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Damping and Tune Shifts in a Transverse Feedback System

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1 Introduction

Recently S. Koscielniak and H. J. Tran[1] have shown that one can obtain more damping in a transverse damping system if the feedback is driven into saturation. The damping in this case is linear (as opposed to exponential) and Koscielniak and Tran derived an approximate expression for the damping rate in this case. The purpose of this note is to derive the exact damping rate and compare it with the damping for the unsaturated case.

2 Theory

At the pickup of the feedback system let (x_p, x'_p) be the transverse coordinates of the centroid of the bunch and let (x_k, x'_k) be the centroid coordinates at the kicker. At the PU

$$x_p = a\sqrt{\beta_p} \cos\psi_p \, . \tag{1}$$

At the kicker

$$x_k = a\sqrt{\beta_k} \cos\psi_k \, . \tag{2}$$

$$x'_k = \frac{a}{\sqrt{\beta_k}} \left[-\sin\psi_k - \alpha_k \, \cos\psi_k \right] \, . \tag{3}$$

The phase advance between the PU and the kicker is $\Delta\psi$ so

$$\psi_k = \psi_p + \Delta\psi \, . \tag{4}$$

The kick $\delta x'_k$ given to the bunch at the kicker will be some function of the pickup signal:

$$\delta x'_k = K(x_p) \, . \tag{5}$$

From Eqs. (2) and (3) the amplitude a is given by the standard formula

$$a^2 = \gamma_k \, x_k^2 + 2\alpha_k \, x_k \, x'_k + \beta_k \, x_k'^2 \, . \tag{6}$$

Using Eqs. (5) and (6) the change in a due to a kick is

$$\delta a = \frac{\alpha_k\, x_k + \beta_k x'_k}{a}\, K(x_p)\,. \tag{7}$$

Assuming that $|K(x_p)| \ll a/\sqrt{\beta_k}$ the effect of the kicks can be ignored to 0^{th} order. To 0^{th} order then ψ_k is uniformly distributed over the interval 0 to 2π and the average change a is obtained by averaging Eq. (7) over ψ_k. Using this with Eqs. (2) and (3) gives

$$\langle \delta a \rangle = \frac{-1}{2\pi} \int_0^{2\pi} d\psi_k \, \sqrt{\beta_k}\, \sin\psi_k \; K(x_p)\,. \tag{8}$$

The average phase advance due to the kicks can be similarly computed: From Eqs. (2) and (3) the phase angle is given by

$$\tan\psi_k = -\frac{\beta_k\, x'_k + \alpha_k\, x_k}{x_k}\,. \tag{9}$$

Using Eqs. (5) and (9) the change in ψ_k due to a kick is

$$\delta\psi = \frac{-\sqrt{\beta_k}\;\cos\psi_k}{a}\, K(x_p)\,. \tag{10}$$

In analogy with Eq. (8) the average phase advance per turn due to the kicks is

$$\langle \delta\psi \rangle = \frac{-1}{2\pi} \int_0^{2\pi} d\psi_k \, \frac{\sqrt{\beta_k}\;\cos\psi_k}{a}\, K(x_p)\,. \tag{11}$$

Equations (8) and (11) are the basic formulas for deriving the damping and tune shifts due to the feedback.

3 Proportional Kick Case

For a non-saturated feedback system the kick is

$$\delta x'_k = k\, x_p\,. \tag{12}$$

Using Eqs. (1) (4), and (12) in Eqs. (8) and (11) gives

$$\frac{\langle \delta a \rangle}{a} = \frac{-k}{2} \sqrt{\beta_p \beta_k}\, \sin\Delta\psi\,, \tag{13}$$

$$\langle \delta\psi \rangle = \frac{-k}{2} \sqrt{\beta_p \beta_k}\, \cos\Delta\psi\,. \tag{14}$$

The damping is exponential as expected and is at a maximum when $\Delta\psi = \pi/2$. The phase shift is in general non-zero except at the optimum phase advance of $\Delta\psi = \pi/2$.

4 Saturated Kick Case

For a saturated feedback system the kick is

$$\delta x'_k = \begin{cases} C & x_p > 0 \\ -C & x_p < 0 \end{cases} . \tag{15}$$

Using Eqs. (1) and (4), and (15) in Eqs. (8) and (11) gives

$$\langle \delta a \rangle = \frac{-2C}{\pi} \sqrt{\beta_k} \sin \Delta\psi , \tag{16}$$

$$\langle \delta \psi \rangle = \frac{-2C}{a\,\pi} \sqrt{\beta_k} \cos \Delta\psi . \tag{17}$$

The damping is linear and is at a maximum when $\Delta\psi = \pi/2$. Like the non-saturated case the phase shift is in general non-zero except at the optimum phase advance of $\Delta\psi = \pi/2$.

To compare the damping in the proportional kick case to the damping in the saturated kick case consider the situation where the system is just starting to saturate. For a proportional kick the maximum kick is obtained from Eqs. (1) and (12) to be $\delta x'_k|_{max} = k\, x_p|_{max} = k\, a\sqrt{\beta_p}$. When the system is just saturating this maximum kick is just $\delta x'_k|_{max} = C$. Using this in Eqs. (13) and (16) gives the damping ratio between the saturated and non-saturated cases

$$\frac{\langle \delta a \rangle_{sat}}{\langle \delta a \rangle_{nonsat}} = \frac{4}{\pi} . \tag{18}$$

Thus the saturated system gives about 25% more damping. The real advantage of the saturated system comes about when $C > k\, a\sqrt{\beta_p}$. That is, it always pays to increase the gain of the system up to the point where the kicks are comparable to the bunch oscillation amplitude. At this point the assumption made to derive Eq. (7) that $|K(x_p)| \ll a/\sqrt{\beta_k}$ breaks down and the feedback will act as a source of noise for the beam (*i.e.* the feedback will act as a 'heat bath' keeping the beam's amplitude roughly constant). Another way of looking at this is to note that when the kicks become comparable to the bunch oscillation amplitude the closed loop gain of the system is approaching 1. For the proportional gain case this occurs when $k \approx 1/\sqrt{\beta_p \beta_k}$ and for the saturated case it is when $C \approx a/\sqrt{\beta_k}$.

Given the above considerations one can design the optimum kick function which would have the largest linear slope for small x_p consistent with the requirement that the closed loop gain be smaller than 1. This 'optimum' kick function would then be

$$\delta x'_k = \begin{cases} \dfrac{g}{\sqrt{\beta_p \beta_k}}\, x_p & |x_p| \le \frac{\sqrt{\beta_p \beta_k}\, C}{g} \\ C & x_p > \frac{\sqrt{\beta_p \beta_k}\, C}{g} \\ -C & x_p < \frac{-\sqrt{\beta_p \beta_k}\, C}{g} \end{cases} . \tag{19}$$

Where $g > 1$ is some factor to keep the closed loop gain below unity, say $g \sim 10$.

References

[1] S. Koscielniak and H. J. Tran "Properties of a Transverse Damping System Calculated by a Simple Matrix Formalism," 1993 IEEE Particle Accelerator Conference, Dallas, TX.