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ODYSSEUS: A Dynamic Strong-Strong Beam-Beam Simulation

E.B. Anderson and J.T. Rogers

We have developed a simulation of the beam-beam interaction in e^+e^- storage ring colliders which is specifically intended to reveal the dynamic collective behavior of the colliding beams. This program is a true 6-dimensional strong-strong simulation in which the electromagnetic fields of longitudinal slices of the colliding beams are recalculated for each slice collision. Broadband wake fields are included. No constraints are placed on the distribution of particles in the beams. This simulation makes it possible to characterize the beam-beam modes, to see whether the saturation of the beam-beam parameter is sometimes due to collective motion, to determine the thresholds for wake field induced instabilities in the presence of the beam-beam interaction, and to determine the Landau damping rate for any beam-beam mode.

1 Introduction

The beam-beam interaction is of fundamental importance to storage ring colliders because it limits the attainable luminosity. However, several aspects of the beam-beam interaction, especially those involving collective motions of the beam, are incompletely understood.

In CESR the observed instability threshold for the $m = -1$ head-tail instability is lower when beams are in collision. We expect that, in general, *any* wake field induced instability will have a different threshold for colliding beams, for three reasons. First, the beam-beam force produces a coupling between the colliding bunches, so the modes of oscillation are the coupled-bunch modes. Second, the beam-beam force is typically much stronger than wake field forces, and does not possess the same symmetry. The internal modes of the bunches will be different from those of the non-colliding bunches. Third, the nonlinearity of the beam-beam force produces Landau damping. The Landau damping rate may be different for each beam-beam mode.

Collective effects arising from the beam-beam force alone can limit luminosity. The flip-flop instability, in which the steady state has beams of unequal transverse sizes, is commonly observed in e^+e^- storage ring colliders. The DCI storage ring at LAL, Orsay, France, had four colliding beams in which the e^+ beam charge was compensated by the e^- charge, but the beam-beam limit was not significantly different from that for uncompensated beams [1]. The beam-beam limit in DCI was attributed to a collective beam-beam instability. This suggests that the beam-beam limit for two-beam collisions may also be due, in some cases, to a collective instability.

We wish to characterize the beam-beam modes; to determine whether the saturation of the beam-beam parameter is due, under some conditions, to collective oscillations of the bunches; to find the thresholds for wake field induced instabilities in the presence of the beam-beam interaction; and to determine the Landau damping rate for any beam-beam mode.

Because the beam-beam force is strongly nonlinear in transverse position, exact calculations are difficult and particle tracking simulations are a suitable method for studying the beam-beam interaction. Weak-strong simulations, in which probe particles in the "weak" beam are tracked through a "strong" beam with a fixed charge distribution, have been used extensively and successfully to predict single particle motion, such as the growth in transverse amplitude leading to the formation of beam tails. Strong-strong simulations, in which the force on each beam from the opposing beam is calculated, are capable of generating self-consistent charge distributions and have been used to examine the collective behavior of round beams [2], charge-compensated round beams [3], and beams of arbitrary ellipticity [4]. These simulations are very time-intensive because of the need to repeatedly calculate the electromagnetic field of each beam, and previous strong-strong simulations have included only transverse degrees of freedom. To our knowledge, ODYSSEUS is the first 6-dimensional strong-strong beam-beam simulation in which no constraints are placed on the beams, and the first to include wake fields. This enables us to investigate any mode of oscillation of the colliding beams.

Unlike previous simulations, ODYSSEUS is capable of rapidly calculating the electromagnetic field of a beam divided into many longitudinal slices because it adaptively chooses from a variety of different field computation methods. Different algorithms are used for the core and transverse tails of the beam, and for longitudinal slices with large or small charge. The parameters of the program can be changed to model flat or round beams. The inclusion of the longitudinal degrees of freedom and wake fields allows us to probe previously inaccessible physics. ODYSSEUS is designed to serve as a flexible, efficient, and portable tool for investigating beam-beam effects.

2 Field calculation

For purposes of calculating the electromagnetic field from the beam, each beam is divided into longitudinal slices. Although the number of slices can be set arbitrarily, around twenty are typically used. The field from each slice, integrated over the length of the slice, is calculated independently. The beams are assumed to be ultra-relativistic, so the field due to each slice is transverse and affects only particles within the region of that slice.

The calculation of the electromagnetic field of each beam is adaptive, to maximize the speed of the program. Different methods are used depending on whether the field is calculated for the region of the beam core or for the beam tails, whether the number of macroparticles within a slice N is large or small, and whether the number of grid points N_g used in the field calculation is large or small.

2.1 Beam core

2.1.1 Small N

If the number of macroparticles N within a slice is very small, the field (integrated over the length of the slice) at a (probe beam) macroparticle is calculated from the exact radius vector from each opposing (source) beam macroparticle. The field must be calculated at the position of each macroparticle in the probe beam, so the number of calculations goes as N^2, making this method efficient only for very small N. In practice, this method is only used when N is less than fifty.

2.1.2 Large N, small N_g

For larger values of N, the electromagnetic field is calculated on a rectangular grid, using precalculated Green's functions for charges on the grid points. The beam charge is assigned to the grid points using an area-weighted technique known as 'cloud-in-cell' [5]. For small values of the number of grid points, the convolution of the charge density and Green's function is done as a summation in real space. The number of calculations required for this convolution goes as N_g^2, where N_g is the number of grid points. The portion of the code whose speed is dependent on the number of macroparticles is now only linear in N. This technique is used only when the number of grid points is quite small, under two hundred.

2.1.3 Large N, large N_g

For larger values of N_g, the convolution of the Green's functions and charge density is done as a simple multiplication in wavenumber space. The speed of this method is limited by the speed of the necessary Fourier transform to wavenumber space and inverse transform back to real space. The number of calculations goes as $N_g \log_2 N_g$. To suppress edge effect problems in the Fourier transforms, the size of the wavenumber space is doubled in both directions and padded with zeros [5].

2.2 Beam tails

The tails of the beam, typically taken to be particles with a displacement of more than $(10/3)\sigma$ in the horizontal, vertical, or longitudinal directions, are treated differently than the core particles. The tail particles have very little effect on the beam-beam force. They do, however, respond to the beam-beam force and must be tracked to determine the beam lifetime. Performing a strong-strong calculation for the beam tails with the grid method is computationally inefficient and unnecessary, so we use a weak-strong calculation.

2.2.1 Longitudinal tails

Longitudinal tail particles are subject to forces from the core of the opposing beam. A full calculation of the field from the opposing beam slice is performed, as described above for the beam core. This is a weak-strong calculation, however. The tails are assumed to have no effect on the other beam.

2.2.2 Transverse tails

The transverse tail particles are subject to a beam-beam force of similar magnitude to that experienced by the core particles. The fine structure of the charge distribution of the core has little influence on the field in the transverse tails, so the field in the transverse tails is calculated for a 2-D Gaussian charge distribution with the same 0th, 1st, and 2nd moments as the charge distribution of the slice. The field from this Gaussian charge distribution is calculated using the rational approximation of Talman and Okamoto [6] to the complex error function solution of Bassetti and Erskine [7].

2.3 Wake fields

Longitudinal and transverse wake fields are included in the simulation. One of the program inputs is a list of longitudinal and transverse resonators with values for the resonant frequencies, shunt impedances R/Q, and quality factors Q of the resonators. The wake functions are therefore the sum of exponentially damped sinusoids. The transverse wakes are calculated from the 1st moments of the slice charge distribution, and are assumed to be uniform over each slice. The longitudinal wake has a uniform term that is dependent on the 0th moment of the preceeding slice, and a term that is dependent on the 1st moments of the preceeding slice that changes linearly with particle position.

3 Particle tracking

3.1 Single-turn loop

On each turn through the machine, from the collision point and back again, each macroparticle is propagated through the linear optics of the storage ring, including chromaticity, synchrotron radiation excitation and damping, RF phase focusing, and wake field deflections. The full phase space distribution of macroparticles is written to a file at intervals of many turns. The 0th, 1st, and 2nd transverse moments of the entire bunch are written to a file on each turn. Individual macroparticles undergoing longitudinal oscillations may migrate from slice to slice, so on each turn the macroparticles are sorted and assigned to slices.

Because the motion from slice to slice is slow, heapsort is an effective sorting algorithm [9]. During collisions, each macroparticle is assumed to remain within its slice. Macroparticles which have migrated past a transverse aperture are removed from the simulation.

3.2 Collisions

During its passage though the opposing bunch, the transverse position of each macroparticle changes appreciably, because the beta function (in CESR, β_V^*) is comparable to the bunch length. In the simulation we collide each pair of slices sequentially, updating the transverse positions and momenta of each macroparticle after each pairwise slice collision. For each slice collision, the 1st and 2nd transverse moments of the slice charge distribution and the slice electromagnetic field are calculated. The structure of the program is shown in Fig. 1.

4 Postprocessing

We are interested in the coherent oscillations of the bunch, so postprocessing to analyze the bunch spectrum is necessary. Postprocessing is done in a *Mathematica* [8] notebook. Beam-beam modes may be identified in the Fourier transform of the turn-by-turn moments of the bunch. Growth and damping rates of these modes are determined by windowing around the mode frequency in the frequency domain, then performing an inverse Fourier transform into the time domain, where the amplitude is fitted by an exponential. Analytic estimates are also displayed within the *Mathematica* notebook, allowing for easy comparison to the computational experiment.

5 Status

Initial trials of ODYSSEUS using standard CESR colliding beam parameters reproduce the expected single beam and colliding beam spectra. The program is sufficiently fast to allow runs of several radiation damping times in duration on a PC with an Alpha microprocessor. Trials including broadband wakefields are underway. In future months we plan to write a version of ODYSSEUS for parallel processing.

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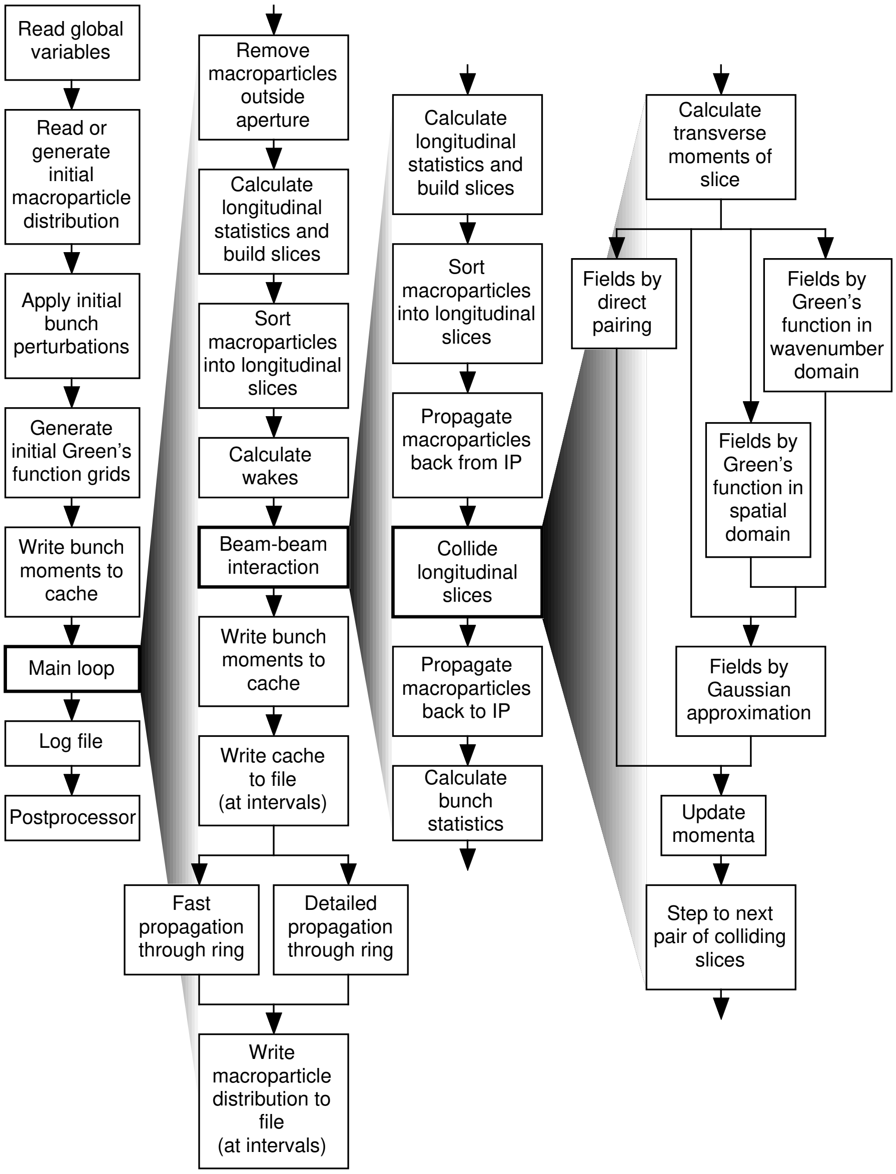

Figure 1: Structure of ODYSSEUS.