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The Incoherent Long Range Beam-Beam Interaction in CESR. [1]

A. Temnykh and D. Sagan
Cornell University, Ithaca NY 14953, USA

INTRODUCTION

CESR has reached the world's highest luminosity[1] using mutibunch operation. The key to this operation is a horizontal 'pretzel' separation of the beams in the arcs and a crossing angle at the IP. The pretzel separation is needed since the counter rotating bunches share the same beam pipe and there is thus a long range beam–beam interaction (LRBBI) between the beams. The effects of the LRBBI can be divided into coherent and incoherent parts. The coherent part involves motion of the bunches as a whole. In CESR the coherent LRBBI is strong enough to cause operational problems in terms of tune shifts and orbit displacements[2] but it does not lead directly to instabilities. The incoherent LRBBI, on the other hand, involves individual particles of one beam (henceforth called the "probe" beam) interacting with the other "strong" beam. Particles of the probe beam are destabilized when their horizontal oscillation amplitude is large enough to pass near, and feel the full effect of, the strong beam. Because of the possible lifetime problems, it is the incoherent LRBBI that sets the limit for the minimum practical pretzel separation between beams and it is the incoherent LRBBI that is the subject of this paper.

Future plans call for increases in current as well as the number of bunches. Since more current will mean a stronger beam-beam kick and more bunches will mean more parasitic crossing points, it is important to understand how the LRBBI destabilizes particles. Temnykh, Welch and Rice[3] have studied the LRBBI experimentally and have put forward phenomenological models to predict the minimum separation achievable for a 50 minute beam lifetime. The criteria from these models have been incorporated into the CESR optic design

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to minimize the effect of the LRBBI[1].

To go beyond these phenomenological models the way in which the LRBBI destabilizes particles must be understood. It is shown in this paper using a simple numerical simulation that, with CESR conditions, and with the beams separated horizontally, the LRBBI leads to vertical beam tail growth and loss of particles in the vertical plane. Furthermore, the threshold of this instability depends on the vertical size of the opposite beam. An increase of the vertical size of the opposite beam leads to an increase of the allowed beam intensity for a given separation and for a given lifetime. These conclusions are shown to be supported experimentally.

1 Theory

1.1 Tune shift

To illustrate how strong the LRBBI is consider the linear tune shift that would be present if the beams where brought into head on collision at one parasitic crossing point. The tune shift is given by the standard formula[4]

$$\Delta\nu_{x,y} = \frac{N_p r_e \beta_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)} \tag{1}$$

Here $\sigma_{x,y}$ are vertical and horizontal beam sizes, N_p is the number of particles, r_e is the classical electron radius, β is the beta function, and γ the usual relativistic factor. Using typical CESR parasitic interaction parameters: $N_p = 1.6 \cdot 10^{11}$, $\beta_{x,y} = 15\,\text{m}$, $\sigma_x = 2.5\,\text{mm}$, $\sigma_y = 0.25\,\text{mm}$, and $\gamma = 10^4$, the above expression gives:

$$\begin{aligned} \Delta\nu_x &= 0.017 \\ \Delta\nu_y &= 0.17 \end{aligned} \tag{2}$$

The tune shift in the vertical is 10 times the tune shift in the horizontal and gives a good indication why the LRBBI destabilizes vertically. The vertical tune shift is also much greater than the maximum tune shift obtained with the beam-beam interaction at the IP of around 0.05. This shows why in the arcs the beams need to be kept well separated.

Parameter	Symbol	value
Horizontal beta	β_x	10 m
Vertical beta	β_y	20 m
Horizontal tune	Q_x	.538 m
Vertical tune	Q_y	.602 m
Horizontal sigma	σ_x	1.50 mm
Vertical sigma	σ_y	0.25 mm
Bunch-bunch offset	x_{sep}	9.75 mm
Strong bunch current	I	10 mA

Table 1: Parameters used in simulating the LRBBI.

1.2 Tracking

Using a tracking simulation program a previous study[3] found that the horizontal and synchrotron motion did not exhibit instabilities. Only the vertical motion was seriously affected by the LRBBI. At the time it was not clear why this was so. A simple explanation for this can be constructed as follows: Consider a simple particle tracking model where a particle is first transformed from a parasitic collision point back to the parasitic collision point using a linear matrix:

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = \begin{pmatrix} \cos\mu_x & \beta_x \sin\mu_x & 0 & 0 \\ -\sin(\mu_x)/\beta_x & \cos(\mu_x) & 0 & 0 \\ 0 & 0 & \cos\mu_y & \beta_y \sin\mu_y \\ 0 & 0 & -\sin(\mu_y)/\beta_y & \cos\mu_y \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} \tag{3}$$

where $\mu_{x,y}$ are horizontal and vertical betatron phase advances and x and y are the distances from the probe beam center. After the ‘arc’ transport the LRBBI kick is given by

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \mathcal{F}_x(x - x_{sep}, y) & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \mathcal{F}_y(x - x_{sep}, y) & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} \tag{4}$$

Here x_{sep} is the horizontal separation of the beams at the parasitic interaction point and the kick functions $\mathcal{F}_{x,y}(x, y)$ for a bi–Gaussian strong bunch are obtained from the standard Bassetti and Erskine formula[6]. In the tracking program we have used a subroutine for the beam-beam kick extracted from the MAD program developed in CERN[7].

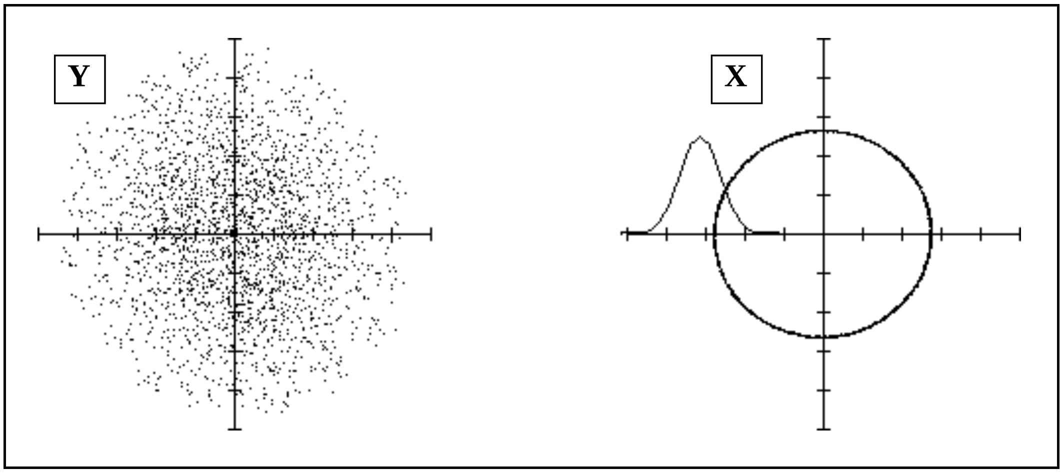

Figure 1: Tracking simulation in phase space using the parameters of table 1. The starting point was $x = 5.5\sigma_x$, $x' = 0$, $y = 1.0\sigma_y$, $y' = 0$. The scale in the figures is $2\sigma_{x,y}$ per division. The gaussian curve in the X plot is the horizontal strong bunch profile.

Figure 1 shows the results of tracking using parameters appropriate for the minimum acceptable separation used for CESR operation as given in table 1. The beams were separated by a distance $x_{sep} = 9.3\,\mathrm{mm}\,(6.3\sigma_x)$ and a single particle with initial conditions $x = 5.5\sigma_x$ (corresponding to a 50 min lifetime) and $y = 1.0\sigma_y$ was tracked for 1000 turns. As clearly seen in the figure the horizontal motion is stable and practically unperturbed. The vertical motion, on the other hand, looks chaotic. This simple simulation agrees well with a previous, more detailed simulation[3].

2 Resonance Structure

As a function of the distance from the strong bunch $x_{offset} \equiv x - x_{sep}$ the vertical kick is a maximum when the the distance from the strong bunch is $x_{offset} \sim 1\sigma_x$ and past $1\sigma_x$ the vertical kick decreases roughly as $1/x_{offset}$. Thus, for a typical separation of $x_{sep} \simeq 6.5\sigma_x$, and a horizontal oscillation amplitude of, say, $A_x \sim 5.5\sigma_x$, the vertical kick is strongly influenced by the horizontal oscillations. This coupling from horizontal to vertical makes the analysis of the vertical motion extremely complicated.

With this being said, in order to try to understand the vertical motion consider the following drastic simplification: Consider the motion of a particle in vertical phase space due to the LRBBI assuming that the x–position of the

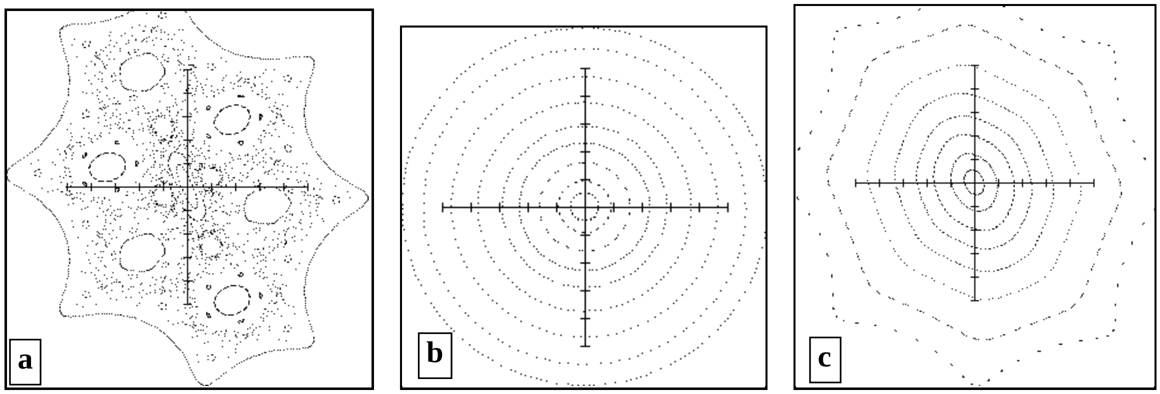

Figure 2: Vertical phase space for particles with fixed x_{offset}. The strong beam intensity and other parameters are the same as for the tracking shown on figure 1. *a*) $x_{offset} = 0.86\sigma_x$. *b*) $x_{offset} = 9.90\sigma_x$. *c*) $x_{offset} = 3.76\sigma_x$.

particle is frozen at some constant value. For purposes of illustration consider the particle tracked in figure 1. On the first turn it had $x_{offset} = 0.86\sigma_x$, on the second turn x_{offset} was $9.90\sigma_x$ and on the third turn one had $x_{offset} = 3.76\sigma_x$. The vertical phase space with x_{offset} fixed at these 3 values is shown in figures 2a, b, and c respectively.

With $x_{offset} = 0.86\sigma_x$ the difference in vertical tune between small and large amplitudes is large enough such that two low order nonlinear resonances can be observed. Here one can see the island chains associated with 4th and 6th order resonances. An 8th order resonance is also seen at large amplitudes. Overlapping of these resonances results in the appearance of a stochastic region[9] from $1\sigma_y$ up to $15\sigma_y$. In this region there is strong diffusion and particles experience fast and unpredictable changes in amplitude. In figure *b* corresponding to $x_{offset} = 9.9\sigma_x$ there are stable, practically unperturbed, trajectories. The last case, *c*, is intermediate between the first two.

If we now include the effect of the horizontal motion we see that a particle passes near the center of the opposite beam, due to the stochastic motion seen on figure 2*a* the vertical amplitude will be unpredictably changed. This will cause diffusion in vertical phase space as has been seen in figure 1. While this picture is, of course, greatly oversimplified it does bring out some of the physics behind the LRBBI. In tracking simulations it has been seen that there is a good correspondence between when a particle with a given horizontal amplitude shows unstable behavior and the onset of chaos in the vertical phase space with the horizontal position of the particle fixed at the extreme end nearest the strong beam.

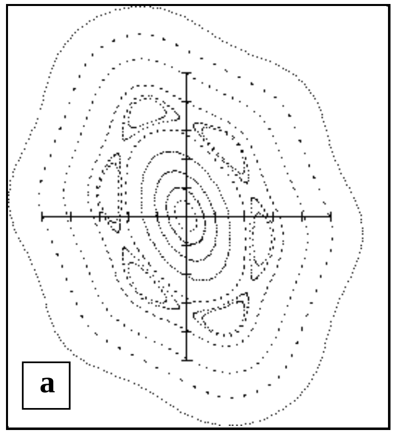

Figure 3: Vertical phase space for particles with the same fixed horizontal position as in figure 2a but with a strong beam vertical size of 0.75 mm. Other parameters are the same as in Table 1.

The condition for resonances overlapping, seen on figure 2a, is determined by the strength of the resonance harmonics and by the betatron tune dependence on the amplitude. This is affected by vertical strong beam size and by the strong beam intensity. An increase of the vertical beam size for a given intensity reduces the strength of the resonance harmonics as well as tune difference between large and small amplitude. This results in a reduction of the number of resonances between these amplitudes and the disappearance of the stochastic region. If the foregoing discussion is correct this will stabilize the vertical motion.

Figures 3 and 4 illustrate this. Figure 3 shows the vertical motion using the same parameters as used for figure 2a except with a strong vertical size of $\sigma_y = 0.75\,\text{mm}$ which is 3 times larger than what was used for figure 2a. The figure shows regular trajectories with little sign of chaotic behavior. Here the difference between tunes for large and small amplitudes is smaller so that only one resonance structure is seen. Figure 4 shows the 2 dimensional tracking and in contrast with figure 1 there is no hint of unstable motion. Thus with an increase in vertical beam size we can expect that for a given separation it should be possible to reach higher beam current. This is indeed seen experimentally as discussed in the next section.

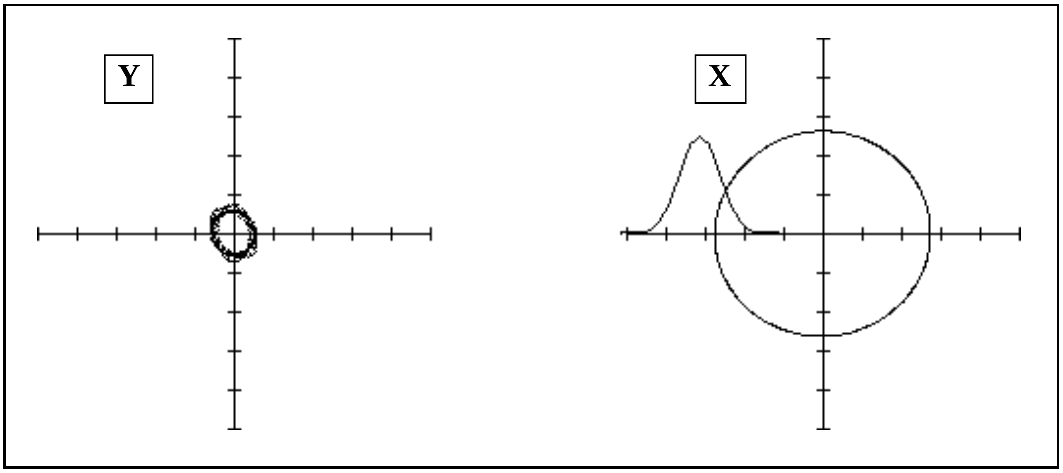

Figure 4: 2 dimensional tracking with a large strong vertical size of $\sigma_y = 0.75\,\mathrm{mm}$. Other parameters identical to used for tracking shown in figure 1. The Gaussian curve in the X plot is the horizontal strong bunch profile.

Parameter	PC1	PC2
β_x(m)	9.3	10.3
β_y(m)	33.2	20.4
η(m)	0.10	1.19
σ_x(mm)	1.26	1.51
x_{sep}(mm)	9.16	9.78
x_{sep}/σ_x	7.26	6.47

Table 2: Parameters at the parasitic crossing points.

3 LRBBI Beam Tails Measurement

In order to experientially see how particles are destabilized by the LRBBI the enlargement of the beam tails due to the LRBBI was measured experimentally. This was achieved by using the standard technique of monitoring beam lifetime verses the position of a scraper[8]. One bunch per each beam were filled such that the bunches did not collide at the IP but interacted at two opposing parasitic crossing points in the arcs. Table 2 gives parameters of the two crossing points.

Figures 5 and 6 show the results for the horizontal and vertical respectively. Each figure shows loss rate as a function of scraper position with and without the strong bunch. For the horizontal, the presence of the strong bunch only

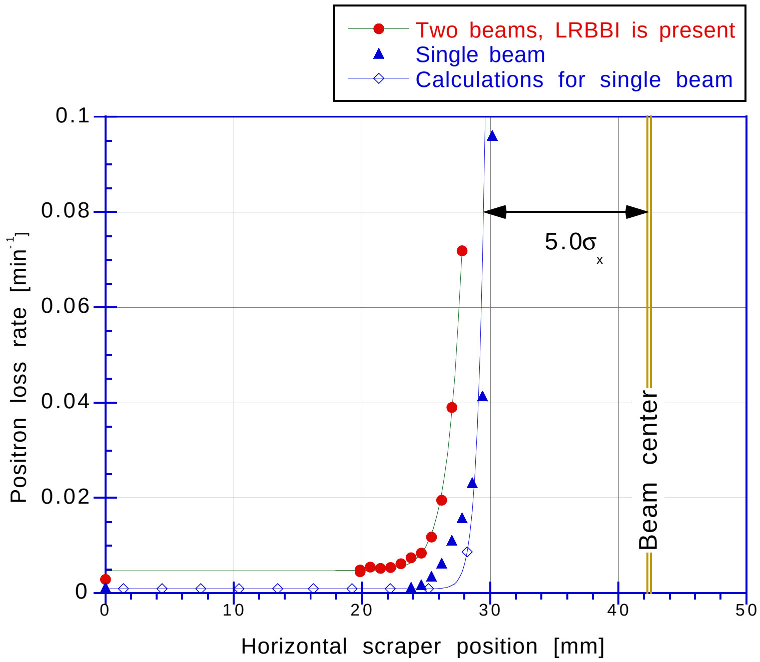

Figure 5: Positron beam loss rate versus horizontal scraper position. Triangles and diamonds show single beam measurements and calculation, circles show loss rate with the LRBBI.

enlarged the tails by 20% or so. For the vertical the enlargement of the tails was dramatic such that the size of the vertical tails became similar to that of the horizontal. This is in qualitative agreement with the previous section and shows that the instability is indeed a vertical one.

4 Dependence of the LRBBI Effect on Strong Beam Size

To test empirically the conclusion of the previous section that enlarging the strong beam size would reduce the affect of the LRBBI, experiments were preformed where the lifetime of a probe bunch was measured as a function of current for fixed vertical strong beam size. The vertical beam size of the strong bunch was varied without affecting the probe bunch by taking advantage of the fact that the beams follow different trajectories. It was thus possible to change the tune of one beam but not the other by using sextupole magnets. By tuning the strong bunch to be near the coupling resonance sideband $Q_x - Q_y + Q_s = n$ the strong bunch beam size could be increased without affecting the probe

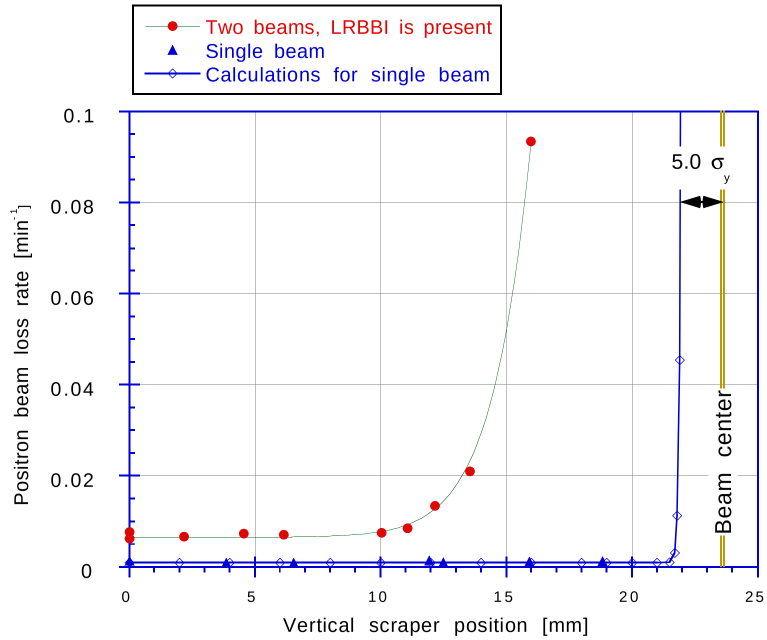

Figure 6: Positron beam loss rate versus vertical scraper position. Triangles and diamonds show measurements and calculations with a single beam. Circles show loss rate with with two beams so that the LRBBI is present.

bunch and without causing lifetime problems for the strong beam. Figure 7 shows σ_y as a function of horizontal tune near the coupling resonance sideband.

The procedure used to study the dependence of the minimum separation (as determined by the given minimum lifetime) on vertical beam size was similar to the method used by Temnykh, Welch and Rice[3]. One bunch per each beam was filled such that the bunches did not collide in the main IP but interacted at two opposite crossing points in the arcs. Table 2 gives parameters of the two parasitic crossing points.

The probe positron bunch had a current of 3 to 5 mA while the strong electron bunch had currents up to 18 mA. Care was taken to keep beam tunes, beam-beam separation, etc. constant. In particular the horizontal emittance of the strong beam was constant in spite of the presence of the coupling sideband resonance used to vary the vertical beam size.

Figure 8 shows the positron probe beam loss rate for two different strong beam sizes. As can be seen from the figure the loss rate of the probe beam is very dependent on the strong vertical size. For example with an electron current of, say, 12 mA with a small vertical emittance, the probe beam loss rate was 0.042/min corresponding to a 25 min lifetime. With a large strong

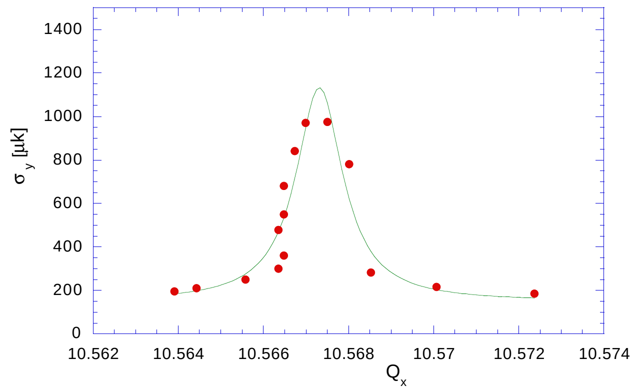

Figure 7: Vertical beam size versus horizontal tune, Q_x, crosses resonance $Q_x - Q_y + Q_s = 1$. The tunes were $Q_y = 9.618$ and $Q_s = 0.051$

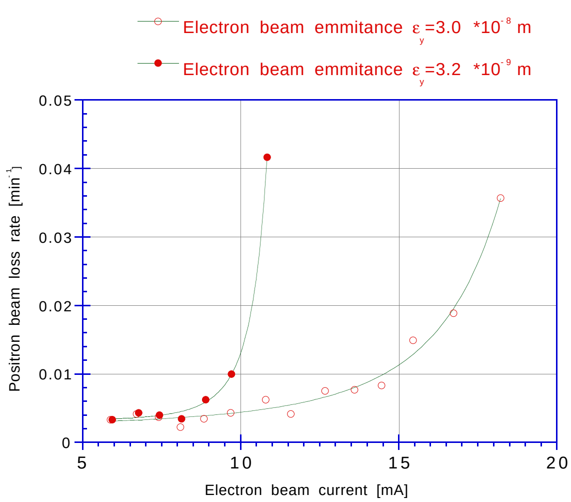

Figure 8: Probe beam loss rate versus strong beam current. Solid points — Strong beam vertical emittance $\epsilon_y = 3.2 \cdot 10^{-9}$ m. Open points — Strong beam on resonance with $\epsilon_y = 3.0 \cdot 10^{-8}$ m.

bunch emittance, on the other hand, the loss rate was 10 times smaller and essentially equal to the background lifetime. To put it another way, for equal loss rates at, say, 0.01/min, an increase in vertical emittance allows us have 50% more current in the opposite beam for a given separation.

5 Conclusions

We have found good qualitative agreement between a simple simulation model and experiment which indicates that the incoherent LRBBI may be simply simulated with good reliability. In particular the understanding of the nature of the diffusion in the vertical plane should lead to the development of more reliable criteria for the minimum acceptable separation of the beams.

The effect of a large vertical beam emittance on the LRBBI will benefit operation in the case where counter rotating bunches that share the same beam pipe need to have a separation at the parasitic crossing point.

6 Acknowledgments

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