# Forces on Interaction Region Quadrupoles and Dipoles Due to a Detector Solenoid Magnet *

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One feature of today's high energy physics experiments is a detector with a large scale solenoid magnet for measurement of transverse momenta of charged particles [1]- [6]. Another is the demand for very high luminosity. To meet luminosity demands accelerator designers need to put insertion magnets close to the interaction point, even within the detector solenoid field. Such designs have lower chromaticity, smaller peak beta functions, require less aperture and basically allow tighter focussing of the beams. However, the insertion magnets become more difficult and complicated to design since they are so closely coupled with the detector magnetically and mechanically.

Only insertion magnet designs that use superconductors or rare earth permanent magnets are practical within the detector volume. Detector magnetic fields are typically 1.5 T and can overload any steel placed within them. Furthermore, steel within the solenoid field would generate large perturbations of the fields that would be difficult to predict or control accurately due to hysteresis. Permanent magnets tend to have less interaction with the detector solenoid than superconducting magnets. The fringe field of permanent magnets rapidly falls off with radial distance, especially if a large number of segments are used [7]. With superconductors there are typically no special coils to cancel the external fields, (though in principle that would be possible at the cost of substantial radial space and complication), so the problems of interaction with the solenoid are maximal. In this paper we will only discuss the forces on superconducting magnets.

Superconducting magnets are often designed to support large internal Lorentz forces generated by their self-fields. However when immersed in the external field of a solenoid the Lorentz forces can produce large net forces and torques on a magnet as a whole. This leads to problems supporting the magnet, coupling the magnet to the helium vessel, and supporting the cryostat. Support is critical because the beam is particularly sensitive to motions of insertion magnets due to the relatively high gradients and large beta functions in the interaction region.

## Detector Solenoid Field

We will only discuss detector solenoids which have a main component $B_z$ as well as a ra-

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dial fringe field component $B_r$, but no azimuthal component. Both components can produce substantial Lorentz forces on the accelerator magnet coils. In the body of the magnet coils, where the current is only in the $\pm z$ direction, the solenoid $B_z$ component generates no force — only the radial component does. However both components can contribute to generate forces in the coil ends.

As an example, consider the CLEO detector solenoid. The calculated $B_z$ and $B_r$ components are shown in Figure 1. Also shown in the plot are outlines of insertion quadrupoles and concentrically wound dipoles which are being built for the CESR phase III upgrade [2]. The forward quadrupole magnet Q1 is entirely immersed in an almost uniform solenoid field with only a small radial component near the outermost end. The second magnet coil Q2 starts where the radial component of the solenoid field is near maximum and $B_z$ has dropped to about one-half its central value.

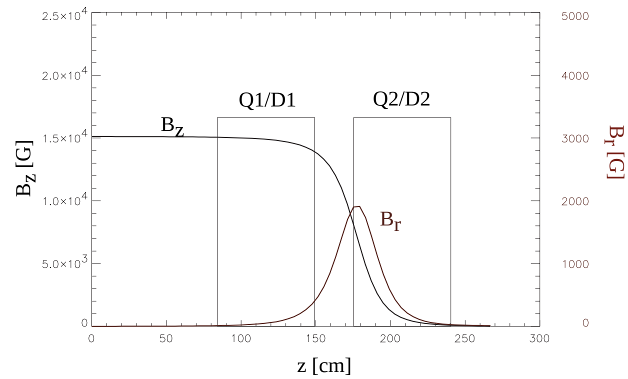


Figure 1: The calculated longitudinal and radial components of the magnetic field of the CLEO solenoid are plotted as a function of the distance from the interaction point, for a constant radii equal to the average coil radius. Superimposed are the locations of the superconducting quadrupole magnets.

## Symmetries

The symmetries of external electromagnetic forces acting on dipole and quadrupole coils are shown in Figures 2—5. Forces due to the $B_z$ and $B_r$ components are separately plotted for clarity. One force vector is shown for the end of each coil and one is shown for each magnet straight section. Only near-side forces are shown in each view.

### Dipole Symmetries

In Figure 2 we can see that from the longitudinal component of the solenoid field a dipole coil experience forces only at the coils ends. The resulting torque can be large since the end forces are separated by the entire length of the magnet.

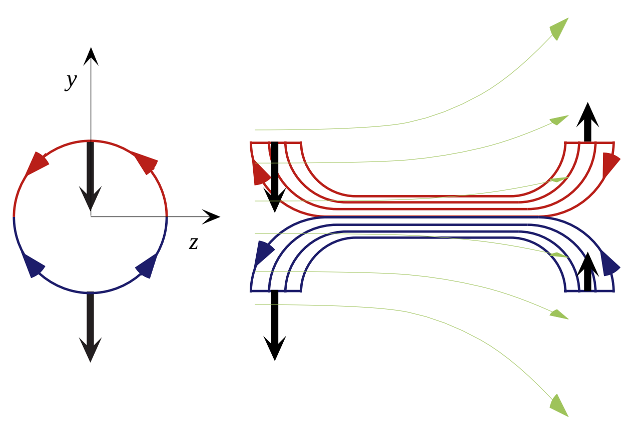


Figure 2: Forces acting on a dipole magnet due only to the $B_z$ component of the solenoid field

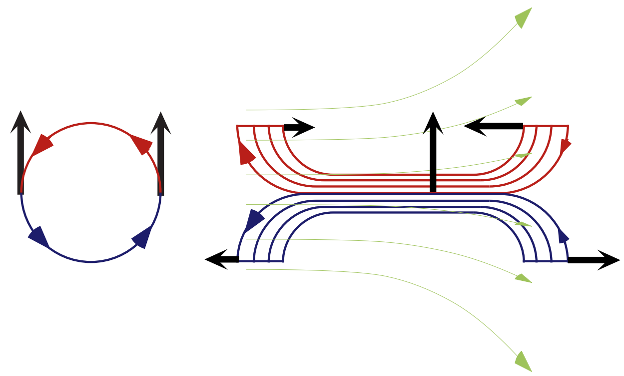

Figure 3: Forces acting on a dipole magnet due only to the $B_r$ component of the solenoid field

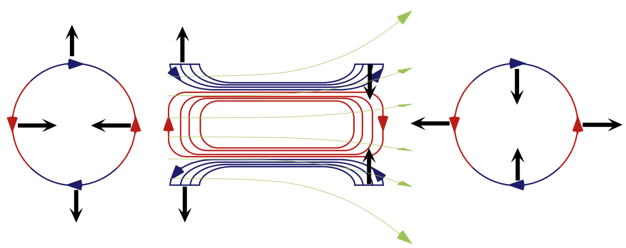

Figure 4: Forces acting on a quadrupole magnet due only to the $B_z$ component of the solenoid field

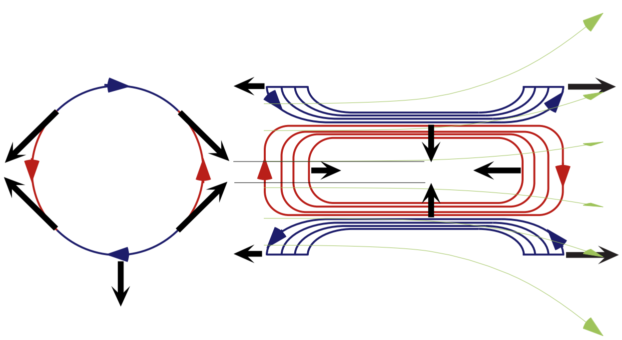

Figure 5: Forces acting on a quadrupole magnet due only to the $B_r$ component of the solenoid field

There will also be a net force to the extent that $B_z$ is different at the two ends. The direction of the force is either parallel or anti-parallel to the field the dipole generates.

In Figure 3 the dipole coils shown produce a dipole field downward just as in Figure 2. One can see that $B_r$ produces an upward force in the body which is centered at a location determined by $\int zB_r dz$. This force is only partially canceled by the forces generated in the ends by the $B_z$ component shown in Figure 2. The $B_r$ component will also generate torques in the coil ends which partially cancel each other to the extent that $B_r$ is the same at the two ends.

## Quadrupole Symmetries

Perfectly aligned quadrupole coils experience no net forces or torques due to $B_z$ or $B_r$. This is easily seen in Figure 4 and Figure 5. Generally the force on each coil part is canceled by an equal and opposite force on the corresponding part of the opposing coil.

Even though there are no net forces or torques, the forces can be important to the magnet design. The net solenoid generated force on the ends may be $\sim 1/3$ as much as the quadrupole self-field generated forces. [1] But the symmetry of the solenoid generated force on the ends is much more unfavorable. The solenoid force tends to crush the end of the coil while the internally generated forces attempt the much more difficult operation forcing the coils into a square shape. Adequate collar stiffness has to be provided to resists this crushing force.

[1]Typical solenoid fields are 1.5 T and typical quadrupole self-field is $\sim 5$ T.

### Misaligned Magnets

A substantial force can be generated for a quadrupole asymmetrically placed with respect to nearby detector steel (not shown). For example the Q2 magnet in Figure 1 is surrounded by the CLEO solenoid yoke steel. Two dimensional ANSYS calculations indicate that for a 3 mm offset of the quadrupole center with respect to the hole in the pole, an 1100 lb force is generated which tries to pull the magnet toward the steel. This force could be in any transverse direction and would be proportional to the current in the quadrupole coils. Similarly large forces can be generated by asymmetrically adding large holes to the steel yoke even if the quadrupole is perfectly positioned.

If the solenoid field is tilted with respect to the quadrupole or dipole axes, or vice versa, many more forces and torques are introduced.

## Calculation of Forces

Direct 3D integration of the Lorentz force per unit volume over the coil geometry can provide values for the total forces and torques on elements of the coils. Forces due to $B_r$ are best obtained by this method, though the integration over the coil end geometry can be difficult. However, within the approximation of $\vec{B} = B_0\vec{e}_z$ where $B_0$ is constant there are several simple analytic methods which can be quite accurate in determining forces on the coil ends.

We start by calculating the forces on a single turn. In this case there is no force on the wire in the straight portion of the coil because the current and the solenoid field are in the same direction — all the forces are generated in the ends. The force on a infinitesimal length of a wire $d\vec{s}$, with current $I$ in the uniform field is:

$$\begin{aligned} d\vec{F} &= IB_0 d\vec{s} \times \vec{e}_z && (1) \\ &= IB_0(-dx\vec{e}_y + dy\vec{e}_x) && (2) \end{aligned}$$

The total force on the coil end is obtained by integrating $d\vec{F}$ along the wire around the end. The integral is trivial and the result is simply that the net force on the single wire is $\vec{F} = IB_0(-\Delta x\vec{e}_y + \Delta y\vec{e}_x)$, where $\Delta x(\Delta y)$ is the net change in the $x(y)$ coordinate of the wire. The actual path the wire takes does not matter, only the net change in position contributes to the total force on the end. Applying this result to the coil as a whole we have that the total force on a coil end is:

$$\vec{F} = \sum_{wires} IB_0(-\Delta x\vec{e}_y + \Delta y\vec{e}_x) \qquad (3)$$

So the problem remains only to add up the contributions from all the wires. This is best done with a set of $x, y$ positions of all the wires. Such data can be put into a spreadsheet and the resulting force immediately calculated via equation 3.

Even more convenient would be a simple analytic expression. We will derive four. The first two are for a quadrupole idealized as a $\cos 2\theta$ current sheet or as a $30°$ sector current sheet. (To better represent the real coil the radial dependence should be taken into account.) In the former case the density of turns per unit angle $n(\theta) = 2N\cos 2\theta$, where $N$ is the number of turns per coil. We will express $\Delta x$ and $\Delta y$ in terms of the radius of the current sheet $R$ and $\theta$. Assuming the coil is wound symmetrically about a central post, $\Delta x = R(\cos(\frac{\pi}{2} - \theta) - \cos\theta)$ and $\Delta y = -\Delta x$. The force on the coil may now be obtained by integrating over $\theta$ from 0 to $\pi/4$.

$$\vec{F}_{coil\ end} = -\frac{2}{3}NIB_0R(\vec{e}_y + \vec{e}_x) \qquad (4)$$

$$F_{radial,\ coil\ end} = \frac{2\sqrt{2}}{3} NIB_0R \tag{5}$$

which is the desired result for the $\cos 2\theta$ quadrupole.

For example, a quadrupole with $NI = 450,000$ ampere turns, with a coil radius of $R = .092$ m in a solenoid of $B_0 = 1.5$ T, the radial force on each pole is 58,548 N or 13,130 lbs.

For 30 degree sector idealization of a quadrupole the density of turns is simply a constant: $n(\theta) = 6N/\pi$. The expressions $\Delta x$ and $\Delta y$ are the same so the force integral is just

$$\vec{F}_{coil\ end} = -\frac{B_0NIR3(\sqrt{3}-1)}{\pi}(\vec{e}_y + \vec{e}_x) \tag{6}$$

$$F_{radial,\ coil\ end} = -\frac{3\sqrt{2}(\sqrt{3}-1)}{\pi} NIB_0R \tag{7}$$

The force is $\frac{9}{2\pi}(\sqrt{3}-1) = 1.05$ times larger than for the $\cos 2\theta$ approximation.

Expressions for the force on dipole coils can be derived in exactly the same manner using equation 3 and current distributions for a dipole. For a $\sin\theta$ dipole coil idealization we have that $n(\theta) = N\sin\theta$. For a symmetrically wound dipole producing a horizontal field, $\Delta x = 0$ and $\Delta y = 2R\sin\theta$. Replacing the summation with an integral and integrating yields

$$\vec{F}_{dipole\ coil} = B_0 I \int_0^{\pi/2} N\sin\theta 2R\sin\theta \vec{e}_x \tag{8}$$

$$= \frac{\pi}{2} B_0 RNI\vec{e}_x \tag{9}$$

For the 60 degree sector approximation for a dipole we have $n(\theta) = 3N/\pi$. This distribution yields

$$\vec{F}_{dipole\ coil} = \frac{3\sqrt{3}}{\pi} B_0 RNI\vec{e}_x \tag{10}$$

For illustration, consider a dipole at radius 0.144 m and with $NI = 28,400$ ampere turns, in a uniform solenoid field of 1.5 T. The $\sin\theta$ idealization yields a horizontal force per coil end of 9636 N or 2161 lbs, for total force on the magnet end of 4,322 lbs. The 60 degrees sector approximation result is again only about 5% higher.

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