Effects of the CHESS Wigglers on a Beam with an Angular Offset

James J. Welch, *Cornell University, Ithaca NY* *

Introduction

The magnetic field due to a single pole of a wiggler is, by normal accelerator magnet standards, both very strong and very nonuniform. Because the poles alternate sign and are made to have the same strength the effects of the nonuniform fields on the beam largely cancel from one pole to another. In an ideal case where there are no mechanical or magnetic errors, the net integrated dipole field along a path parallel to the wiggler axis and on the midplane would be exactly zero.

In this note I estimate the integrated field along a path not parallel to the axis but still in the midplane. Such a path is typical of the closed orbit during electron injection or luminosity optics. When the path of integration is at an angle to the wiggler axis, the horizontal or vertical position in one pole is not precisely the same as the horizontal or vertical position in any other pole. Because of the relatively strong variation of field strength with position the integration over one pole does not cancel with the integration over another pole. Based on approximate expressions for the field in the wiggler, I will show that the size of this effect is substantially smaller than the actual integrated dipole field measured parallel to the axis. This means the beam dynamical effects due to the angle the closed orbit takes through the wiggler are probably not important compared with those due to the actual field errors due to mechanical and magnetic imperfections, at least for the present wigglers and optics.

An expression for integral of the vertical magnetic field along a horizontally tilted straight path $x(z)$ is:

$$\int_{\frac{-L_w}{2}}^{\frac{L_w}{2}} B_y(x(z))dz =$$

$$\int_{\frac{-L_w}{2}}^{\frac{L_w}{2}} \left\{ B_y)_{x=0} + \frac{\partial B_y}{\partial x}\,)_{x=0}\, x + \frac{1}{2}\frac{\partial^2 B_y}{\partial x^2} x^2 + \ldots \right\} dz \quad (1)$$

For a perfect wiggler the odd order x derivatives are zero for $x = 0$ by symmetry. For a wiggler with an even number of poles, such as we now have at CESR, the vertical magnetic field is an odd function of z. When $x = x_0 + x'z$ is put into equation 1 the only terms left are:

$$\int_{\frac{-L_w}{2}}^{\frac{L_w}{2}} B_y(x(z))dz =$$

$$\int_{\frac{-L_w}{2}}^{\frac{L_w}{2}} \frac{\partial^2 B_y}{\partial x^2}\,)_{x=0}\, x_0 x' z + \frac{1}{3}\frac{\partial^4 B_y}{\partial x^4}\,)_{x=0}\, x_0 x' z + \ldots \quad (2)$$

In our case the lowest order term $\partial^2 B_y / \partial x^2$ is dominate and we will no longer consider the higher order terms.

A simple estimate of the integral may be made from an approximate expression for the wiggler field. This is what I will develop in the next section assuming that we have N identical periods ($2N$ poles), and the end effects are neglible. Later I will deal with the end effects.

In general the magnetic field in the wiggler may be derived from gradient of a scalar potential with $\nabla^2\Phi = 0$ everywhere of interest. From the symmetry the lowest order term must be of the form

$$\Phi \sim \sinh(k_y y)\sin(k_x x)\sin(k_z z) \quad (3)$$

Applying $\nabla^2\Phi = 0$ and utilizing the independence of the x, y, z coordinates, yields:

$$k_y^2 = k_z^2 + k_x^2 \quad (4)$$

So the magnetic field of a wiggler with an even number of identical poles can be approximately described,

$$B_y = B_0 \cosh(k_y y)\cos(k_x x)\sin(k_z z) \quad (5)$$

Now, using the equation 5 we can evaluate the integral equation 2. On the wiggler axis $\partial^2 B_y/\partial x^2 = -k_x^2 B_y$ so we have,

$$\int_{\frac{-L_w}{2}}^{\frac{L_w}{2}} B_y(x(z))dz = -k_x^2 \int_{\frac{-L_w}{2}}^{\frac{L_w}{2}} B_y(x=0,z)x_0 x' z dz \quad (6)$$

$$= -k_x^2 x_0 x' B_0 \int_{\frac{-L_w}{2}}^{\frac{L_w}{2}} z\sin(k_z z)dz \quad (7)$$

$$= k_x^2 x_0 x' B_0 L_w^2 \frac{(-1)^N}{2\pi N} \quad (8)$$

I have used the relation that:

$$\int_{\frac{-L_w}{2}}^{\frac{L_w}{2}} z\sin(k_z z)dz = -\frac{L_w^2(-1)^N}{2\pi N} \quad (9)$$

which can be verified by direct substituion of $k_z = 2\pi N/L_w$ and integrating.

Equation 8 represents an estimate for a wiggler with N identical periods. The wigglers at CESR have 22 more or less identical poles with two poles the ends with roughly

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half the strength. The effect of these end poles can be estimated by assuming the magnetic field contribution from the end has the same shape as a normal strength pole. To see how to include this effect first consider how much adding one more period of *normal strength* poles would increase the total field integral. Using equation 8 the change would be:

$$\frac{k_x^2 x_0 x' B_0 L_w^2}{2\pi}\left[\left(\frac{N+1}{N}\right)^2 \frac{(-1)^{N+1}}{N+1} - \frac{(-1)^N}{N}\right] \tag{10}$$

The weaker end poles would produce a proportionally weaker change to the integral since I assume the shape is the same. [1] To get the addition to the integral of the field due to the weaker end poles I simply replace B_0 with B_{end} in equation 10 where B_{end} is the peak magnitude of the field in the end poles. The change in the integral caused by adding the end poles to N identical poles reduces to

$$-k_x^2 x_0 x' L_w^2 B_{end} \frac{(-1)^N}{2\pi}\frac{2N+1}{N} \tag{11}$$

The net integral of N full strength poles plus the change due to two end poles of strength B_{end} is obtained from equations 11 and 8:

$$\int_{total} B_y(x(z)) = k_x^2 x_0 x' L_w^2 B_0 \frac{(-1)^N}{2\pi}\frac{1}{N}\left(1 - \frac{B_{end}}{B_0}(2 + \frac{1}{N})\right) \tag{12}$$

Measurements of the field indicate that $k_x \approx 6\ m^{-1}$. [2] The longitudinal wavenumber k_z is about 32 m^{-1} and is determined by the spacing between the poles. Therefore k_y works out to 33 m^{-1}. The peak vertical field is $B_0 = 1.2\ T$ at the nominal closed gap of 4 cm.

Some typical numbers for the present crossing angle conditions and existing wigglers at CESR are:

x	0.01	*m*
x'	0.0005	*radians*
L_w	2.3	*m*
B_0	12000	*G*
B_{end}	7600	*G*
N	11	*full strength poles*

For these values the net integral for one full strength adjacent poles, equation 12,

$$\int_{total} B_y(x(z)) = -5.3 \times 10^{-2}\ Gm \tag{13}$$

per wiggler. This is substantially smaller than the variation in the measured integrated field over for both the east and west wigglers. Over the interval from $x = 0$ to $x = 1\ cm$ the variation of the integrated field for the wigglers measured parallel to the axis is: [3] [4]

East	$\sim 1-2$	Gm
West	$\sim < 1$	Gm

An analogous argument can be made for vertical offsets and angles. The result is obtained by replacing k_x with k_y in equation 8 and substituting y for x and y' for x'. Thus the sensitivity to vertical offsets and angles is higher by a factor of $(k_y/k_x)^2 \approx 30$ but the size of such offsets is quite a bit smaller than those due to the pretzel. For example, a 1 mm offset in y with and angle of 5×10^{-4} would give a net integrated field of only -0.127 Gm.

Hence the beam dynamical effects due to the angle and offset of the closed orbit through the wiggler are probably not as important as those due to physical and magnetic imperfection, for the present crossing angle lattice. In the future the angular and offset effects can be made stronger or weaker proportional to the product $x_0 x'$.

[1] The iron pole shape is identical though the permanent magnet material is less in the end poles and is perturbed by a field clamp.

[2] This value was derived from measurements reported by Ken Finkelstein in a memo by him dated October 23, 1992

[3] Measurements of the west wiggler are reported in CBN 93-7, (1993) by *D. Frachon, I. Vasserman from Advanced Photon Source, ANL* and *J. Welch and A. Temnykh, CESR*

[4] Measurements of the west wiggler are reported in CON 94-16 (1994) by *A. Temnykh and J. Welch, CESR*