Synchrotron Radiation and Pretzeled Orbits

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Turning on a 'pretzel' , or any closed orbit distortion, can substantially alter the path of synchrotron radiation (SR), in some instances causing it to strike components not designed to absorb the radiation. In this paper I develop equations useful in determining the source position and angle of dipole SR as a function of the pretzel amplitude and angle. Formula for linear power deposition and rough estimates of peak temperatures are also included.

Geometry of Synchrotron Radiation Illumination

Figure 1 shows the design orbit, labeled 'ref', going from a dipole where it has constant bend radius ρ to a straight section. A pretzeled orbit is also shown in the bend magnet only. The amplitude and angle of the pretzeled orbit are measured from the design orbit with positive horizontal values referring to the ring outward direction. The pretzeled orbit shown has a negative pretzel amplitude $-p$ but a positive slope p'. The angle θ is measured along the design orbit and is equal to the design orbit path length from the end of the dipole to the source point, divided by the bend radius. SR is emitted at angle $\theta + p'$ relative to the straight section axis. At some distance l from the end of the dipole the emitted radiation reaches a point y away from the straight section axis where it may be absorbed.

I first calculate y in terms of the design orbit angle θ. After some approximation I then invert the expression to find the θ in terms of y and l. By inspection of figure 1:

$$y = (l + \rho \sin\theta + p \sin\theta)\tan\theta + p' - \rho(1 - \cos\theta) + p\cos\theta \tag{1}$$

Bear in mind that the pretzel shown has a negative amplitude. Equation 1 gives the radial trajectory of the ray emitted from a pretzeled orbit. Usually we have a known radial location and want to know what the source point is.

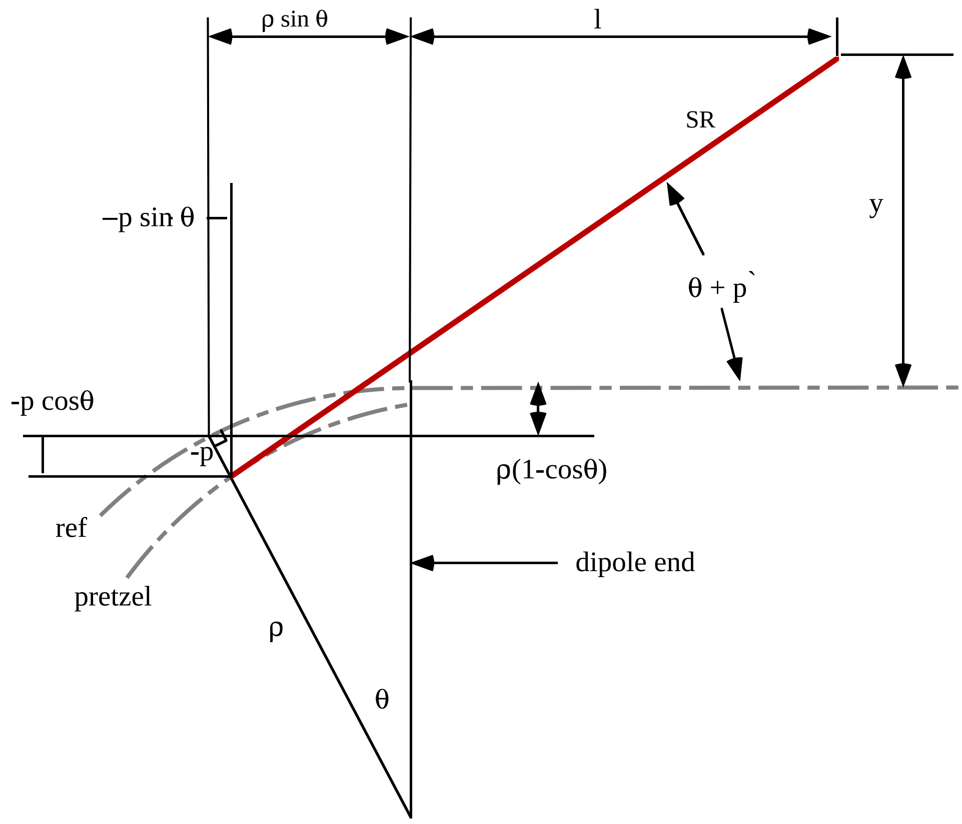

Figure 1: The figure shows the geometrical realtions between emission and 'absorption' of a ray a synchrotron radiation from a dipole magnet next to a straight section

As it stands the equation cannot be inverted (at least by me). [1] However, by approximating the trigonometric terms it is possible to reduce the expression to one which is second order in θ and can be inverted. Carrying out the calculation to second order in θ is important because terms such as $l\theta$ are of the same magnetude as terms such as $\rho\theta^2$. The result of this approximation is

$$\boxed{y \approx l\theta + \frac{\rho\theta^2}{2} + p + (l + \rho\theta)p'} \tag{2}$$

As values for θ are usually 0.02 radians or less, a second order calculation is pretty accurate. These geometrical relations have been confirmed by direct construction using a CAD layout and in some instance compared with output from a completely independent program written by Roberto Kersevan at LNS.

Equation 2 can be inverted easily to yield the useful expression given below:

$$\boxed{\theta = -(\frac{l}{\rho} + p') + \sqrt{(\frac{l}{\rho} + p')^2 - 2(\frac{p + lp' - y}{\rho})}} \tag{3}$$

Note that θ is the angle of the point on the design orbit adjacent to the source point on the pretzeled orbit. The emitted radiation therefore has angle $\theta + p'$ and not just θ unless the pretzel happens to be flat at the emission point. With no pretzel the expression reduces to

$$\theta = \frac{l}{\rho}\left(\sqrt{1 + \frac{2y\rho}{l^2}} - 1\right) \tag{4}$$

The quantity $2y\rho/l^2$ is often not as small as you might first think, so expanding the square root may not be very accurate.

The maximum pretzel angle is very roughly the maximum pretzel amplitude divided by the average horizontal beta function. For CESR this is around 1 mr. Typical θ for CESR are 10 - 20 mr so the direct effects of pretzel angle are usually small. The effects of the pretzel amplitude are not small. In fact a change in the pretzel amplitude has the same effect as changing the radial position of the absorber by the same amount.

[1] Roberto Kersevan has shown me a derivation of the exact relation for zero pretzel. His result is:

$$\theta = \arctan\frac{y + \rho}{l} - \arcsin\frac{\rho}{\sqrt{l^2 + (y + \rho)^2}}$$

His derivation can probably be extended to include pretzels.

Power and Temperature Calculations

Normally for storage rings, the SR is absorbed over a stripe with very little vertical height compared to the width. Consequently the temperature distribution tends to assume a two dimensional profile. The peak temperature occurs at the vertical center of the stripe. Also typical for storage rings, is the fact that the vertical height is much smaller than any other physical size of relevance: distance to cooling channel, thickness of plate, etc. Often the full width half maximum of the distribution is only tenths of millimeters compared with millimeters or more to the cooling channels. For this reason the highest thermal gradients occur very close to the center of the stripe and the overall temperature rise is only weakly determined by the physical sizes of the absorbing structure. Under these conditions, to calculate the approximate peak temperature rise in an absorber of a given material, it is only necessary to evaluate the linear power density.

Power

The average power radiated by beam of electrons per angle of bend is:

$$\boxed{\frac{dP}{d\theta} = \frac{88.5E^4I}{2\pi\rho}} \tag{5}$$

where E is in GeV, I is the average beam current in mA, and ρ is the bend radius in meters, [1]. To get the total SR power absorbed one only needs the angular range of emitted radiation which reaches the absorber. The minimum and maxium angles that can strike an absorber are easily computed by evaluating equation 3 for the maximum and minimum radial extents of the absorber. Because the power emitted per unit angular bend is constant, the total power absorbed is simply the angular size multiplied by $dP/d\theta$.

In the case of an absorber whose absorbing surface is perpendicular to the beamline the linear power density is

$$\frac{dP}{dy} = \frac{dP}{d\theta}\frac{d\theta}{dy} \tag{6}$$

From equation 2 we have $dy/d\theta \approx l + \rho(\theta + p')$. The linear power density striking a perpendicular absorber is then:

$$\frac{dP}{dy} = \frac{88.5E^4I}{2\pi\rho}\frac{1}{l + \rho(\theta + p')} \tag{7}$$

Similarly if the absorber is parallel to the design orbit so that y is constant and l varies, the linear power density can be calculated by evaluating

$$\frac{dP}{dl} = \frac{dP}{d\theta}\frac{d\theta}{dl} \tag{8}$$

Equation 3 can be differentiated and multiplied by equation 5 to give

$$\frac{dP}{dy} = \frac{88.5E^4 I}{2\pi\rho}\left\{\frac{-1}{\rho} + \frac{2l}{\rho^2}\left[(\frac{l}{\rho} + p')^2 - 2(\frac{p + lp' - y}{\rho})\right]^{-1/2}\right\} \tag{9}$$

Temperature rise

Taking advantage of the relative simplicity of the SR stripe mentioned above, I make a reasonably accurate approximations which yields a relatively simple formula for peak temperature rise at the center of the stripe. This calculation assumes the bulk of the material is cooled by ordinary means and that the peak temperature results only from the heat flux and thermal conductivity of the material the SR strikes.

Consveration of energy requires

$$J = -K\nabla T \tag{10}$$

where J is the heat flux (energy per unit area per unit time), K is the thermal conductivity, and T is the temperature. In steady state conditions,

$$\nabla \cdot J = \rho \tag{11}$$

where now ρ is the heat energy per unit volume, not the bend radius. These two equations imply

$$\nabla^2 T = -\frac{\rho}{K} \tag{12}$$

which has the same form as the Poisson equation of electrostatics.

I will choose a particularly simple geometry that might be representative of the SR absorption problem. Refer to figure 2. For this geometry the problem is easy to solve because it is just like a line charge in two dimensions.

$$\int J da = P \rightarrow P = K\pi R\frac{dT}{dR}\delta z \tag{13}$$

where P is the total power striking a length along the beamline δz, R is the distance to a constant temperature surface (the water cooling for instance), and $w << R$ is the effective width of the source. So we have,

$$\frac{dT}{dR} = \frac{(P/\delta z)}{\pi R K} \tag{14}$$

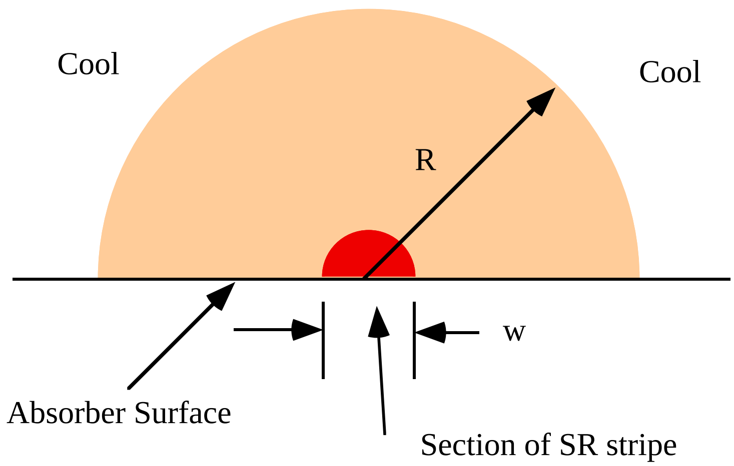

Figure 2: Simplified geometry of heat generated by absorbed SR.

and finally integrating from R to w,

$$\boxed{\delta T = \left(\frac{P}{\delta z}\right)\frac{1}{\pi K}\ln\frac{2R}{w}} \tag{15}$$

As an example consider copper with $K = 3.4\ W/cm°C$. The vertical sigma of the beam stripe is about 0.2 mm so I will take $w = 0.04$ cm. If the distance to the cooling source is about 1 cm and the linear power density is a rather high 1 kW/cm, then

$$\delta T = 366\ °C$$

Note that the temperature rise is proportional to the linear power density but only varies quite slowly with the ratio vertical size to the distance to the isothermal cooling surface due the the ln term. For example if the beam size in the example were doubled the calculated temperature rise is 301 $°C$. Similarly the distance to the isothermal cooling source can vary a factor of two with the same small effect on the peak temperature rise.

References

[1] O. Grobner, et. al., 'Studies of photon ...', Vacuum, vol 33, No. 7, p 397, (1983)