TITLE: Exploring Kepler

AUTHOR:
David Teitelbaum
Dteitelbaum@sidney.k12.ny.us
Seth Mead
Sm91@mac.com

APPROPRIATE GRADE LEVEL: 9

NYS STANDARDS:

- Standard 1 – Mathematical Analysis – 2
- Standard 1 – Scientific Inquiry – 2
- Standard 1 – Scientific Inquiry – 3
- Standard 1 – Engineering Design – 1
- Standard 2 – Information Systems – 1
- Standard 6 – Models – 2
- Standard 6 – Equilibrium and Scale – 3

ABSTRACT: the purpose of the lab is twofold. First, as an introductory activity, the students will create an ellipse. After having drawn the ellipse, the students must measure out the arcs using a "cutout" method. Students will then compare the masses of these cutouts to make an inference about the area of each cutout. Also in this first part of the lab, the students will calculate the eccentricity of the ellipse that was drawn. Second, as a reinforcement to the first activity, using real world data plot the orbit of their chosen object. By plotting the students would be prove Kepler's 1st law. Once the students have plotted this data, they will need to calculate the area that the object sweeps. This information will be used to prove Kepler's 2nd law. And using the data that they have gathered, students will calculate and compare the orbital period and average radius of the celestial object. This data will be used to arrive at Kepler's 3rd law.

# EXPLORING KEPLER

OBJECTIVES:

The students will be able to:

1. Draw an ellipse and calculate the eccentricity using the eccentricity of an ellipse equation from the reference tables.
2. Geometrically prove Kepler's 1st law.
3. Prove Kepler's 2nd law by relating the masses of two areas and by calculating the area.
4. Calculate the period of revolution for a chosen celestial object (using Kepler's 3rd law).
5. Using polar graph paper, construct and plot points creating an ellipse.
6. Using ordinary graph paper, construct and plot points creating an ellipse.

CLASS TIMED REQUIRED: 200 minutes

TEACHER PREPARATION TIME: Photocopying and materials gathering

MATERIALS NEEDED (per pair):

- String (20 cm long)
- 1 Piece cardboard (8.5in x 11in)
- Thumbtacks (2)
- Polar graph paper (2)
- Digital scale accurate to 0.01 g (1 per classroom)
- Protractor (1)
- Sharpened pencil (2)
- Metric ruler (1)
- Calculator (1)
- Colored pencils

BACKGROUND INFORMATION (student):

- Before conducting this exercise, students will need to have a basic understanding of geometry, graphing and algebra.

BACKGROUND INFORMATION (teacher):

- Plan to conduct a small discussion regarding ellipses after the conclusion of part one. Possible questions for discussion include;
  - How does eccentricity change the appearance of the ellipse?
  - How does the distance between the foci relate to the shape of the ellipse?
  - Where do we encounter ellipses in our day to day life?
- At the conclusion of the part 2, the teacher should be prepared to conduct a brief discussion about Kepler's laws.
  - Who was Kepler?
  - What was Brahe's data collection like?
  - What were his findings i.e. what are Kepler's three laws?
  - Why do you suppose that there are no perfectly circular orbits?

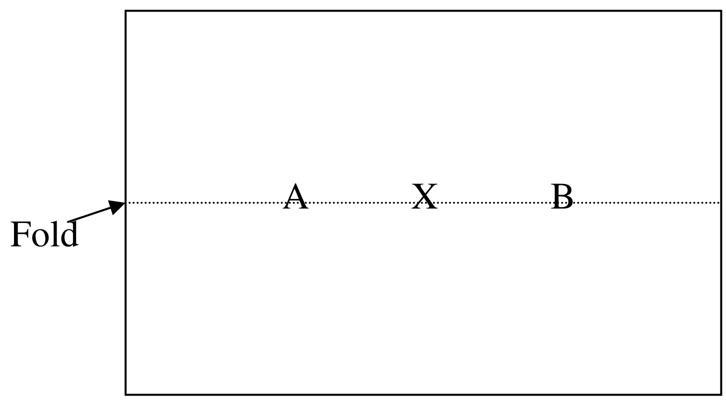
## EXPLORING ELLIPSES

We will begin with an activity where you will create an ellipse. What is an ellipse?… Well just wait and you shall see for your self.

## PART 1

*PROCEDURE:*

Cut a piece of string about 30 cm long and tie the ends together forming a loop. Take a piece of standard printer paper (8.5" X 11") and fold it in half, bisecting the 8.5" side. You should now have a piece of paper that is divided into two 4.25" X 11" sections. Unfold the paper and find the exact center of the fold line and mark it with an "x". Now choose an interval between 2 and 5 centimeters and record it here;______. Measure the interval of your choice in both directions along the crease and mark them with an "A" and a "B". The result should look something like this:




Use masking tape to tape your paper your piece of paper to the cardboard. Now place a pushpin through the foci ("A" and "B") on your paper. Next drop your string loop over the pins to encircle them. One person in the group needs to hold the pushpins in with their fingers while the other group member does the following; Take a pencil and place it inside the loop with the point down toward the paper and move the pencil outward until it has pulled the string taut around the pushpins. Describe the shape that the string makes:

Now move the pencil around while keeping the string taut, and it will draw a shape on the paper. Draw everywhere you can get to while maintaining a taut string. What shape have you created?

## ANALYSIS

*Place all Answers on Report Sheet1*

1) Now describe your own homemade definition of an ellipse and record it here. Be sure to use terms like foci ("A" and "B") and circular:

2) What can you think of in nature that might be in the form of an ellipse?

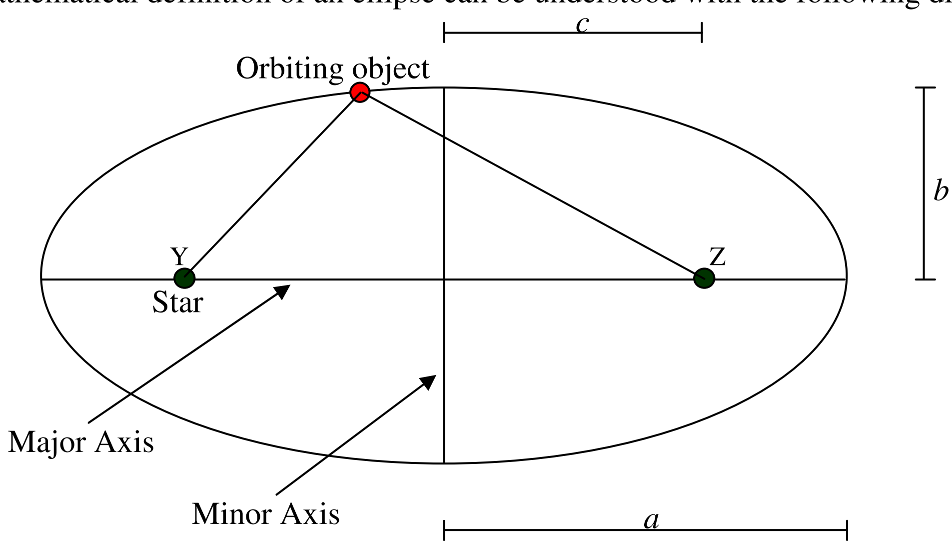
3)  How could you make your ellipse more round or more flat using a similar process as the one above with strings and pushpins?

4)  Try your alteration of the process and describe how the shape was altered.

5)  Take a moment to look at what other groups are doing.  What component have they altered and what was the result?

6)  What would happen if you did exactly the same process as you did in part 1 except that you only used one pushpin?

A mathematical definition of an ellipse can be understood with the following diagram




Every ellipse has a characteristic that scientists call ECCENTRICITY.  Eccentricity can be understood mathematically as the ratio of the half the distance between the foci to half the length of the major axis.  Or if equations are better for you; $e = \frac{c}{a}$ .

7)  Now take a moment and calculate the eccentricity of the ellipse that you created and record it here.  The number should be between 0 and 1.

8)  What is the relationship between the eccentricity value and the shape of the ellipse?

9)  Why must the eccentricity value be between zero and one?

# PART 2:

## INTRODUCTION:

In this experiment, you will use heliocentric data tables to plot the positions of Mercury on polar graph paper and draw the orbit of Mercury. Then you will plot the positions of a celestial object that you will select from Table 2, on regular graph paper and draw the orbit of the object. With the celestial object, you will also calculate the period of revolution.

## PROCEDURE:

1. The distance from the sun (radius vector) is compared to Earth's average distance from the sun (1 astronomical unit or 1 AU). The angle (longitude) between the planet and a reference point in space is measured from the zero degree point (vernal equinox). Orient the polar graph so that the zero degree point is on your right side as you view the graph paper. The sun is located in the center of the paper. Label the sun without covering the center mark. Move about the center point in a counter-clockwise direction as you measure and mark the angle of each longitude line. REMEMBER: *how many degrees are in a circle?*
2. Use the following scale for the radius vectors: every 3$^{rd}$ circle is equal to 0.1 AU.
3. Table 1 provides the heliocentric positions of mercury over a period of several months.
4. Starting with October 1$^{st}$, locate the given longitude on the polar graph paper. Then, measure out along the longitude line an appropriate distance, on the polar graph paper, for the radius vector for this date.
5. Make a small dot at this point to represent Mercury's distance from the sun. Write the plot number next to this point.
6. Repeat the procedure, plotting all given longitudes and associated radius vectors.
7. After plotting all the data, carefully connect the points of Mercury's positions and sketch the orbit of Mercury.

## PART 2 ANALYSIS:

Place all answers from this section into Report Sheet 2
In this portion we will verify Kepler's 2$^{nd}$ law by looking in depth at the graph of Mercury's orbit.

Using a ruler, draw a line from the sun to Mercury's position on October 1. Count 2 more points in the counter clockwise direction and draw a second line from the sun to Mercury's position. The two lines and Mercury's orbit describe an area swept by an imaginary line between Mercury and the sun during a 9-day interval of time.
Choose another 9-day interval and draw the lines that connect the sun to the positions at the beginning and the end of the 9-day interval.
Without damaging the rest of the graph, **very carefully** cut out the two pieces of the pie that you made lines around.

1) Take the mass (measuring to the nearest hundredth of a gram 0.01g) of each piece and record the results on report sheet 1. Include the dates for each area.

2) If they were exactly equal in area, how would their masses compare?

Devise a method to test the accuracy of massing as a method for evaluating area.

3) Explain your method.

Now take your original graph paper with the ellipse on it and choose two additional 9-day periods. Over a small portion of an ellipse, the area can be approximated by assuming the ellipse is similar to a circle. The equation that describes this value is.

$$area = (\theta / 360)\pi r^2$$

Where: r is the average radius of the orbit.

Determine θ by finding the difference in degrees between the lines of your first 9-day period. Measure the radius at a point midway in the orbit between the two dates.

4) Calculate the area in AUs for both 9-day periods of time and show work and results.

# PART 3:

1. Find the data for a celestial object on Table 2.
2. Using a piece of regular graph paper, construct an X-Y axis. Make sure that the origin is in the center of the graph paper. Develop a scale that incorporates all of your data points.
3. Carefully, plot the points on your graph.
4. After plotting all the data, carefully connect the points and sketch the orbit of the celestial object.

**PART 3 ANALYSIS:**

Place all answers from this section into Report Sheet 3

1) How would you describe, in your won words, the eccentricity of your ellipse? Explain your reasoning.

2) Once you have constructed the objects orbit, as a group, compare your graphs with other groups. What are some observations that you can make? Your answer should include but not limited to perihelion and aphelion distances.

3) Using your knowledge of ellipses, give a logical prediction of what the period of revolution will be for your celestial object.

4) Calculate the average radius for your celestial object and record this on the report sheet 3. Using this information and the following equation, calculate the period of revolution for your celestial object. Be sure to show all work.

$$\frac{T_E^{\,2}}{T_o^{\,2}} = \frac{R_E^{\,3}}{R_o^{\,3}}$$

Where: $T_E$ = Period of revolution of Earth (measured in Earth years)

$T_o$ = Period of revolution of celestial object (measured in Earth years)

$R_E$ = Mean distance from Earth to the sun (measured in AU)

$R_o$ = Mean distance from celestial object to the sun (measured in AU)

The Earth's average radius is 1 AU and it takes 365.25 days to revolve around the sun once.

5) How close was your prediction to your actual calculation? What might have been a source of error?

6) Calculate the percent error of your calculation. You will need to get the accepted value from the teacher.

**EXTENSION:**

1. Using the same method as PART 2 of the procedure and the data in Table 3, plot the radius vectors and corresponding longitudes for Mars on a second piece of polar graph paper.

**EXTENSION ANALYSIS:**

*Place all answers from this section into Report Sheet 4.*

1) Select three different areas with the same time interval (16-days) and calculate the area per section. Show work.

2) Does the orbit you drew support Kepler's Law of ellipses?

3) Does Kepler's law of areas apply to your model of Mars? Explain.

4) Using your knowledge of ellipses, give a logical prediction of what the period of revolution will be for Mars.

5) Calculate the average radius for Mars and record this on the report sheet 4.

6) Calculate the percent error of your calculation. You will need to get the accepted value from the reference tables.