Estimate of Persistent Current Magnetization Effects in CESR Phase III IR Quadrupoles

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In this note, I describe estimates of the persistent magnetization currents in coils of the CESR Phase III IR quadrupoles. A general result is derived for the magnetization, and a rough estimate given for the magnitude of the effect compared to the main quadrupole field. This rough estimate indicates that the effect is very small, so the detailed expression is not evaluated.

1. Filament and Wire Magnetization

Figure 1 illustrates one filament in a superconducting cable. We consider here only the case in which the induced currents in the superconductor fill the entire volume of the filament at the maximum current density; this is the situation in Type II superconductors for applied fields greater than a few tenths of a Tesla.

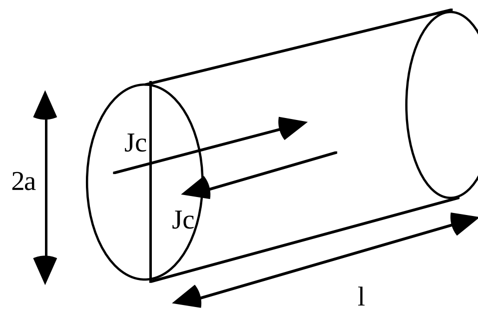

Fig. 1

Referring to Fig. 1, the magnetic moment of the current loop of length l is

$$m = \int dm = \int P dI \tag{1}$$

in which

$$P = 2rlCos(\phi) \tag{2}$$

is the area of an increment current loop at dI (r,ϕ) (See Fig. 2, end-on view of Fig. 1)

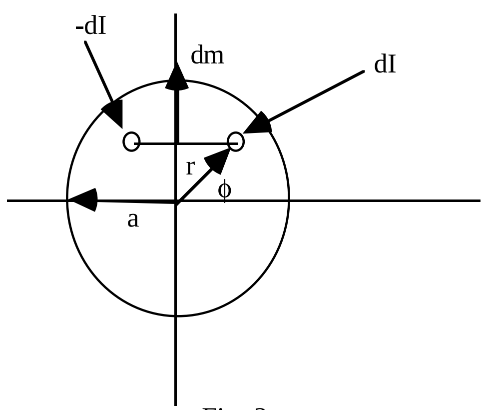

Fig. 2

and

$$dI = J_c r dr d\phi \tag{3}$$

so

$$m = 2lJ_c \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi Cos(\phi) \int_0^a r^2 dr = \frac{4}{3} l J_c a^3 \tag{4}$$

For a cable with N filaments, the total magnetic moment is Nm. If the total cable area is A, then

$$\varepsilon A = N\pi a^2 \tag{5}$$

where ε is the fraction of the cable which is superconductor. The total magnetic moment per unit length is then

$$m_l = \frac{Nm}{l} = \frac{4}{3\pi} \varepsilon A J_c a \tag{6}$$

The critical current density is a function of the applied field B; the induced magnetic moment is opposite to the direction of the applied field. So

$$\vec{\mathbf{m}}_{\mathbf{l}} = \hat{B} \frac{4}{3\pi} \varepsilon A J_c(B) a \tag{7}$$

2. Magnetization field of a cable winding

We now consider the field created by this magnetic moment. We take the cable to be arranged in the form of a thin shell of radius R (fig. 3), of thickness w, which extends from $\phi=0$ to $\phi=\phi_0$.

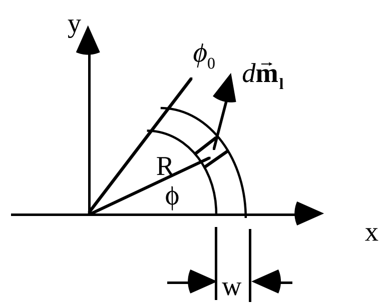

Fig. 3

The incremental magnetic moment per unit length at ϕ is

$$d\vec{\mathbf{m}}_{\mathbf{l}} = \hat{B} \frac{4}{3\pi} \varepsilon J_c(B) a w R d\phi \tag{8}$$

.

The two-dimensional magnetic field $\vec{\mathbf{B}}_d(r,\phi)$ of a magnetic moment per unit length $\vec{\mathbf{m}}_{\mathbf{l}}$, located at (r', ϕ') is given by (Fig. 4)

$$\vec{\mathbf{B}}_d(r,\phi) = \frac{\mu_0}{4\pi\rho^2}\left[2(\vec{\mathbf{m}}_{\mathbf{l}} \bullet \hat{\rho})\hat{\rho} - \vec{\mathbf{m}}_{\mathbf{l}}\right] \tag{9}$$

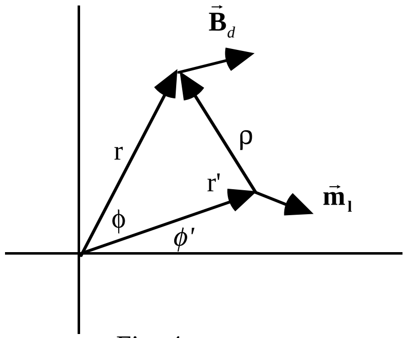

Fig. 4

Here

$$\vec{\rho} = \vec{r} - \vec{r}\,'$$
$$\rho^2 = r^2 + r'^2 - 2rr'\,Cos(\phi - \phi') \qquad (10)$$

Applying this to the case of Fig. 3, we have

$$d\vec{\mathbf{B}}_d(r,\phi) = \frac{\mu_0}{4\pi\left(r^2 + R^2 - 2rRCos(\phi - \phi')\right)}\left[2(d\vec{\mathbf{m}}_\mathbf{l} \bullet \hat{\rho})\hat{\rho} - d\vec{\mathbf{m}}_\mathbf{l}\right] \qquad (11)$$

$$\vec{\rho} = \vec{r} - \vec{R} \qquad (12)$$

So the total field due to the current shell magnetization is

$$\vec{\mathbf{B}}_d(r,\phi) = \frac{\mu_0 \varepsilon awR}{3\pi^2}\int_0^{\phi_0} d\phi' \frac{J_c(B)\left(2(\hat{B} \bullet \hat{\rho})\hat{\rho} - \hat{B}\right)}{\left(r^2 + R^2 - 2rRCos(\phi - \phi')\right)} \qquad (13)$$

The applied field $\vec{B}$ depends on ϕ' and so cannot be taken outside the integral. To compare this field to the main quadrupole field, we evaluate it at the reference radius r_0

$$\vec{\mathbf{B}}_d(r_0,\phi) = \frac{\mu_0 \varepsilon awR}{3\pi^2}\int_0^{\phi_0} d\phi' \frac{J_c(B)\left(2(\hat{B} \bullet \hat{\rho})\hat{\rho} - \hat{B}\right)}{\left(r_0^2 + R^2 - 2r_0RCos(\phi - \phi')\right)}. \qquad (14)$$

The error harmonics from this field can then be obtained by Fourier analyzing this expression as a function of ϕ. We also need to add the contributions from the other coil shells.

3. Estimate of the magnitude of the effect.

To obtain a rough estimate of the size of the effect from Eq. (14), we make the following approximations:

$$r^2 + R^2 - 2rRCos(\phi - \phi') \approx (R - r_0)^2$$
$$\left(2(\hat{B} \bullet \hat{\rho})\hat{\rho} - \hat{B}\right) \approx 1$$
$$\int_0^{\phi_0} d\phi' J_c(B) = \phi_0 J_c(\overline{B}) \qquad (15)$$

in which $\overline{B}$ is the average value of the applied field over the cable shell. Then, neglecting the effect of the other coil shells, we have

$$\overline{B}_d(r_0,\phi) \approx \frac{\mu_0 \varepsilon awR}{3\pi^2}\frac{\phi_0 J_c(\overline{B})}{(R - r_0)^2} \qquad (16)$$

For N_t turns of wire, each of which has area A_w, we have for the total area of the cable block

$$wR\phi_0 = N_t A_w \tag{17}$$

and the critical current in the wire is

$$I_c(\overline{B}) = J_c(\overline{B})\varepsilon A_w \tag{18}$$

so we have

$$\overline{B}_d(r_0,\phi) \approx \frac{\mu_0 N_t a}{3\pi^2}\frac{\varepsilon J_c(\overline{B})A_w}{(R-r_0)^2} = \frac{\mu_0 N_t I_c(\overline{B})}{3\pi^2}\frac{a}{(R-r_0)^2} \tag{19}$$

The cases of interest are the magnetization fields produced by the main quadrupole, skew quadrupole, and dipole coils, in the presence of the combined field (in practice, this is roughly the quadrupole field). The critical current scaling with B is taken as

$$\begin{aligned} I_c(B) &= I_c(4)\frac{B_{c2}-B}{B_{c2}-4} \quad B>4 \\ &= \frac{2I_c(4)}{1+\frac{B}{4}} \quad B<4 \end{aligned} \tag{20}$$

The field B_{c2} =10.4 T. In Table 1, the relevant quantities are shown for each coil. The reference radius is taken as r_0= 50 mm. B_{app} is the average applied field, called $\overline{B}$ above, and B_{quad} is the quadrupole field at r_0. Eqs. (19) and (20) have been used. In all cases, the error field is much less than one unit. The main effect is due to the quadrupole coil itself.

Coil	R(coil) (mm)	a(microns)	Ic(Bref) (A)	Bref (T)	Bapp (T)	Ic(4)(A)	Ic(Bapp) (A)	Turns	B(d)(T)	Bquad (T)	B(d)/B quad (units)
Dipole	140	13.5	67	5.6	2.6	89.33	108.28	710.00	5.4E-06	2.42	0.022
Skew Quad	135	3.1	1100	4	3.5	1100.00	1173.33	216.00	4.6E-06	2.42	0.019
Main Quad	113	4.4	1500	6.4	4.5	2400.00	2212.50	364.00	3.8E-05	2.42	0.157

This note benefited from conversations with Jim Welch.