Betatron Transients Caused by Rapid Changes in the Closed Orbit

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Introduction

During a normal cycle of operation of CESR there are three occasions when the closed orbit is deliberately changed: Electrostatic separators create the appropriate pretzel for injection, pre-collision and luminosity conditions. At the same time the separator voltages are varied corrector magnets create orbit distortions that help reduce radiation sources. In addition to the deliberate changes there are occasional undesired closed orbit alterations due to separator high voltage sparking or magnet power supply anomalies. The analysis in this paper shows that a rapid change of the closed orbit, whether intentional or not, generates betatron motion. The amplitude of the betatron motion is such that at least twice the aperture is required to make a rapid change than a slow one.

Analysis

For clarity, in this analysis I describe the motion of interest as horizontal, but it could equally well have been vertical. Imagine an instantaneous change in the closed orbit caused by suddenly turning on and leaving on a angle kick $\Delta\theta$ at a location where the betatron phase is $\phi = \phi(s) = \phi_i$ and the beta function is $\beta = \beta(s) = \beta_i$. The new closed orbit measured with respect to the old closed orbit is constructed by finding a betatron trajectory which has $x(\phi_i + 2\pi\nu) = x(\phi_i)$ and $x'(\phi_i + 2\pi\nu) = x'(\phi_i) - \Delta\theta$. The well known result is:

$$x_c(\phi - \phi_i) = \Delta\theta\sqrt{\beta\beta_i}\,\frac{\cos(\phi - \phi_i - \pi\nu)}{2\sin\pi\nu} \tag{1}$$

where $\phi - \phi_i$ is evaluated *modulo* $2\pi\nu$, and ν is the phase advance per turn. The closed orbit is then periodic in ϕ with period $2\pi\nu$. It has a 'kink' in it at $\phi = \phi_i + 2\pi\nu N$, $(N = 1, 2, ...)$ where the kick is applied but otherwise is a smooth free betatron orbit. After several damping times a particle which had no betatron amplitude before the kick was turned on closely follows the new closed orbit given by equation 1. However, immediately following the turn on, the particle is not on the new closed orbit and instantaneously gets a betatron amplitude about the new closed orbit.

When the angle kick is first turned on and the particle has passed the kick point only once, the motion may be well described as a free betatron oscillation about the original closed orbit In this case the net about the original closed orbit for the first turn only is,

$$x = \Delta\theta\sqrt{\beta\beta_i}\sin(\phi - \phi_i) \tag{2}$$

assuming there was no betatron motion present before the kick was turn on. Since $x = x_\beta + x_c$, for $0 \le (\phi - \phi_i) \le 2\pi\nu$ the particle motion about the new closed orbit is,

$$\begin{aligned} x_\beta &= \Delta\theta\sqrt{\beta\beta_i}\sin(\phi - \phi_i) \\ &- \Delta\theta\sqrt{\beta\beta_i}\,\frac{\cos(\phi - \phi_i - \pi\nu)}{2\sin\pi\nu} \end{aligned} \tag{3}$$

Here I have dropped the *modulo* operation implicit in equation 1. Nevertheless equation 3 is valid for all $\phi > \phi_i$. This expression for x_β has been constructed to give the correct initial value and slope at the kick point. It clearly solves the betatron equations of motion for all $\phi - \phi_i > 0$ as it is made of two pieces each of which is a valid betatron trajectory. So equation 3 must be valid for all $\phi - \phi_i > 0$, not just the first turn.

Now we can easily compute the net position of the particle for all $\phi - \phi_i > 0$ using equations 1 and 3 and $x = x_\beta + x_c$, but we must keep in mind that equation 1 is evaluated with $\phi - \phi_i$ *modulo* $2\pi\nu$. An simple example of these motions assuming constant β is shown in figure 1.

Amplitude of Transient Motion

To calculate the effects of the transients it is useful to know the size of its invariant betatron amplitude. Once that is know the maximum excursion at any point in the ring can be calculated quite easily.

In a constant guide field and with no radiation, particles follow trajectories about a closed orbit of the form

$$x_\beta = a\sqrt{\beta}\cos(\phi - \phi_0) \tag{4}$$

where a and ϕ_0 determine the particular trajectory and a is the invariant betatron amplitude. Under these conditions at any point in the ring the peak value of x_β will eventually be reached when the argument of the cosine happens to be near 0. The peak amplitude is then simply $a\sqrt{\beta}$.

I will directly calculate the invariant amplitude about the new closed orbit just after the kick is applied. As before, I assume that before the kick is turned on the particle is on the design orbit with no betatron motion. Just after the kick, we have at $\phi = \phi_i$ using equation 1

$$x(\phi_i) = 0, \quad x'(\phi_i) = \Delta\theta$$

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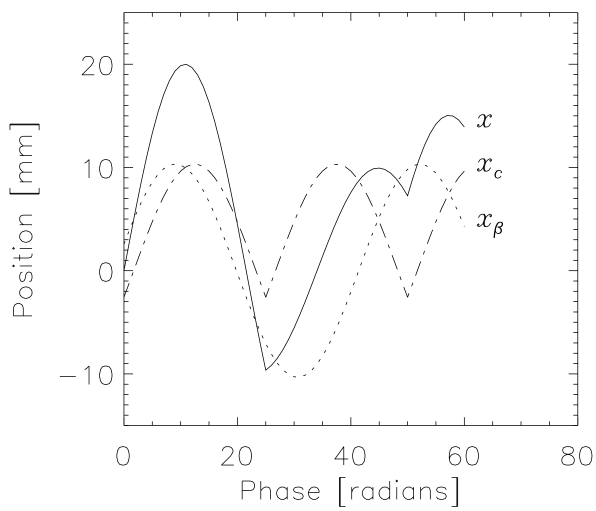

Figure 1: The closed orbit, betatron motion and the total transient motion are plotted as a function of $\phi - \phi_i$ with constant $\beta = 20\ m$ assumed. A kink occurs in x and x_c once each turn. Note that the peak excursion of x is about twice the peak closed orbit excursion. The fractional part of the tune is .58, the kick angle is 0.001 radians, and the initial position is $x(\phi_i) = 0$.

$$\begin{aligned} x_c(\phi_i) &= \Delta\theta\frac{\beta_i}{2}\cot\pi\nu \\ x_\beta(\phi_i) &= x(\phi_i) - x_c(\phi_i) \\ &= -\Delta\theta\frac{\beta_i}{2}\cot\pi\nu \end{aligned} \tag{5}$$

An expression for $x'_\beta(\phi_i)$ may be derived by differentiating $x_\beta = x - x_c$. The algebra is somewhat tedious but the result is

$$x'_\beta(\phi_i) = \frac{\Delta\theta}{2}(1 - \frac{\beta'_i}{2}\cot\pi\nu) \tag{6}$$

Another handy formula, borrowed from Sands but simple to derive is

$$a^2 = \frac{x_\beta^2}{\beta} + \beta(x'_\beta - \frac{\beta'}{2\beta}x_\beta)^2 \tag{7}$$

Plugging 5 and 6 into 7 yields:

$$|a| = \left|\frac{\Delta\theta}{2}\sqrt{\beta_i}\csc\pi\nu\right| \tag{8}$$

This result can also be obtained with less algebra if one imagines turning on the kick just after the passage of the particle. Then for the rest of the turn $x = 0$ so $x_\beta = -x_c$. This leads to $|a| = |a_c|$ where a_c is the invariant betatron amplitude of the closed orbit trajectory equation 1.

Equation 8 gives the invariant betatron amplitude about the new closed orbit after the closed orbit has been suddenly changed by a single kick of $\Delta\theta$ and before damping or feedback have had any effects. Note I have also assumed that the there was no betatron motion before the closed orbit was changed. If there were no further changes to the closed orbit after the kick was applied, at each point in the ring the particle position would 'oscillate' about the new closed orbit eventually approaching peak values of

$$\hat{x}_\beta(s) = \left|\frac{\Delta\theta}{2}\sqrt{\beta_i\beta(s)}\csc\pi\nu\right| \tag{9}$$

Using equations 1 and 9 we have the ratio of peak betatron motion to the closed orbit at every point in the ring:

$$\frac{\hat{x}_\beta(s)}{|x_c|} = \left|\frac{1}{\cos\phi - \phi_i - \pi\nu}\right| \geq 1 \tag{10}$$

So $\hat{x} \geq 2x_c$ in general, where $\hat{x}$ is the maximum value that x can reach at each point in the ring. Near a closed orbit maximum, the ratio is close to 2.

After a few turns, the number depending on the choice of tune, the phase of the betatron motion and that of the closed orbit distortion combine to give a maximum orbit excursion which is at least twice the peak closed orbit change. This means that a rapid change in the closed orbit at requires twice the aperture of a slow change.

Examples

When a separator sparks in CESR the voltage goes very rapidly to zero, recovering a few tens of milliseconds later. The size of the transient generated depends on which separator sparks. To calculate the transient we first calculate the change to the closed orbit by setting one separator at zero voltage, then add the betatron transient motion from equation 9, (see figure 2). Note that in this case the transient amplitude is not greater than or equal to the original pretzel because only one of four separators sparked. Nevertheless the transient still brings the beam center out well beyond the original pretzel maximum to over 30 mm in the region of the north interaction point ($s = 384$ m). The vacuum chamber wall is at 45 mm.

A table of maximum amplitudes for various configurations of CESR undergoing sparking any of four separators is shown in table 1. Electron injection conditions are the most dangerous for the beam. A spark at 8E would swing the beam center within 7 mm of the vacuum chamber wall near the east IR. This is shown in figure 3.

Beam loss is also likely during pre-collision if the 8E separator sparks. The excursions are calculated for the positron beam.

An even more dangerous situation occurs if the separator voltages rise too fast. Then the amplitude of the transient is indeed more than twice the amplitude of the full pretzel and the peak excursion will be at least 32 millimeters. However the high voltage power supplies are not capable of raising the voltage arbitrarily fast and some damping reduces the maximum excursion the beam reaches.

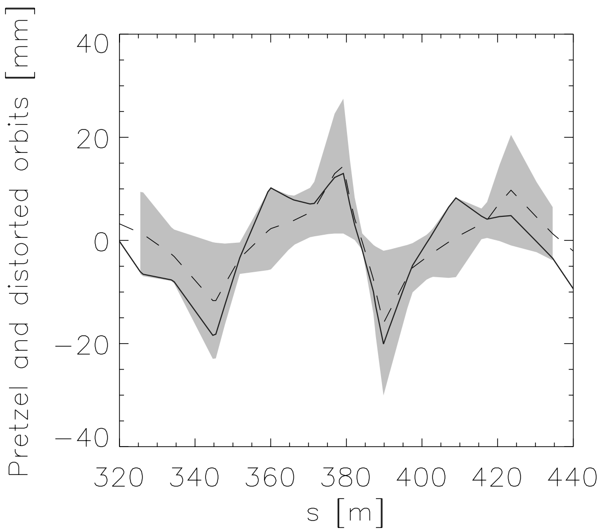

Figure 2: The original pre-spark pretzel (solid line) and the distorted post-spark pretzel (dashed line) are plotted with the resulting betatron motion represented by shading. Here the separator at 45W was assumed to spark during e^- injection.

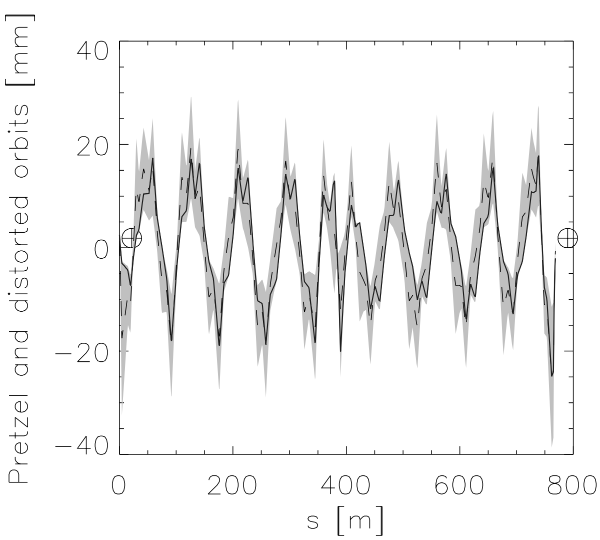

Figure 3: A spark in the separator at 8E would cause this distortion of the pretzel (dashed line) and transient betatron motion about the distorted pretzel (shaded region). The pre-spark pretzel is shown in solid line. The interaction point is marked with a $\oplus$.

Condition	Location of spark			
	8E	45E	45W	8W
e^- injection	38.6	25.5	29.9	22.6
pre-collision	33.3	22.6	26.0	27.8
collision	18.3	20.2	18.8	25.3

Table 1: The maximum excursion in millimeters of the beam center including the transient from a separator spark at various locations. Note that magnetic bumps were not taken into account and could further increase the maximum excursion. The vacuum chamber wall is at 45 millimeters.

Acknowledgements

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