Construction Error Tolerances for the CESR Phase III IR Quadrupoles

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In this note, I estimate the error harmonics produced by variations in the coil geometry parameters from the design values. The techniques for the calculation of the error harmonics are based on the formalism outlined in CBN 96-5: The methods outlined in CBN 96-5 have been generalized to allow estimates of "non-allowed" error harmonics due to deviations from quadrupole symmetry. The details of this generalization are presented in an Appendix, available from the author on request. With these techniques, the harmonics may be calculated with different geometry parameters, and these harmonics may be used to evaluate the tolerances required on the geometry of the coils.

The coils comprising the two-dimensional cross section of the quadrupole may be broken down into "coil segments" labeled as shown in Fig. 1.

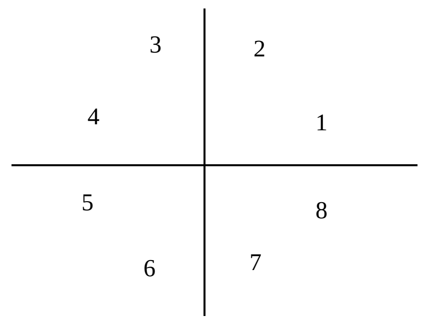

Fig. 1 Coil segment designations

In the final design, each of the 8 coil segment is formed from three blocks, each spanning a different azimuthal range. The geometry of a single block is defined by the parameters shown in Fig. 2.

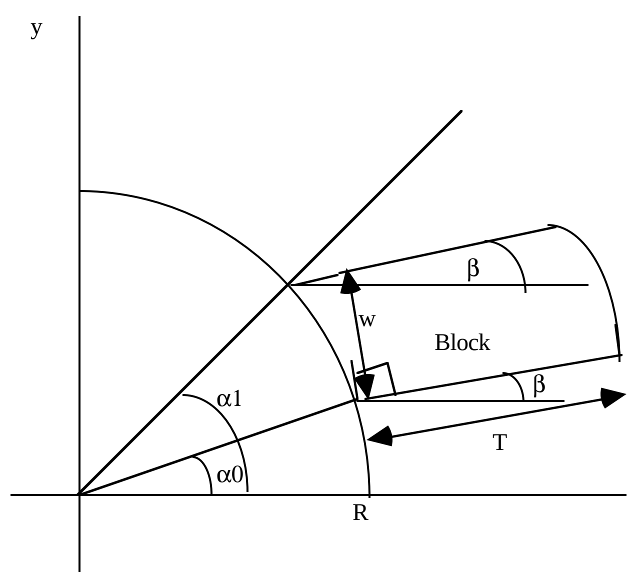

Fig. 2: Coil block geometry; taken from CBN 96-8

In the ends, coil segments 1 and 2, 3 and 4, 5 and 6, and 7 and 8 join. There are additional parameters describing the geometry of the ends: z_0, the extension of the straight part of the coil segment beyond nominal z=0; and e, the half-axis along the longitudinal direction of the ellipse which the coil segment forms as it goes around the end to join its partner . (See CBN 96-8 for more details). Additional constraints on the end geometry are provided by the constant-perimeter requirement, as discussed in CBN 96-8. The design coil segment geometry of the quadrupole is presented in Table 1.

Block #	1	2	3
R_1 (mm)	92.05	92.05	92.05
T(mm)	38.68	38.68	38.68
w(mm)	22.1	14.7	11
NI(kAmp)	205.8	137.2	102.9
α_0(rad)	0.0005	.262	.461
β(rad)	0.0	.2281	.5170
e (mm)	94.71	64.81	36.89
z_0 (mm)	29.24	9.87	4.33
Overall length(mm)	767	643	555.6

Table 1: Design coil segment geometrical parameters; taken from CBN 96-8

By design, of course, all the coil segments designated 1-8 in Fig. 1 are the same. I will define as "systematic" errors those field harmonic errors which are caused by the deviation of one or more of the geometrical parameters from the design values shown in Table 1, which occurs in *all* the coil segments Such deviations have quadrupole symmetry, so they will only generate "allowed" harmonic errors (the first allowed harmonic after m=2 is normal m=6).

There may also be deviations of a coil segment from the design which occurs only in one of the 8 segments shown in Fig. 1, or only in one of the 4 ends. I will define as "random" those field harmonic errors (which may be normal or skew, of any harmonic) caused by such a deviation, which occurs *only in one* of the coil segments.

Systematic errors and related construction tolerances

In Table 2 below, the systematic errors (normal, m=6) produced by the specified variations in the block geometry parameters are given. The last column indicates the tolerance on each parameters associated with a 1 unit change in the allowed b_6 harmonic.

Block	Parameter	Variation	Change in b_6 (units): Body	Change in b_6 (units): Ends	Tolerance for 1 unit change in b_6
All	**Inner radius R_1**	-0.5 mm	-0.3	-0.6	.45 mm
1	**w**	1 mm	-5.2	-1.2	.16 mm
1	α_0	10 mrad	-7.5	-2.2	1 mrad
1	β	50 mrad	-4.9	-1.3	8.1 mrad
1	T	1 mm	-2.4	-0.3	.37 mm
1	z_0	2 mm	1.2	.1	1.5 mm
1	**e**	5 mm	0.4	0.6	5 mm
2	**w**	1 mm	-3.5	-0.5	.25 mm
2	α_0	10 mrad	-6.8	-0.8	1.3 mrad
2	β	50 mrad	-3.6	-1.0	10.9 mrad
2	T	1 mm	1.0	0.5	.67 mm
2	z_0	2 mm	-0.5	0	4 mm
2	**e**	5 mm	-0.3	-1.3	3.1 mm
3	**w**	1 mm	0.3	0	3.3 mm
3	α_0	10 mrad	0.1	0.1	50 mrad
3	β	50 mrad	0.7	-0.5	>50 mrad
3	T	1 mm	1.7	.3	2 mm
3	z_0	2 mm	.9	0	2.2 mm
3	**e**	5 mm	-.4	-1.4	2.8 mm

Table 2: Construction tolerances from systematic errors

Random errors and related construction tolerances

A simple description of the tolerance to random errors is difficult, because such errors may produce many harmonics. To assess the situation, each parameter characterizing the geometry of each block has been independently varied, and the resulting error harmonic calculated, using the formulas given in the appendix. The results are given in Tables 3, 4 and 5. The entry for "R-0.5 mm" in Table 3 actual corresponds to a change in the inner radius of *all three* blocks by the same amount, not just that of block 1.

Case	m=1	m=1	m=2	m=2	m=3	m=3	m=4	m=4	m=5	m=5	m=6	m=6	m=7	m=7	m=8	m=8	Sum
	Body	Ends	Body	Ends	Body	Ends	Body	Ends	Body	Ends	Body	Ends	Body	Ends	Body	Ends	m>2
w+1 mm	7.07	.18	6.32	.44	4.15	.64	2.37	.06	1.23	.41	.65	.29	.26	.07	.24	0	10.37
α_0**+10 mr**	13.29	2.77	11.96	1.15	7.98	.59	4.67	.09	2.53	.77	1.29	.54	.63	.11	.37	0	19.57
R-0.5 mm	10.54	2.49	1.23	.55	2.39	.65	2.09	.06	1.34	.33	.71	.15	.32	.004	.13	0	8.174
β+**50 mr**	11.01	.39	9.27	2.28	5.76	1.86	3.12	.11	1.55	.67	.73	.33	.35	.03	.19	0	14.70
T+1 **mm**	7.61	.53	1.20	1.89	.68	1.04	.77	.04	.53	.09	.30	.07	.15	.06	.07	0	3.80
z_0**+2 mm**	1.31	1.45	1.05	1.58	.66	.58	.46	.23	.28	.07	.15	.02	.07	0.	.03	0.	2.55
e+5 mm	14.58	10.75	5.63	11.25	2.22	3.78	1.11	.16	.48	.18	.18	.14	.06	.11	.02	.005	8.445

Table 3: Harmonic errors due to perturbations of block 1 geometrical parameters
The numbers correspond to the absolute value of the largest harmonic error (in units) for each m, due to the indicated geometry change in a single coil segment (i.e., 1/8 of the coil for the body, 1/4 for the end)

Case	m=1	m=1	m=2	m=2	m=3	m=3	m=4	m=4	m=5	m=5	m=6	m=6	m=7	m=7	m=8	m=8	Sum
	Body	Ends	Body	Ends	Body	Ends	Body	Ends	Body	Ends	Body	Ends	Body	Ends	Body	Ends	m>2
w+1 mm	4.92	.37	3.84	.49	1.92	.26	1.31	.02	.85	.13	.44	.12	.18	.05	.05	0	5.33
α_0**+10 mr**	9.22	.365	7.35	.27	3.85	.04	2.29	.02	1.55	.17	.86	.20	.39	.09	.25	.01	9.72
β+**50 mr**	6.69	1.12	.84	1.69	2.31	.92	1.55	.05	.93	.34	.46	.25	.18	.08	.13	0	7.20
T+1 **mm**	2.80	.07	.45	.46	.69	.41	.57	.03	.35	.18	.13	.12	.10	.03	.06	0	2.67
z_0**+2 mm**	1.28	.79	.43	.86	.47	.32	.27	0	.14	.04	.07	.01	.04	0	.03	0.	1.39
e+5 mm	7.27	4.79	2.87	5.74	1.04	2.48	.40	.14	.17	.59	.06	.33	.23	.07	.11	0	5.62

Table 4: Harmonic errors due to perturbations of block 2 geometrical parameters
The numbers correspond to the absolute value of the largest harmonic error (in units) for each m, due to the indicated geometry change in a single coil segment(i.e., 1/8 of the coil for the body, 1/4 for the end)

Case	m=1	m=1	m=2	m=2	m=3	m=3	m=4	m=4	m=5	m=5	m=6	m=6	m=7	m=7	m=8	m=8	Sum
	Body	Ends	Body	Ends	Body	Ends	Body	Ends	Body	Ends	Body	Ends	Body	Ends	Body	Ends	m>2
w+1 mm	3.42	.53	2.28	.56	1.06	.19	.25	.01	.42	.02	.31	.01	.14	0	.1	0	2.51
α_0**+10 mr**	6.8	.54	4.32	.48	1.85	.10	1.31	.0	.49	.04	.55	.04	.18	.01	.18	0	4.75
β**+50 mr**	3.25	1.23	2.43	1.48	1.31	.63	.73	.03	.52	.16	.32	.11	.14	.04	.11	0	4.10
T+1 **mm**	.61	.16	1.0	.03	.86	.08	.43	.01	.35	.08	.21	.07	.09	.03	.05	0	2.26
z_0**+2 mm**	1.89	.38	1.39	.41	.71	.15	.33	0	.22	.02	.11	.01	.04	0	.03	0	1.62
e+5 mm	3.05	2.12	1.37	2.74	.63	1.35	.28	.09	.13	.47	.06	.35	.03	.12	.02	.01	3.54

Table 5: Harmonic errors due to perturbations of block 3 geometrical parameters
The numbers correspond to the absolute value of the largest harmonic error (in units) for each m, due to the indicated geometry change in a single coil segment(i.e., 1/8 of the coil for the body, 1/4 for the end)

In the last column of each table, I have added all the contributions from the harmonics for m=3 to 8, body plus end. This is essentially a worse-case estimate, corresponding to the contributions from all the harmonics adding in-phase. I have omitted the contributions from m=1 and m=2, since such errors should be able to be compensated for by using the dipole and/or skew quad correctors. In Table 6, the results from the final columns of Tables 3-5 are summarized, in a format similar to that of Table 2. The last column indicates the tolerance on each parameters associated with a 1 unit change in the total sum of harmonics >2.

Block	Parameter	Variation	Sum m>2 harmonics (units)	Tolerance for total 1 unit change in the sum
All	**Inner radius R_1**	-0.5 mm	8.2	.12 mm
1	**w**	1 mm	10.4	.1 mm
1	α_0	10 mrad	19.6	0.5 mrad
1	β	50 mrad	14.7	3.4 mrad
1	T	1 mm	3.8	.26 mm
1	z_0	2 mm	2.6	.77 mm
1	**e**	5 mm	8.4	.6 mm
2	**w**	1 mm	5.3	.19 mm
2	α_0	10 mrad	9.7	1 mrad
2	β	50 mrad	7.2	6.9 mrad
2	T	1 mm	2.7	.37 mm
2	z_0	2 mm	1.4	1.4 mm
2	**e**	5 mm	5.6	.9 mm
3	**w**	1 mm	2.5	.4 mm
3	α_0	10 mrad	4.8	2.1 mrad
3	β	50 mrad	4.1	12.2 mrad
3	T	1 mm	2.3	.44 mm
3	z_0	2 mm	1.6	1.25 mm
3	**e**	5 mm	3.5	1.4 mm

Table 6: Construction tolerances from random errors

Sensitivity to coil ovality in the ends

A special case to be considered is the consequences of the deformation of the coil ends due to the interaction of the CLEO solenoid field with the transverse currents flowing in the ends of the quadrupole coils. The resulting forces cause an end deformation resulting in a radius increase (+ΔR) of the coil ends formed from segments 1 and 2, and from segments 5 and 6; and a decrease (-ΔR) in the radius of the coil ends formed from segments 3 and 4, and from segments 7 and 8. Analysis indicates that such a deformation (in both ends, but not in the body) results, to lowest order, in a skew octupole (m=4) error harmonic, of magnitude 0.2 units, for a radial displacement of $\Delta R=0.25$ mm.

Conclusion

In Table 7, the last columns from Tables 2 and 6 are collected, and grouped by block parameter, rather than by block number. The systematic error tolerances are generally larger than that for the random errors. The smallest position tolerance is on the width of the block 1 coil segment: 0.1 mm, or about 4 mils. The smallest angular tolerance is on the block position angle α_0 for block 1: 0.5 mrad. At a radius of 92 mm, this corresponds to a required azimuthal positioning accuracy for the block of .05 mm, or about 2 mils. The quadrupole field specification calls for an overall field quality of better than 5 units, so tolerances larger than those shown in Table 7 can be allowed, provided their net contribution is less than 5 units.

Block	Parameter	Systematic Error Tolerance for 1 unit	Random Error Tolerance for 1 unit
All	**Inner radius R_1**	.45 mm	.12 mm
1	**w**	.16 mm	.1 mm
2	**w**	.25 mm	.19 mm
3	**w**	3.3 mm	.4 mm
1	α_0	1 mrad	0.5 mrad
2	α_0	1.3 mrad	1 mrad
3	α_0	50 mrad	2.1 mrad
1	β	8.1 mrad	3.4 mrad
2	β	10.9 mrad	6.9 mrad
3	β	>50 mrad	12.2 mrad
1	T	.37 mm	.26 mm
2	T	.67 mm	.37 mm
3	T	2 mm	.44 mm
1	z_0	1.5 mm	.77 mm
2	z_0	4 mm	1.4 mm
3	z_0	2.2 mm	1.25 mm
1	**e**	5 mm	.6 mm
2	**e**	3.1 mm	.9 mm
3	**e**	2.8 mm	1.4 mm

Table 7: Overall summary of construction error tolerances for 1 unit of error harmonic