\psi+Q \cdot \phi(s)), \tag{19}
\end{equation*}
$$

where $a$-is an amplitude of transverse displacement at the place where envelope function is $\beta_{0}, \psi$-is constant. In the presence of octupole, $a$ and $\psi$ become a functions of it's amplitude. Using the Bogolubov-Krylov method [9] one can obtain, that the dependence of the tune becomes

$$
\begin{equation*}
\Delta Q \cong \frac{1}{2 \pi} \cdot \frac{\beta_{0}}{(H R)} \cdot \oint \sqrt{\frac{\beta(s)}{\beta_{0}}} \cdot \frac{O(s) \cdot x^{3}(s) \cdot \cos (\psi+Q \cdot \varphi(s))}{a} d s . \tag{20}
\end{equation*}
$$

Substitute here expression for $x$ from (17) and averaging over phases one can find

$$
\begin{equation*}
\Delta Q \cong \frac{1}{2 \pi} \cdot \frac{\beta_{0} \cdot a^{2}}{(H R)} \cdot \oint\left(\frac{\beta(s)}{\beta_{0}}\right)^{2} \cdot O(s) \cdot \frac{3 \pi}{4} d s \approx \frac{3}{8} \frac{\beta_{0} \cdot a^{2} O \cdot l_{O}}{(H R)} \tag{21}
\end{equation*}
$$

where $l_{0}$ - is the length of octupole.
For amplitude $a \cong 1 \mathrm{~cm}, \Delta Q \approx 0.01, \beta_{0} \approx 20 \mathrm{~m}=2000 \mathrm{~cm},(H R) \cong 1.7 \cdot 10^{4} \mathrm{kG} \cdot \mathrm{cm}$, one can obtain from (21), that $O \cdot l_{O} \approx 0.22\left[k G / \mathrm{cm}^{2}\right]$. This gives $O \approx 0.022 \mathrm{kG} / \mathrm{cm}^{3}$ for $l_{O} \cong 10 \mathrm{~cm}$. One can see, that it is a strong octupole. To handle the dynamic aperture, one needs to use a few octupoles around the orbit. For example, ten octupoles will yield the octupole strength ${ }^{7} O \approx 0.0022 \mathrm{kG} / \mathrm{cm}^{3}$. That will give a gradient about $G \cong 3 \cdot O \cdot a^{2} \approx 0.048 \mathrm{kG} / \mathrm{cm}$ at the boundary of aperture, i.e. about $5 \%$ of the main quadrupole.

### 4.3. The field generation.

For generation the field distribution like (18) we used an intermediate approach, similar to one used in sextupole design. Here on vertical wall at $x=a$, the current density distribution must satisfy the equation

$$
\begin{equation*}
j_{s}(y)=\frac{\partial I(y)}{\partial y}=\frac{1}{0.4 \pi} O \cdot\left(a^{3}-3 \cdot a \cdot y^{2}\right), \tag{22}
\end{equation*}
$$

[^2]which yields a parabolic currant distribution here. Again, the ratio of the current density at the midplane to the current density at height $y=b$ will be $\frac{j_{s}(y=b)}{j_{s}(y=0)}=\frac{a^{3}-3 a b^{2}}{a^{3}}=\left(1-\frac{3 b^{2}}{a^{2}}\right)$, i.e. three times bigger, than for sextupole. For octupole proposed $a=3.75 \mathrm{~cm}, b=0.75 \mathrm{~cm}$ and the current variation expected to be $12 \%$. Once again we interrupt the current here and continue the wall with pole of appropriate shape, thereby reducing the current density variation influence. Total current (per one pole) required for generation of the octupole harmonic can be found from (18) as
\[

$$
\begin{equation*}
\int_{0}^{R} B_{r}(r, \varphi) d r=\int O \cdot r^{3} d r=O \frac{R^{4}}{4}=0.4 \pi I . \tag{23}
\end{equation*}
$$

\]

So for octupole $O \cong 0.025 \mathrm{kG} / \mathrm{cm}^{3}$, the current required is about $I \approx 1 \mathrm{kA} \cdot$ turns total.

### 4.4. Design.

Basically design follows the same scheme as quadrupole and sextupole. A single-layer coil per pole is used here. Twelve coils are used totally for each dual bore sextupole magnet. Two halves allow splitting the magnet along the median plane. Similar to sextupole, the poles are made removable and manufactured as parts of a cylinder. They are attached to the yoke by screws. At the ends they also have cuts. Additional metallic screens reduce the cross-interference the fields from neighboring lens. Basically the inner window without poles attached looks like a regular 16-pole shape one. So the space for the poles and coils had chosen the same. The length of each side of this 16 -pole figure is about 15 mm.


Fig.8. Octupole cold mass.

### 4.5. The field quality.

The field quality defined as a result of calculation with MERMAID in $1 / 4$ of whole dual bore yoke. This takes into account the influence of neighboring lens. In Fig. 9 the picture of magnetic lines is shown. One can see from here that some of the lines are moving around both centers. The field and gradient behavior are shown in Fig. 10. For the feeding current $0.5 \mathrm{kA} /$ pole, the field can be represented here as

$$
B_{y}(x)=5.6 \cdot 10^{-6} \cdot x+0.0151 \cdot x^{3}
$$

so at the boundary, $x=2.7 \mathrm{~cm}$, the ratio will be $\Delta B / O \cdot x^{3} \approx 5 \cdot 10^{-5}$.


Fig.9. Octupole field lines. Left aperture in Fig. 8 is shown. Symmetrical case.


Fig.10. Magnetic field $B_{y}(x)$ and it's gradient $G_{y}(x)=\frac{\partial B_{y}(x)}{\partial x}$ as the functions of transverse coordinate.

### 4.6. Parameters.

| Parameter | Value |  |
| :---: | :---: | :---: |
| $\int O(\mathrm{~s}) \mathrm{ds},\left[\mathrm{kG} / \mathrm{cm}^{2}\right]$ | 0.15 |  |
| $O,\left[\mathrm{kG} / \mathrm{cm}^{3}\right]$ | 0.015 |  |
| Current, $A$ | 30 |  |
| Turns/pole | 17 |  |
| Aperture, cm | $5.4(\mathrm{dia})$ |  |
|  |  |  |

## 5. Dipole

### 5.1. The field.

The field distribution could be represented as the following

$$
\begin{equation*}
\bar{B}=B_{x}-i B_{y}=B_{0} \cdot e^{i \varphi} \equiv\left|B_{0}\right| \cdot e^{i \psi} \cdot e^{i \varphi}, \tag{24}
\end{equation*}
$$

where $B_{0}$-is the amplitude, and $\psi$-is the phase, which defines orientation the field vector inside the aperture.

### 5.2. The dipole value.

The angular kick arising from the dipole is

$$
\begin{equation*}
\frac{\Delta p}{p} \cong \frac{B_{0} l_{D}}{(H R)}, \tag{25}
\end{equation*}
$$

where $l_{D}$ a -is an effective length of the dipole. For the beam energy $E \cong 5 \mathrm{GeV}$, $(H R) \cong 1.67 \cdot 10^{4} \mathrm{kG} \cdot \mathrm{cm}, l_{D} \cong 16 \mathrm{~cm}, B_{0} \cong 1.25 \mathrm{k} G$, and for angle kick one can obtain the value of the order $\Delta p / p \cong 1.2 \cdot 10^{-3}$ or 1.2 mrad .

### 5.3. The field generation.

The field generated by two flat saddle-type coils connected in series. Two coils together provide a thin sheet of current at the yoke wall. The current density along the wall must cancel the tangential to the iron component of magnetic field, so

$$
\begin{equation*}
j_{s}(y)=\frac{\partial I(y)}{\partial y}=\frac{1}{0.4 \pi} B_{0} \tag{26}
\end{equation*}
$$

and the current is obviously constant along the wall. From last expression it follows an ordinary formula for magnetic field calculation, $B_{0}=0.4 \pi I / A$, where $A$ - is the aperture, $I-$ is the full current.

### 5.4. Design.

Design is represented in Fig. 11. Basically it was not so simple to achieve the necessary quality due to strong influence, what imperfections in corners gave to the field. Dipole field does not give fast decay for non principal harmonics. One possible solution for this was found in slightly curved poles, see Fig. 12. As the manufacturing of these poles becomes more complicated, the other solution was accepted. That is in arrangement of some controlled gap in the middle of the side winding. The height of this gap is twice as the gap between the last turn of the coil and the top of the corner.


Fig. 11. Dipole corrector cold mass.

### 5.5. The field quality.

The field quality considered with the same numerical code as previous multipoles. Field lines represented in Fig. 12. Here in left aperture the field has the vertical component only, in right -both vertical and horizontal components are equal.


Fig. 12. Example of field generation in neighboring apertures with curved side walls model.

In Fig. 13. There is represented the field behavior as a function of transverse coordinate for $20 A$ of feeding current. Nonlinear behavior is a result of the field dependence on path length in yoke. Material of the yoke - Standard 1008.


Fig.13. Field distribution across the aperture. Maximal field is $0.5 k G$. One division on vertical scale corresponds to $0.5 \cdot 10^{-4}$ of relative deviation. Flat walls with gap compensation.

### 5.6. Parameters.

| Parameter | Value |
| :---: | :---: |
| $\int H d s,[k G \cdot \mathrm{~cm}]$ | 20. |
| $H, k G$ | 1.25 |
| Current, $A$ | 50 |
| Turns/pole | 75 |
| Aperture, cm | $5.4($ dia $)$ |
| Angular kick, mrad | 1.2 |

6. Multipoles in cryostat.

Multipoles in cryostat are represented in Fig. 14. The cold mass enveloped by copper shield cold down to nitrogen temperature. Nitrogen is going through tubing 29,56,57 in Fig. 14. High temperature rods made on BSCCO marked HTS in Fig. 14, used for reduction of the heat leakage to the cold mass


Fig. 14. Multipoles in the cryostat.
held under temperature of boiling Helium. Total number of these rods may vary from 6 to 16, depending on the feeding independence required. Minimal number of the rods (6) required when quadrupoles connected in series, same for sextupole and the only one component of the field is required from dipole corrector. Cold mass and nitrogen shield wrapped by superinsulation.


Fig. 15. Cryostat. The quadrupole cold mass is represented at the right. Multipoles are inserted into the same cold mass envelope. Copper nitrogen shield is shown inserted into the cryostat cabinet.

## 7. Conclusion

We represented here some general properties of multipoles for dual aperture storage ring. Sextupole considered a little bit more detailed. In the present design a single layer coils is used for the multipoles. A two layer option will reduce the feeding current, keeping the field quality on the level required. The thickness of two layer coil about 1 mm (instead of 0.5 mm for a single layer coil) allows an easy accommodation into the present design concept.
3D consideration each of the multipoles will be described in separate publications.

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[^0]:    ${ }^{1}$ Supported by National Science Foundation.
    ${ }^{2}$ Where it is $\left[B_{y}(x)-G \cdot x\right] / G \cdot x \leq 4 \cdot 10^{-4}$.

[^1]:    ${ }^{5}$ We used a SI units here. For practical units: $G, A, c m,\left(1 / \mu_{0}\right)$ should be replaced by $1 /(0.4 \pi)$
    ${ }^{6}$ Constant for a quadrupole, as $B_{y}(x=a)=G a$

[^2]:    ${ }^{7}$ For the same tune shift.

