SOME PECULIARITIES OF MAGNETIC FIELD BEHAVIOR IN DUAL BORE MAGNETS

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In this brief report we described the field interference in dual bore focusing magnets.

Introduction

● For CESR next stage upgrades the dual bore lenses with superconducting coils were proposed [1]. Each of these lenses is a combination of two quadrupole magnets that share the same iron yoke. Distance between axes required about 80 *mm*. Dual bore superconducting *multipole* (sextupole, skew quadrupole or octupole and dipole) magnets supposed to be installed in series with these quads in the same cryostat. The field quality required for quadrupole lens is about

$$[B_y(x) - G\cdot x]/G\cdot x \le 5\cdot 10^{-4} \ , \tag{1}$$

where $B_y(x)$ is the field across the aperture, G is a gradient, $G = \left.\frac{\partial\, B_y(x)}{\partial\, x}\right|_{x=0} = \left.\frac{\partial\, B_x(y)}{\partial\, y}\right|_{y=0}$.

● For this high field quality required, however, the path way of the magnetic field lines in the iron yoke influence to the field quality in the lens aperture. Really, one can see that the path integral for magnetic field vector over closed loop L around the current I can be split in two parts (in practical units *Gs*, *cm*, *Ampere*)

$$\oint_L \vec{H}(\vec{r})d\vec{l} = \int_{IRON} \vec{H}(\vec{r})d\vec{l} + \int_{AIR} \vec{H}(\vec{r})d\vec{l} = 0.4\pi \ I \tag{2}$$

In the air $\vec{H} \equiv \vec{B}$ and boundary conditions for $\vec{B}(\vec{r})$ are $\left.\vec{B}_n(\vec{r})\right|_{IRON} = \left.\vec{B}_n(\vec{r})\right|_{AIR}$, and $\left.\vec{H}_t(\vec{r})\right|_{IRON} = \left.\vec{H}_t(\vec{r})\right|_{AIR}$, where indices n and t stand for normal and tangential to the iron components of the vector, so one can obtain

$$\oint_L \vec{H}(\vec{r})d\vec{l} = \int_{AIR} \vec{B}(\vec{r})d\vec{l} + \int_{IRON} \frac{\vec{B}(\vec{r})}{\mu(B)} d\vec{l} = 0.4\pi \ I \tag{3}$$

For quadrupole field this yields

$$\frac{G\cdot R_0^2}{2} + \frac{G\cdot R_0\cdot L_{IRON}}{\dot{\mu}} = \frac{G\cdot R_0^2}{2}\cdot\left(1 + \frac{L_{IRON}}{\dot{\mu}\, R_0}\right) \cong 0.4\pi \ I \ , \tag{4}$$

where R_0 stands for the radius of inscribed circle, $\dot{\mu}$ is the effective magnetic permeability along the line in iron. We would like to have the second term in the brackets as small as possible. In typical magnet, however, $L_{IRON} \approx R_0$, so from (4) yields even for relatively high magnetic permeability as $\dot{\mu} \approx 1000$

$$\frac{G\cdot R_0^2}{2}\cdot\left(1 + 10^{-3}\right) \cong 0.4\pi \ I \ \ , \tag{5}$$

or noticeable change of gradient with changing L_{IRON} . Situation becomes much worse, when the iron yoke becomes saturated. This effect, however does not influence to the *symmetry* of problem if the lens is a single aperture one. One can trim the poles in symmetrical way to adjust the field. For modeling the problem calculation within 45° is enough for mostly cases.

Cross interference of the fields

- When dual bore lenses shared the same yoke and placed close one to another, some magnetic lines travel around both centers, Fig.1. So in the presence of neighboring lens, magnetic lines *lose its quadrupole symmetry in the yoke*, and, hence in all lens. Now *not* only quadrupole-associated harmonics allowed. Dipole and sextupole components are the strongest among allowed now. The same happens, sequentially, for the whole lens.
- So one needs to have ability for effective compensation of this effect. This mechanism needs to be simple and effective. The lenses, where the poles *and* the coils acting *together* for the field generation were chosen finally for the next stage of CESR upgrade [2]. This type of lens was suggested in [3]. The superconducting coils are a racetrack type of single layer windings.
- We will consider asymmetrical case, when the gradients in both neighboring lenses have the same sign and about the same value. This means that the neighboring currents on the sides of the septum have the same value and there is no, hence, vertical component of magnetic induction in the iron septum.
- The shortest ways have the lines that travel around currents in the center of the lens. Example of such behavior is clearly seen in Fig. 1.

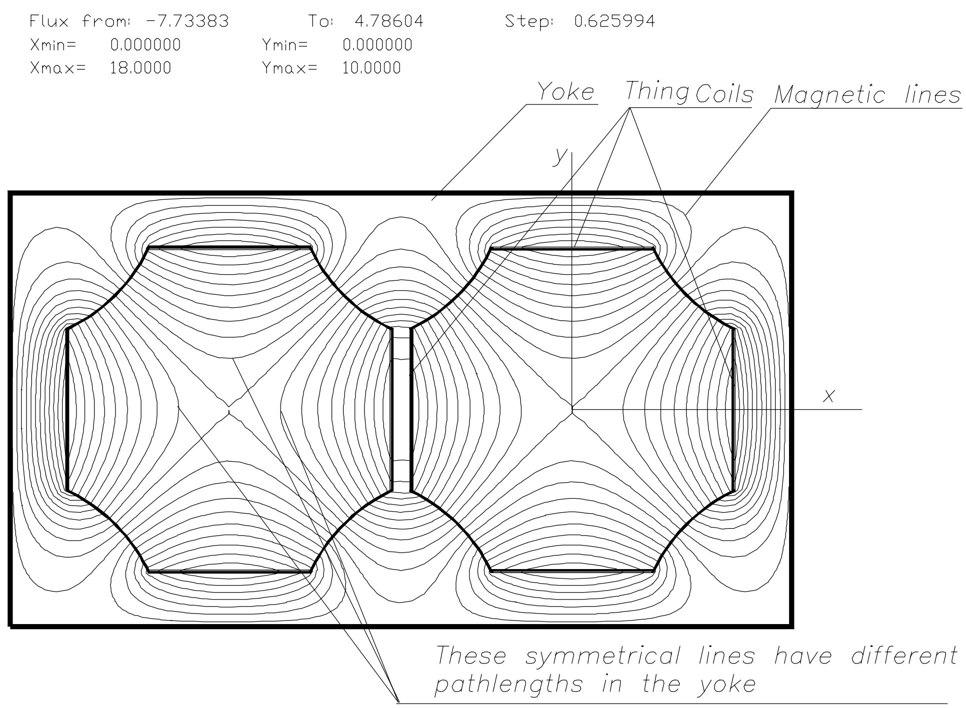

Fig. 1. The magnetic lines of dual bore lens. Windings are single layer racetrack like coils, which are thin in the drawing scale. They occupy the whole straight sections. The gradients in both lenses have the same sign. The distance between quadrupole axes is 79 *mm*. Yoke septum is 4 *mm*. Printout from MERMAID [4].

Here in Fig. 1 the example with superconducting coils is represented. The similar behavior, however, has any dual bore symmetrical lenses. One can also see from Fig. 1, that the lines spaced at the same distance from the center of each quad, have different path lengths in the material of yoke. So, as we mentioned above, the quadrupole symmetry is broken here. This effect yields a *sextupole* (and so on) component of the field at the center of each quadrupole.

- How big is this sextupole and how to compensate it? To estimate the sextupole value one can compare the difference in the path for central and side lines, what is approximately equal to the height a of vertical side.

So the difference in field values will be $\Delta B \approx GR_0 \frac{1}{\mu}\frac{a}{R_0} = \frac{Ga}{\mu}$ and sextupole

$$S \approx \frac{\Delta B}{(2\cdot R_0)^2} \approx \frac{Ga}{4\cdot\mu\cdot R_0^2} \approx \frac{G}{4\cdot\mu\cdot R_0} \tag{6}$$

where we supposed that $a \approx R_0$. At the distance $x \approx R_0$ relative field variation will be

$$\frac{\delta B}{B} \cong \frac{B_y - G\cdot R_0}{G\cdot R_0} \cong \frac{S\cdot R_0{}^2}{G\cdot R_0} \cong \frac{G\cdot R_0^2}{4\cdot\mu\cdot R_0\cdot G\cdot R_0} = \frac{1}{4\cdot\mu} \tag{7}$$

Substitute here again $\mu \approx 1000$ one can estimate relative variation as $2.5\cdot 10^{-4}$ what affects the field quality required.

● To compensate this effect, some appropriate non-symmetrical deformation of the poles required. Let us consider sextupole component. The pole profile could be found from expression for complex potential which must be constant on the pole surface, [3]

$$\mathrm{Re}\{(-i)\left[G\cdot z^2 + S\cdot z^3\right] = Const\,, \tag{8}$$

where G and S stand for quadrupole and sextupole coefficients correspondingly, $z = x + iy$, $i^2 = -1$, x and y are Cartesian coordinates, Fig.1. The last expression (8) yields the formula for the pole profile as

$$2xy + \frac{S}{G}\cdot(y^3 - 3x^2y) = R_0^2\,, \tag{9}$$

where R_0 stands for the radius of inscribed circle. At the vertical level equal to the radius of aperture R_0, $y = R_0$ the equation (9) becomes

$$2x + \frac{S}{G}\cdot(R_0^2 - 3x^2) = \pm R_0\,, \tag{10}$$

and relative deviation of the pole from hyperbola $2XY = R_0^2$ could be estimated as

$$\frac{(x - X)}{R_0} = \frac{\delta}{R_0} \approx \frac{3\cdot S\cdot R_0}{8\cdot G} = \frac{3\cdot S\cdot R_0^2}{8\cdot G\cdot R_0} \equiv \frac{3\cdot S\cdot R^2}{8\cdot G\cdot R}\cdot\frac{R}{R_0}\cdot\frac{R_0^2}{R^2} = \frac{3}{8}\frac{B(sext)}{B(grad)}\frac{R_0}{R}\,, \tag{11}$$

where $B(sext)$ and $B(grad)$ stand for the sextupole and quadrupole field amplitude on the radius R[1] correspondingly. If we require that the ratio $B(sext)/B(grad) \approx 5\cdot 10^{-4}$ at the radius R = 27 *mm*, then for the radius of inscribed circle of $R_0 \approx 35mm$, controlled deformation of the pole profile must be of the order $\delta \approx \frac{3}{8}5\cdot 10^{-4}\cdot\frac{35}{27}\cdot 35 = 0.0085\,mm$, or 8.5 *micrometers* at the vertical level $y = R_0$. The same considerations could be applied to the radial disposition $x = R_0$.

● We can also treat the consideration represented above in a different way. Namely, if we fix, that the accuracy of the field required is of the order $B(sext)/B(grad) \approx 5\cdot 10^{-4}$, then the accuracy of *fabrication* must be $\delta \approx 8.5\cdot 10^{-3} mm$ for the point around $y = R_0$. One can see, that if we fixed the sextupole field variations on the radius R and we would like to know what is the deviation of the profile allowed at bigger radius, then we can scale the absolute deviation $\delta = x - X$ *quadratically* with the radius increase. This example gives an idea for the accuracy of fabrication required.

[1] Radius R, where the fields arising from different harmonics are compared, called normalization radius.

● So one can see that the profile is rather complicated. In simplest case, however, when hyperbola is approximated by an arc of constant radius, the arc radius of the left and right poles could be made slightly *different*, keeping the vertex of the arcs in the same place.

● One can see also, that sextupole is not the only multipole allowed by broken quadrupole symmetry. From the other side, for the lens described [3], if it is far from neighboring lens, the field quality could be obtained *ideal* (confirmed by computer calculations down to numerical noise of the computer, what is $5\cdot10^{-6}$) in all cross-section of the lens. So we chose a different approach to the problem solution. Namely we keep the radiuses of all poles *the same*, but different from optimal, obtained, when the lens is far apart from neighboring one.

● Below we represent the graphs of *absolute* deviation of the field for different pole radius[2].

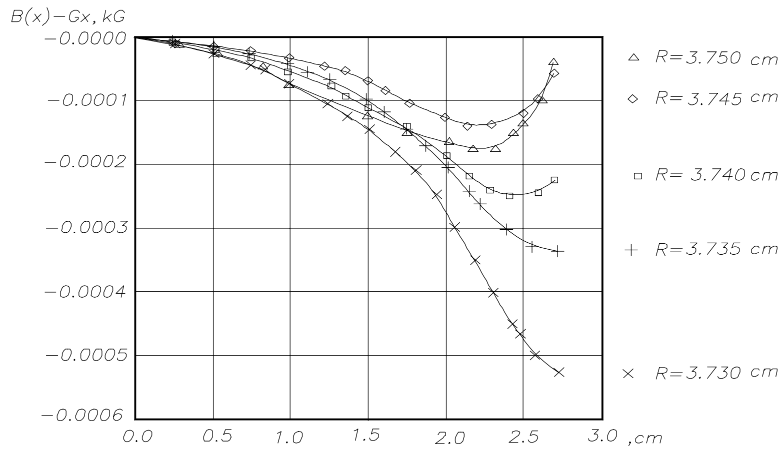

Fig. 2. Calculated field deviation (in *kG*) from linear behavior for different pole radius.

The smallest deviation from linear law indicates the lens with the pole radius of 37.45 *mm*. Maximum deviation of 0.15 *Gauss* occurs at distance 2.25 *cm*. Magnetic field value, associated with pure quadrupole having gradient $G = 1.075 kG / cm$ (for this particular radius and for 6 *kA*/*coil*) is 2.4187 *kG* . So, relative deviation in this point is $0.15 / 2418.7 = 6.2 \cdot 10^{-5}$. Namely this radius was chosen for manufacturing.

● The lens was fabricated and tested. Results of measurements are represented in Fig. 3.

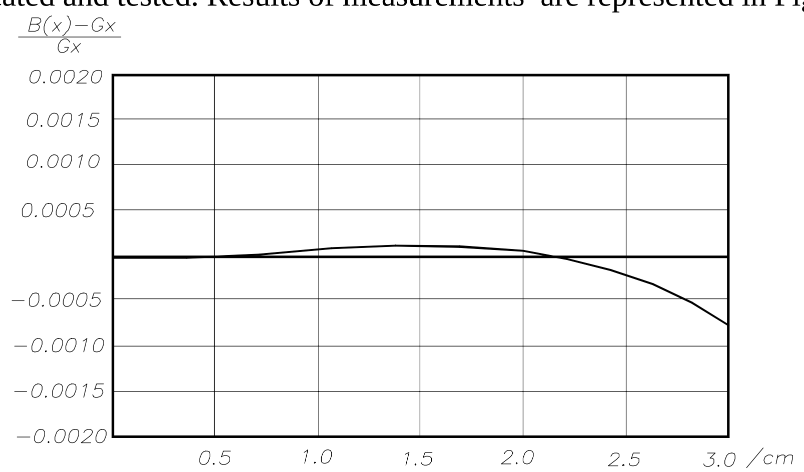

Fig. 4. Measured relative field deviation in one of the dual bore aperture.

[2] We would like to mention that absolute value of gradient is also slightly different in each case.

The graph plotted is a reconstruction of a vector sum of different harmonics measured in assembled dual bore magnet[3].

LHC example

• In much more difficult situation were designers of LHC dual bore quad (and dipole). As the exact coil dimensions are not available, we can make only some general remarks in this case, see Fig. 3. We took dimensions from the publication [5][4].

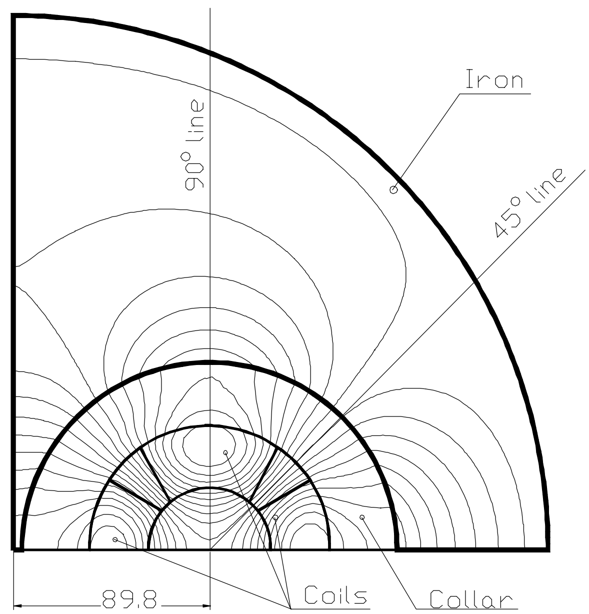

Fig. 3. LHC dual bore quadrupole lens. ¼ of the lens is shown. Diameter of the inner space is 56 *mm*. Outer diameter of the coils is 110 *mm*. Outer diameter of the collar is 172 *mm*. This fraction is enough for correct calculations. Indeed, calculation within 45^o or 90^o is not enough for the correct answer.

• One can see that the symmetry is broken here also. We considered the case with equal gradients in both apertures. The case with opposite gradients is even more difficult from the point of perfect field generation. We suggested current-turns in a single coil as high as 150 *kA*. In Fig. 4 there is represented a difference between calculated gradient for the model, represented in full Fig. 3 and the model limited by 90^o line in Fig. 3. Roughness on the curve explained by the mesh size chosen for modeling. One can see that the difference is bigger at the center of the lens. This has a natural explanation in fact that the influence of the iron becomes more significant for magnetic lines, what is closer to the center of the lens, see Fig. 3.

• The gradient line behavior as a function of transverse position is much better for 90^o model[5]. Typically, this variation is keeping zero on Fig. 4 down to center of the lens. At big displacements the gradient behavior defined mostly by the coils, so there is no big difference in gradient behavior at distance's ~1/3 of full aperture.

• As the gradient is about 11.4 *kG/cm,* the relative variation is about $0.06/11.4 \approx 5.2 \cdot 10^{-3}$. Of cause this value can be easily compensated by slight motion of left up coil in azimuth direction counter-clockwise and left lower coil clock wise.

[3] Measurements were carried out in a standard cryostat. Warm *long* coil was rotated inside the space arranged with two coaxial stainless steel tubes going through the magnet. Tubes in between were vacuumed and wrapped with superinsulation, see [2].

[4] Which might be out of date, of case. This may slightly affect the absolute value of effect.

[5] As one can expect.

- One can see, that this is, basically, the same problem what is present in our lens.

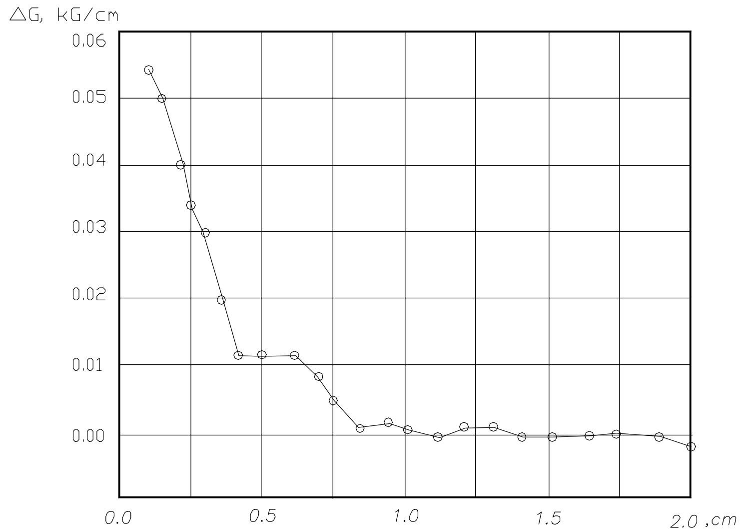

Fig. 4. The difference in absolute gradients for different modeling areas for dual bore LHC lens as a function of radial position. Full gradient is about 11.4 *kG/cm*. For 90^o model there is no absolute gradient deviations at the center.

Conclusion

- The nonlinarities arisen from broken quadrupole symmetry into iron can be taken into account by proper choice of problem for modeling. The level of nonlinarities required (1) yields a strong dependence of field on magnetic properties of the yoke material.
- We also took into consideration the magnetization of the iron[6] (not described here) arising from circulating current.
- The same ideas were applied to other multipole elements (such as sextupole, skew quadrupole, octupole and dipole corrector) of the ring.
- Serially installed quads and sextupoles in each module also may give the way for adjustment the resulting field quality for all unit.
- *However* this is the only small part among peculiarities of dual bore magnets. Mostly strong interference occurs at the magnet edge. Solution of this problem in application to the dual bore magnet designed was found, proved experimentally and will be described in other place.

References

[1] D. Rubin, M. Tigner, "Shared bends and Independent Quadrupoles", Cornell CON 94-28, 1994.
[2] A. Mikhailichenko, D. Rubin, " Concentric Ring Colliding Beam Machine with Dual Aperture Quadrupoles", Cornell CLNS 96-1420, 1996.
[3] A. Mikhailichenko," 3D Electromagnetic Field. Representation and measurement", CBN 95-16, Cornell, 1995.
[4] **MERMAID** --**ME**sh oriented **R**outine for **MA**gnet **I**nteractive **D**esign, SIM Limited, Novosibirsk, P.O. Box 402, Russia.
[5] G. Brianti, T. Tortschanoff, "Superconducting magnets for accelerators", in "Advanced Accelerator Physics and Technologies", vol.12, Edited by H. Shopper, p. 318.

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[6] The current $\approx 3A$, circulating in the ring generates the field in the iron $\approx 3[A]/20[cm] \cong 0.15\,Oe$ (for our geometry).