Cornell CBN 96-06 May 23, 1996

Using a Horizontal Kicker to Damp Longitudinal Oscillations

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At first blush it might seem silly to try to damp longitudinal motion using a horizontal kicker since a transverse kick does not immediately affect a particles energy $\delta \equiv \Delta E/E$ or its longitudinal position z. Yet a horizontal kicker can be used for damping and it is relatively simple to explain how it works.

The first step is to understand how horizontal motion couples into the longitudinal. Consider a particle with horizontal displacement x from the nominal orbit going through a bend of bending radius $r \equiv 1/G$. The difference dl in path length between the nominal orbit and the actual path the particle takes over a distance $ds = d\theta/r$ is (Sands[1] Eq. (2.15))

$$\begin{aligned} dl &= (r + x)d\theta - r\,d\theta \\ &= x\,G\,ds\;. \end{aligned} \tag{1}$$

Now consider the effect of a constant horizontal kick $\Delta x'$ at $s = 0$. This shifts the equilibrium orbit (Sands Eq. (2.92))

$$x_c(s) = \frac{\Delta x'(0)\sqrt{\beta_x(0)\beta_x(s)}}{2\sin\pi\nu_x}\cos(\overline{\phi}_x(s) - \pi\nu_x)\;, \tag{2}$$

where ν_x is the tune, and $\overline{\phi}_x(s) \equiv \phi_x \bmod 2\pi\nu_x$ is the phase advance modulo the tune. Using Eq. (1) the change in path length ΔL of the new equilibrium orbit is

$$\Delta L = \oint ds\,G(s)\,\frac{\Delta x'(0)\sqrt{\beta_x(0)\beta_x(s)}}{2\sin\pi\nu_x}\cos(\overline{\phi}_x(s) - \pi\nu_x)\;. \tag{3}$$

Compare this with the formula for the dispersion η at $s = 0$ (Sands Eq. (3.6))

$$\eta(0) = \frac{\sqrt{\beta_x(0)}}{2\sin\pi\nu_x}\oint ds\,G(s)\,\sqrt{\beta_x(s)}\,\cos(\overline{\phi}_x(s) - \pi\nu_x)\;. \tag{4}$$

Combining Eqs. (3) and (4) gives the deceptively simple equation

$$\Delta L = \Delta x'(0)\,\eta(0)\;. \tag{5}$$

In equilibrium the RF keeps the path length constant so the change in path length given in Eq. (5) must be compensated by a change in energy δ which gives a change in path length of (Sands Eq. (3.10)):

$$\Delta L_\delta = \alpha_P\,L_R\,\delta\;, \tag{6}$$

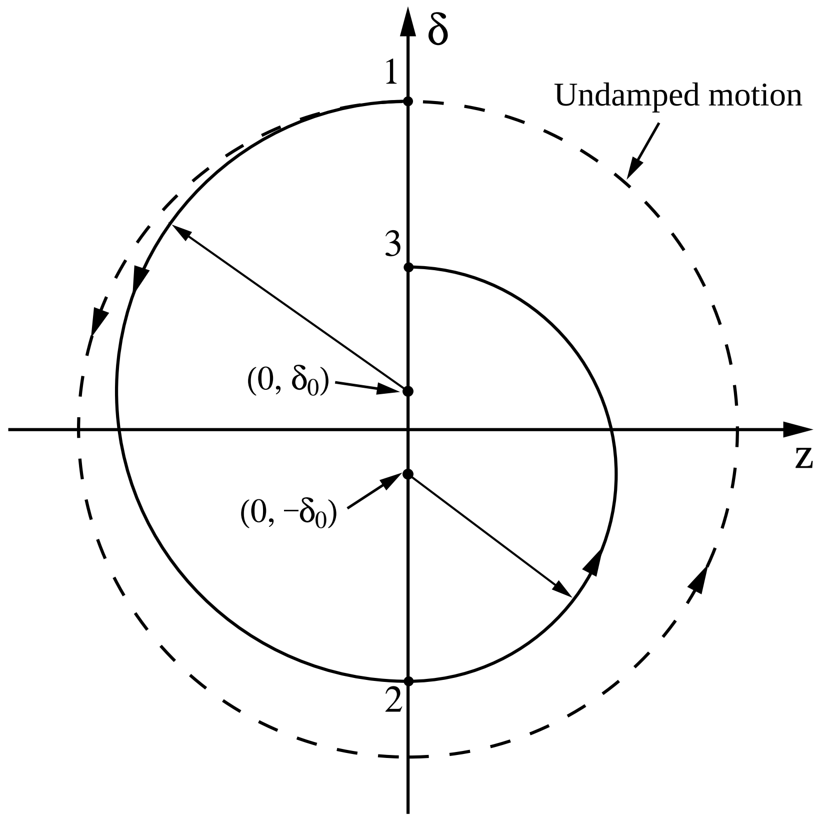

Figure 1: Damping in phase space achieved by shifting the equilibrium point.

where L_R is the ring circumference, and α_P is the momentum compaction factor. Setting $\Delta L + \Delta L_\delta = 0$ gives for the equilibrium energy change[2]

$$\delta_{\rm eq} = -\frac{\eta(0)\,\Delta x'(0)}{\alpha_P\, L_R}\,. \tag{7}$$

Now consider a particle oscillating in longitudinal phase space with oscillation amplitude $\delta_{\rm amp} = (\delta^2 + z^2)^{1/2}$ as shown in figure 1. When the particle reaches point 1 of maximum δ the kicker is turned on to shift the equilibrium point from the origin to a point $(0, \delta_0)$ as shown in the figure. For 1/2 a synchrotron cycle we let the particle oscillate around this point until it reaches point 2. We now reverse the sign of the kicker so that the equilibrium point shifts to $(0, -\delta_0)$. After another 1/2 cycle the particle has reached point 3. Thus, in 1 cycle, we have reduced the oscillation amplitude by $4\delta_0$ so that the damping rate γ_d per unit time is

$$\begin{aligned}\gamma_d &\equiv \frac{d\delta_{\rm amp}}{dt} \\ &\frac{4\eta(0)}{\alpha_P\, L_R T_s}\,\frac{\Delta x'(0)}{\delta_{\rm amp}}\,,\end{aligned} \tag{8}$$

where T_s is the synchrotron period. This is the damping rate when the kicker has a square wave output. When the kicker output is instead proportional to the oscillation amplitude, as shown in appendix A, the factor of 4 in the numerator gets replaced by a factor of π.

Note the fundamental difference here in how damping is achieved as opposed to, say, the usual damping of transverse oscillations. With transverse oscillations you apply a kick to directly change x' and this kick is in phase with the x' oscillations. Here it is not δ that is directly affected but $\delta_{\rm eq}$ and one must wait for the synchrotron motion to move the particle to a position closer to the origin. Also in this case the shift in $\delta_{\rm eq}$ is out-of-phase with the δ oscillations.

The only problem with the above analysis is that if the kicker is suddenly turned on then the transverse motion will only reach equilibrium in a time period large compared with a synchrotron oscillation period. This being the case does it really make sense to use Eq. (7)? To see why this is valid consider the following: After the kicker is suddenly turned on the particle will oscillate about the new closed orbit:

$$x(s,n) = x_c(s) + A_x \sin(\phi_x(s) + 2\pi n \nu_x) \,, \tag{9}$$

where n is the turn number after the kicker is tuned on. The effect of the horizontal motion given in Eq. (9) on the longitudinal can thus be broken down into 2 parts: One part due changes in $x_c(s)$ has been analyzed above. The second part due to the second term in Eq. (9) is due to the free oscillations at frequency ν_x. Since ν_x and ν_z are not commensurate, that is, we cannot find small integers k, l, and m such that $k\nu_x + l\nu_z = m$, the effect of the free oscillations upon the longitudinal motion will, over many turns, average to zero. Thus, over many turns, Eq. (8) gives the correct answer.

Appendix: Damping for a Proportional Kick

Let $\mathbf{z}$ be the complex representation of the position of a particle in longitudinal phase space:

$$\mathbf{z} \equiv z + i\delta \,. \tag{10}$$

The equation of motion of $\mathbf{z}$ is simply

$$\dot{\mathbf{z}} = i\omega_s(\mathbf{z} - \mathbf{z}_0) \,, \tag{11}$$

where $\omega_s = 2\pi/T_s$ is the synchrotron frequency and $\mathbf{z}_0$ is the equilibrium position. With $\mathbf{z}_0 = 0$ Eq. (11) is easily solved and is pure oscillatory as expected.

$$\mathbf{z} = Ae^{i\omega_s t} \,. \tag{12}$$

To find the effect of an non–zero $\mathbf{z}_0$ we assume that $\mathbf{z}_0$ is "small" and use first order perturbation theory. If the amplitude varies as

$$|\mathbf{z}| \sim e^{-\gamma_d t} \,. \tag{13}$$

Then from Eq. (11)

$$\gamma_d = \frac{1}{2|\mathbf{z}|^2}\frac{d|\mathbf{z}|^2}{dt} = \left\langle \frac{i\omega_s(\mathbf{z}\mathbf{z}_0^* - \mathbf{z}^*\mathbf{z}_0)}{2|\mathbf{z}|^2} \right\rangle , \tag{14}$$

where $\langle\cdots\rangle$ is an average over many cycles to average out short term fluctuations and $*$ denotes the complex conjugate. Let $\mathbf{z}_0$ oscillate along the imaginary axis, out of phase with $\mathbf{z}$, and with an oscillation amplitude proportional to $|\mathbf{z}|$:

$$\mathbf{z}_0 = -i\,\alpha\frac{\mathbf{z} + \mathbf{z}^*}{2} , \tag{15}$$

where α is the proportionality factor. Since we are using first order perturbation theory we use the zeroth order oscillation for $\mathbf{z}$ as given in Eq. (12). Using this with Eq. (15) in Eq. (14) gives

$$\gamma_d = \frac{\omega_s\,\alpha}{2} . \tag{16}$$

This is equivalent to Eq. (8) with a factor of π replacing the factor of 4.

References

[1] Matthew Sands, *The Physics of Electron Storage Rings, An Introduction*, SLAC-121 Addendum, (1970).

[2] This has been derived before. Cf: W. Lou, M. Billing, and D. Rice, *Perturbation of Beam Energy Due to Steering and Pretzel Orbit*, 1995 IEEE Part. Acc. Conf. (1995).