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Coupling of a Shorted Stripline Kicker to an Ultrarelativistic Beam

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We compare the transverse deflection, directionality, beam-induced voltage, and beam impedance of a stripline kicker terminated in its characteristic impedance and one terminated in a short. A shorted stripline kicker has equal coupling to beams propagating in both directions. Except for the loss of directionality, the coupling is the same as that of a stripline kicker terminated in its characteristic impedance.

1 Coupling in the frequency domain

1.1 Transverse deflection

We consider a two-electrode stripline transducer used as a transverse kicker. To produce a transverse deflection, the two plates need to be driven differentially with opposite and equal voltages. The transverse deflection can be calculated by determining the longitudinal kick in the differentially driven mode and applying the Panofsky-Wenzel theorem [1]. We follow the notation of Goldberg and Lamberson [2]. The longitudinal voltage seen by a beam skimming very close to the surface of one of the electrodes is:

$$V(x=b) = \int E_s(s) e^{j\omega t} ds = \int E_s(s) e^{\pm jk_B s} ds$$

where b is the half-gap between the electrodes, ω is the frequency of the voltage applied to the kicker, and $k_B = \omega/\beta c$. Here we have used $t = \pm s/\beta c$, where the sign indicates the direction of the beam relative to the applied wave on the kicker. In the x,s plane the longitudinal voltage is approximately linear in x. For $x << b$,

$$V(x) = g_\perp \frac{x}{b} V(x=b)$$

where $g_\perp$ is a factor of order unity that depends on the transverse geometry of the electrodes. For flat electrodes of width w, $g_\perp$ can be calculated analytically [2]:

$$g_\perp = \frac{\tanh \pi w}{4b}$$

Applying the Panofsky-Wenzel theorem,

$$\frac{j\omega \Delta p_\perp}{e} = -\nabla_\perp V$$

so near the longitudinal axis

$$\Delta p_\perp = \frac{jeg_\perp}{\omega b} V(x=b)$$

We wish to compare the deflection produced by striplines which are terminated in their characteristic impedance or shorted for beams moving in either the $+x$ or $-x$ directions, making four cases in all. $V_f(s)$ is the voltage associated with the forward wave injected into the stripline from a line with the same characteristic impedance. The opposite stripline electrode is, of course, driven with voltage $-V_f(s)$. The powered end of the stripline is at $s = 0$, and the other end is at $s = l$.

Case I: Stripline terminated in matched load, beam in $+s$ direction

$$V(x=b) = -V_f(0) + V_f(l)e^{jk_B l} = -V_f(0)[1 - e^{-j(k_L - k_B)l}]$$

where $k_L = \omega/c$ is the wave number in the stripline. For the ultrarelativistic case, $(k_L - k_B)l = 0$, so $V(x=b) = 0$ and $\Delta p_\perp = 0$.

Case II: Stripline terminated in matched load, beam in $-s$ direction

$$V(x=b) = V_f(0) - V_f(l)e^{-jk_B l} = V_f(0)[1 - e^{-j(k_L + k_B)l}]$$

For the ultrarelativistic case,

$$V(x=b) = V_f(0)[1 - e^{-2j\omega l/c}]$$

$$\Delta p_\perp = \frac{jeg_\perp}{\omega b} V_f(0)[1 - e^{-2j\omega l/c}]$$

Case III: Stripline shorted, beam in $+s$ direction

$$V(x=b) = -V_f(0) = -V_f(0)[1 - e^{-2jk_L l}] = -V_f(0)[1 - e^{-2j\omega l/c}]$$

$$\Delta p_\perp = -\frac{jeg_\perp}{\omega b} V_f(0)[1 - e^{-2j\omega l/c}]$$

Case IV: Stripline shorted, beam in $-s$ direction

$$V(x=b) = V_f(0) = V_f(0)[1 - e^{-2jk_L l}] = V_f(0)[1 - e^{-2j\omega l/c}]$$

$$\Delta p_\perp = \frac{jeg_\perp}{\omega b} V_f(0)[1 - e^{-2j\omega l/c}]$$

Cases I and II show the directionality of the kicker terminated in a matched load. Comparison of Cases II and IV show that the transverse kick is the same for the matched and shorted terminations for the same applied voltage. Case III differs from Cases II and IV only by a sign, showing that the directionality is lost in the shorted kicker and that the magnitude of the kick is the same for beams moving in either direction.

1.2 Deflection in the low frequency limit

The frequency dependence of the shorted kicker deflection can be separated into amplitude and phase factors:

$$\omega^{-1}[1 - e^{-2j\omega l/c}] = 2j\omega^{-1}\sin(\omega l/c)e^{-j\omega l/c}$$

so that

$$\Delta p_\perp = \pm\frac{2eg_\perp}{\omega b}V_f(0)\sin(\omega l/c)e^{-j\omega l/c}$$

In the low-frequency limit

$$\Delta p_\perp = \pm\frac{2eg_\perp l}{bc}V_f(0)$$

indicating that the deflection is proportional to the kicker length. The first null in the kicker response, due to the sine term, occurs at a frequency of $f = c/2l$.

1.3 Voltage on kicker terminals induced by the beam

We examine the outgoing voltage wave $V_o(t)$ induced by a beam current $i(t)$ for the same four cases. The image current induced on a kicker electrode is $gi(t)$, where g is a single-electrode geometrical factor of order 1/2. Both the stripline and output line impedance are Z_0, and the wave on the stripline is assumed to have velocity c.

Case I: Stripline terminated in matched load, beam in $+s$ direction

$$V_o(t) = \frac{gZ_0}{2}i(t) - \frac{gZ_0}{2}i(t - l/\beta c - l/c)$$

For an ultrarelativistic beam,

$$V_o(t) = \frac{gZ_0}{2}[i(t) - i(t - 2l/c)]$$

Case II: Stripline terminated in matched load, beam in $-s$ direction

$$V_o(t) = -\frac{gZ_0}{2}i(t) + \frac{gZ_0}{2}i(t + l/\beta c - l/c)$$

For an ultrarelativistic beam,

$$V_o(t) = 0$$

Case III: Stripline shorted, beam in $+s$ direction

$$V_o(t) = \frac{gZ_0}{2}i(t) - \frac{gZ_0}{2}i(t - 2l/c) = \frac{gZ_0}{2}[i(t) - i(t - 2l/c)]$$

Case IV: Stripline shorted, beam in $-s$ direction

$$V_o(t) = -\frac{gZ_0}{2}i(t) + \frac{gZ_0}{2}i(t - 2l/c) = -\frac{gZ_0}{2}[i(t) - i(t - 2l/c)]$$

Again, the directionality of the stripline is lost, and the signal amplitudes are the same for a stripline with a matched termination operated in the "right" direction and a shorted stripline. For the shorted stripline, the sign of the induced voltage depends on the beam direction.

The expression for the induced voltage has no ω^{-1} term. Because the bunch length is much shorter than the stripline electrodes, the average and peak beam-induced power do not depend on the kicker length.

1.4 Beam impedance of kicker

The voltage across the first (non-shorted) kicker terminal reached by the beam is the same in each of the four above cases, when measured along the direction in which the beam is moving (the voltage across the other terminal is zero):

$$V_{term.}(t) = \frac{g_{\|} Z_0}{4}[i(t) - i(t - 2l/c)]$$

where the geometrical factor $g_{\|} = 2g$ on the kicker axis. The beam voltage is

$$V(t) = -g_{\|} V_{term.}(t)$$

In the frequency domain,

$$V(t) = -\frac{g_{\|}^2 Z_0}{4} i(t)[1 - e^{-2j\omega l/c}]$$

so the longitudinal beam impedance is

$$Z_{\|} = \frac{g_{\|}^2 Z_0}{4}[1 - e^{-2j\omega l/c}]$$

for both terminated and shorted kickers.

2 Deflection in the time domain

The deflection of the of the beam in the time domain can be calculated by applying the time-domain Panofsky-Wenzel theorem:

$$\frac{\partial}{\partial t}\Delta p_{\perp} = -e\nabla_{\perp} V$$

In the case of the shorted kicker, the deflection of a beam moving in the $\pm s$ direction by an impulsive forward going wave $V_f = \delta(t)$ is

$$\Delta p_{\perp} = \begin{cases} 0 & \text{if } t < 0 \\ \pm e g_{\perp}/b & \text{if } 0 < t < 2l/c \\ 0 & \text{if } t > 2l/c. \end{cases}$$

3 Conclusions

We have shown that a shorted kicker produces the same beam deflection as one terminated in a matched load, and deflects beams propagating in both directions. This bidirectionality is useful for applications such as time-domain feedback in that it allows the use of a single power amplifier to drive both beams. The amplifier must, of course, be capable of driving a shorted line. The peak beam-induced voltage present at the amplifier output is the same for a shorted kicker as for one terminated in a matched load. The average beam-induced power per beam is the same as for a terminated kicker, but both beams generate power at the same kicker port. The beam impedance presented by shorted or matched termination kickers is identical. At high frequencies, where the details of the geometry at the ends of the electrodes becomes important, it may be easier to provide a smooth transition to the beampipe with a shorted kicker.

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References

[1] W.K.H. Panofsky and W.A. Wenzel, Rev. Sci. Instr. **27**, 967 (1956).

[2] D.A. Goldberg and G.R. Lamberson, AIP Conf. Proc. 249 (1992) 539.