Cornell CBN 96-10 7/22/96

LONGITUDINAL DAMPING SCHEME WITH TWO TRANSVERSE KICKERS

Alexander Mikhailichenko

In [1] the proposal have been made to damp the longitudinal oscillations of the beam with help of a single transverse kicker. Here we considered a similar scheme with *a pair* of transverse kickers. Additional kicker drastically improves the damping possibilities of the scheme. Here the scheme considered, what can be used for damping the longitudinal beam motion in a damping ring as well as for stochastic cooling the longitudinal beam emittance.

Let us consider two transverse kickers K_0 and K_1 what are installed along the particle trajectory in a damping ring, Fig 1. Let the point $s = s_1$ is a focal point for sine-like trajectory, what starts at point $s = s_0$ where the first kicker is installed. Basically, this means that the distance $s_2 - s_1$ corresponds to an integer and a half of a betatron wavelength in a damping ring.

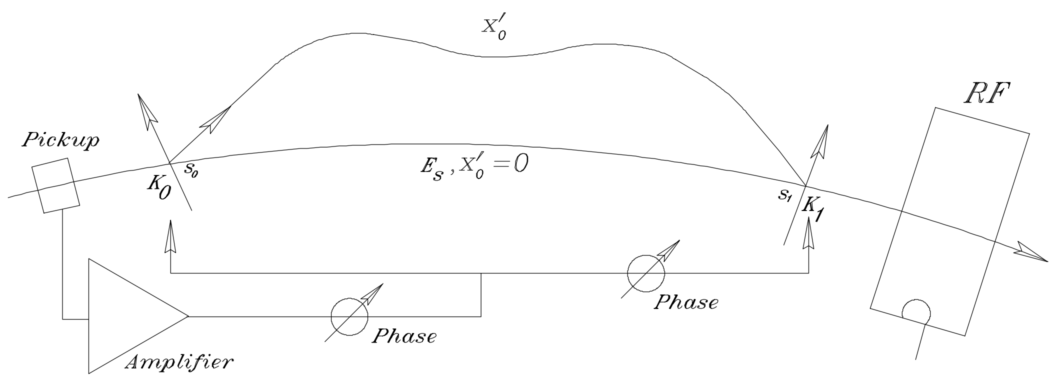

Fig. 1. Kicker K_0 installed at longitudinal position $s = s_0$ and the kicker K_1 installed at longitudinal position $s = s_1$ in the focal point of sine-like trajectory, what starts at the point $s = s_0$. RF is a RF cavity of the ring. This may be also an additional RF cavity operating at higher frequency than the main RF cavity of the ring. There are also shown Pickup, Amplifier and phase adjustment elements (time delay).

Let the amplitudes of the kicks arranged so that there is no residual oscillations after one pass over this system of kickers. Let the transverse motion is represented by the following [2]

$$x(s) = x_0 \cdot C(s, s_0) + x_0' \cdot S(s, s_0) + D(s, s_0)\frac{\Delta p}{p_s},$$

where *C(s)* and *S(s)* -- are cosine and sin-like solutions of equation of motion, *D(s)*-- is the dispersion function and the derivative is taken over longitudinal coordinate *s*. The path length variation between two points can be represented in this case as

$$\Delta l = -x_0 \int_{s0}^{s1} \frac{C}{\rho} ds - x_0' \int_{s0}^{s1} \frac{S}{\rho} ds - \frac{\Delta p}{p} \int_{s0}^{s1} \frac{D}{\rho} ds \ .$$

So if x_0', for example, is modified by the kicker K_0 (and eliminated by K_1), with correspondence to the energy variation of the particle by appropriate way, one can change the phase of arriving into the RF cavity and, hence, tolerate to the phase motion of the (macro)particle[1] without perturbation to the betatron oscillations.

One can obtain the equation of motion for individual (macro)particle in the damping ring in the same manner as description of ordinary longitudinal motion [3]. Namely, variation of the phase and energy deviation from turn to turn will be

$$\phi_{n+1} = \phi_n + \omega_{RF} T_0 \cdot \eta \frac{\Delta E}{E} + \Delta l(x_0, x_0') \frac{\omega_{RF}}{c}$$

$$\Delta E_{n+1} = \Delta E_n + eV(Sin\phi_n - Sin\phi_s) \, ,$$

where T_0 is the period of revolution, and the factor η can be expressed as $\eta \cong \frac{1}{cT_0} \oint \frac{D}{\rho} ds \cong (1/\gamma_{tr}^2 - 1/\gamma^2)$, $\gamma = E / mc^2$, $\gamma_{tr} \cong \alpha^{-1/2}$-- is the gamma factor, corresponding the transition energy. The last term in the first equation arising from the path length variation according to initial conditions at the position of the first kicker. Basically, the general term in our case is

$$\Delta l(x_0, x_1) = -x_0' \cdot \int_{s0}^{s1} \frac{S(s, s_0)}{\rho} ds = -x_0' \cdot I(s_0, s_1) \, ,$$

where we defined the integral $I(s_0, s_1) = \int_{s0}^{s1} S(s, s_0)/\rho \cdot ds$, what is a dimension constant (with the dimension of a length), depending only from positions of initial and final points in a damping ring. Treating the number of turns n as independent variable, one can obtain

$$\frac{d\psi}{dn} = \frac{\omega_{RF}}{c} cT_0 \cdot \eta \frac{\Delta E}{E_s} + \frac{\omega_{RF}}{c} I(s_0, s_1) \cdot x_0'$$

$$\frac{d\Delta E}{dn} = eV \cdot Cos\phi_s \cdot \psi \ .$$

We also suggested, that the difference $\psi = (\phi - \phi_s) << 2\pi$. From the last equations one can obtain

$$\frac{d^2\psi}{dn^2} = \frac{\omega_{RF} T_0 \cdot \eta \cdot eV \cdot Cos\phi_s}{E_s} \psi + \frac{\omega_{RF}}{c} I(s_0, s_1) \frac{d\, x_0'}{dn} \, .$$

If we suppose, that $x_0' \cong k \cdot \psi$, the equation of motion for the phase becomes

$$\frac{d^2\psi}{dn^2} = -(2\pi\nu_s)^2 \psi + \frac{\omega_{RF} I(s_0, s_1) \cdot k}{c} \cdot \frac{d\psi}{dn} \, ,$$

[1] That defined by the bandwidth of the feedback system. Basically the number of the particles in the bandwidth W are $N_W \cong Nc / (\sigma_{\parallel} W)$

where $(2\pi\nu_s)^2 = -\frac{\omega_{RF} T_0 \cdot \eta \cdot eV \cdot Cos\phi_s}{E_s}$. One can see, that the last term describes the decrement. If we define as usual

$$\lambda = -\frac{\omega_{RF} I(s_0, s_1) \cdot k}{2c},$$

then the equation of motion can be rewritten as the following

$$\frac{d^2\psi}{dn^2} + 2\lambda \cdot \frac{d\psi}{dn} + (2\pi\nu_s)^2 \psi = 0.$$

The last equation has a standard solution

$$\psi = c_1 \cdot \exp\{n \cdot [-\lambda - \sqrt{\lambda^2 - (2\pi\nu_s)^2}]\} + c_2 \cdot \exp\{n \cdot [-\lambda + \sqrt{\lambda^2 - (2\pi\nu_s)^2}]\}.$$

If $\lambda > 2\pi\nu_s$ the motion is aperiodic. For this one needs to have $k > 2\frac{cT_0}{I(s_0, s_1)} \frac{\eta \cdot eV \cdot Cos\phi_s}{E_s}$. The factor $cT_0 / I(s_0, s_1)$ is the ratio of the circumference of the damping ring and the path length integral. The last relation can be rewritten as $k > 2\frac{cT_0 \cdot \alpha \cdot eV \cdot Cos\phi_s / E}{I(s_0, s_1)}$, what has a clear physical sense, as the ($cT_0 \cdot \alpha \cdot eV \cdot Cos\phi_s / E$) is the path length difference, arising from the energy variation, produced by one pass through the RF cavity.

Now let us discuss more detailed the nature of the relation $x'_0 \cong k \cdot \psi$. As one can see this term indicates that the kick is proportional to deviation of the bunch position from equilibrium azimuthal position $\psi = \phi - \phi_s$.

So the signal from a pick-up electrode need to be processed through the phase detector, with the RF phase as a reference one. This is a standard technique and we will not discuss it here.

The other possibility is the notch-filter scheme, see Fig. 2.

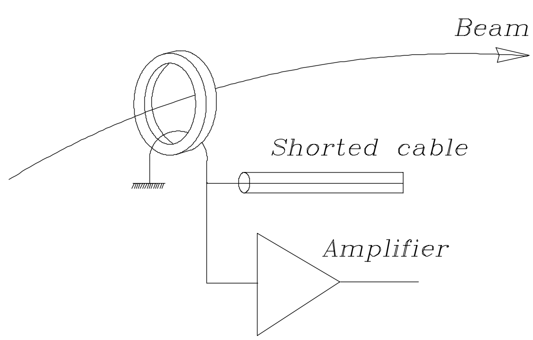

Fig.2. The notch-filter scheme (Lars Thorndahl, CERN, 1975).

In this scheme, the signal, induced by the current $J(t)$, passing through pick-up, induces the voltage $U(t) \cong \int_\omega J_\omega(\psi) Z_\omega e^{-in\omega t} d\omega$ [2]. As the pick-up loop and the cable are connected in parallel,

[2] Details of the scheme depend on the design of the pick-up. We consider the pick-up with inductive type, but considerations still valid in general for any type of pick-up electrode.

$Z_\omega \cong \dfrac{-i\omega L \cdot Z}{Z - i\omega L}$, where L is the inductance of the loop, $Z \cong iZ_0 \tan(\omega l / c)$, l-- is the length of the cable, Z_0 -- is the impedance of the cable. Representing $J_\omega \cong J_{0\omega} \cdot Cos(\nu_s \omega_0 t)$ and considering the signal around harmonic with the number m, $\omega \cong m\omega_0$, where ω_0 -- is the revolution frequency, one can estimate

$$U(t) \cong J_\omega \cong J_{0\omega} \cdot Cos(\nu_s \omega_0 t) \frac{-im\omega_0 L \cdot Z}{Z - im\omega_0 L} e^{-in\omega_0 t}.$$

The length of the cable l is chosen so, that $\dfrac{2l}{c} = T_0$, or $\dfrac{l}{c} \cong \dfrac{\pi}{\omega}$,

$$Z \cong iZ_0 \tan[(m\omega_0 \pm \nu_s \omega_0) l / c] \cong iZ_0 \cdot (-1)^m \tan(\nu_s \pi).$$

If inductance of the loop is small enough, so the $Z_0 \cdot \tan(\nu_s \pi) \geq m\omega L$, than the amplitude of the signal from pick-up is proportional $U(t) \cong J_\omega \cong -iJ_{0\omega} m\omega_0 L \cdot Cos(\nu_s \omega_0 t) \cdot e^{-in\omega_0 t} \propto Cos(\nu_s \omega_0 t) \propto \psi$.

Discussion

One can see, that there is no visible restriction for the speed of damping in this scheme, depending only on bandwidth of the pick-up, amplifier and kickers. So this scheme can be easily implemented for stochastic cooling of the longitudinal emittance as well.

The scheme considered, does not excite the transverse motion of the beam after passing the pair of kickers. Small residual transverse oscillations can be eliminated by tuning the amplitude of the second kicker *in situ*.

One interesting possibility of the scheme described is the following. As one can pick-up the signal what is proportional to the amplitude of phase oscillations and, hence, the instant deviation of the beam energy from equilibrium, $U \propto \psi \propto \Delta p / p$, one can see, that the difference in the path length could me made as following

$$\Delta l = -x_0 \int_{s0}^{s1} \frac{C}{\rho} ds - \left(K \int_{s0}^{s1} \frac{S}{\rho} ds + \int_{s0}^{s1} \frac{D}{\rho} ds \right) \frac{\Delta p}{p},$$

where we supposed, that $x_0' \cong K \dfrac{\Delta p}{p}$, K -- is an appropriate coefficient of proportionality. So if the sum of the terms in the brackets made equal to zero, the channel, connecting s_0 and s_1 will not depend on the energy deviation at all.

One can see, that the scheme proposed in not sensitive to the dispersion at the points of the actual location of the kickers.

References

[1] D. Sagan, M. Billing, "Using a Horizontal Kicker to Damp Longitudinal Oscillations", Cornell CBN 96-06, May 23, 1996.

[2] Klaus G. Steffen, " High Energy Beam Optics", Interscience Publishers, 1964.

[3] D.A. Edwards, M.J. Syphers, " An Introduction to the Physics of High Energy Accelerators", a Wiley-Interscience Publishing 1993.