"Effective field theory and operator mixing"

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REFERENCES

EFFECTIVE FIELD THEORY
- HOLLOWOOD 0909.0859
- MANOHAR hep-ph/9606222
- BURGESS hep-th/0701053
- ROTHSTEIN hep-ph/0308266
- SKIBA 1006.2142

OPERATOR PRODUCT EXPANSION
- BURAS hep-ph/9806471 & Rev. Mod. Phys. 68, 1125
- Peskin CH. 18
- MANOHAR + WISE "Heavy Quark Physics"

OUTLINE
I. A TRIVIAL EXAMPLE
II. A PHILOSOPHICAL INTERLUDE
III. A LESS-TRIVIAL EXAMPLE
IV. SOME CLOSING REMARKS

this course is called

FLAVOR PHYSICS ⟷ but at its heart it is $\boxed{\text{EFFECTIVE FIELD THEORY}}$

↑ MEASURING THE CKM MATRIX

↑ RELEVANT DOF (& EFFECT OF UV ON IR!) this is where "PHYSICAL INTUITION" LIVES

THE MAIN IDEA:

$$\mathcal{L}_{\text{EFF}} = \sum_i C_i \, \mathcal{O}_i$$

LOCAL OPERATORS

WILSON COEFFICIENT ENCODES INFORMATION ABOUT UV PHYSICS
↑ MEASURE EXPERIMENTALLY

LOW ENERGY EFFECTIVE OPERATOR DESCRIBES IR DEGREES OF FREEDOM
↑ FOR MATRIX ELEMENTS

FACTORIZES SHORT & LONG DISTANCE PHYSICS!

NOTE: WE HAVEN'T "THROWN AWAY" ANYTHING!
PHYSICS LIVES EITHER IN C_i OR $\mathcal{O}_i$

... BUT $\mathcal{L}_{\text{EFF}}$ IS ONLY USEFUL (but very useful) FOR ANSWERING QUESTIONS ABOUT LOW E PHYSICS.

<u>Naïve belief</u> : EFFECTIVE THEORIES ARE "MERELY" APPROXIMATIONS.
ie SOMEHOW "LESSER" THAN FUNDAMENTAL THEORIES.

<u>COUNTERPOINT : USES OF EFTs</u>

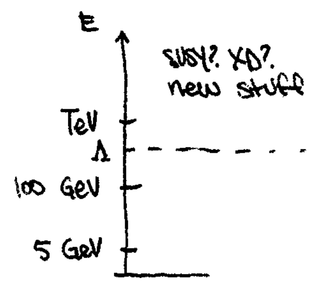
1. <u>BOTTOM-UP PHYSICS</u> ("PHENOMENOLOGY") - UNKNOWN UV PHYSICS
   ESTIMATE SCALE & EFFECT OF HIGH SCALE NEW PHYSICS :
   eg for FERMI THEORY, WE COULD HAVE WRITTEN

$$\mathcal{L}_{\text{eff}} = -\frac{1}{\Lambda^2} J_\mu J^{\dagger\mu} \xrightarrow{\text{EXPERIMENT}} \Lambda = \sqrt{\frac{\sqrt{2}}{G_F}} \approx 350 \text{ GeV}$$

predict "NP" <u>by</u> 350 GeV
(<u>actually</u> shows up @ ~100 GeV)

this is how we know that STRING THY LIVES @ M_{Pl}.

2. <u>SIMPLIFY CALCULATIONS</u> : very important in flavor physics.
   KNOWN UV PHYSICS



PERTURBATIVE ; tree level calc is good

MATCH TO SM EFF OPS (standard set!)

QCD is strong! NEED MULTI-LOOP CALCS.
FORTUNATELY, GERMANS HAVE DONE THIS

otherwise, CSABA WOULD HAVE TO DO 2 & 3
LOOP CALCS FOR EVERY MODEL to MATCH
TO LOW ENERGY CONSTRAINTS!

3. <u>STRONG COUPLING</u> : DESCRIBE EFF DOF OF STRONGLY-COUPLED THEORIES
   INTRACTABLE UV PHYSICS
   eg. MESONS w/ CHIRAL PERT THEORY
   eg. SEIBERG DUALITY
   eg. THEORIES w/ NON-TRIVIAL IR FIXED POINTS (eg d=3)
   <u>LOTS</u> OF "FUNDAMENTAL" THEORY INVOLVED!

   eg. METASTABLE VACUA (COLEMAN WEINBERG V_{eff})

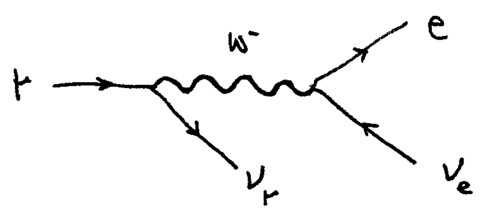
<u>LESSONS</u> : EFTs AREN'T "INFERIOR" APPROXIMATIONS

WE USE EFT IN FLAVOR PHYSICS FOR <u>ALL</u> OF THE ABOVE REASONS!

∃ MANY GOOD REVIEWS ON EFT ; see STRASSLER (th/0309149), MANOHAR (ph/9606222), WEINBERG (0908.1964), BURGESS (th/070153), Hollik (MPI-PH-93-21), ROTHSTEIN (ph/0308266)

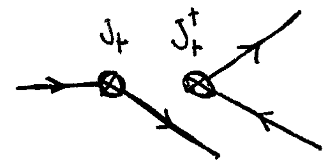
A TRIVIAL EXAMPLE : μ DECAY

non local PROPAGATOR



$\Rightarrow$

LOCAL OPERATOR



<u>SM</u>: $\mathcal{L}_{SM}$ "full theory" $\xrightarrow[\text{zeroth @ in OPE}]{}$ <u>FERMI THEORY</u> : $\mathcal{L}_{EFF} = \underbrace{-\frac{G_F}{\sqrt{2}}}_{C} \underbrace{J_\mu J^{\dagger\mu}}_{\mathcal{O}}$

- TREE LEVEL
- VALID $\forall s$, in principle
- RENORMALIZABLE, by construction

$\Rightarrow$ ALSO AN EFT
(in a slightly different sense)

- "NOT EVEN TREE LEVEL"
- ONLY VALID FOR $s \ll M_W^2$
- NON-RENORMALIZABLE
  $\downarrow$
  "EFFECTIVE THEORY"
  COMES W/ A CUTOFF, $\Lambda \sim M_W$

$$\left(\frac{ig}{\sqrt{2}}\right)^2 \bar{u}\gamma^\alpha P_L u \cdot \frac{-i}{p^2-M_W^2}\bar{u}\gamma_\alpha P_L v$$

$$\underbrace{\frac{G_F}{\sqrt{2}}\left(\bar{u}\gamma^\alpha(1-\gamma^5)u\right)\left(\bar{u}\gamma_\alpha(1-\gamma^5)v\right)}$$

so we <u>define</u> G_F BY <u>MATCHING</u>

HISTORICAL WAY OF WRITING THIS, "V-A" CURRENT.

<u>KEY IDEA</u>: MAP UV INFORMATION (eg. M_W) INTO IR THEORY BY <u>MATCHING</u> THE WILSON COEFFICIENT FROM THE FULL CALCULATION.

<u>QUESTION</u>: AT WHAT ENERGY DO WE MATCH?

A. ... in this case it doesn't really matter
(the difference is RG running w/rt weakly coupled states)

A. IT'S OUR CHOICE, just choice of where to define RG conditions of the EFT.

<u>MATCHING</u>: PICK A SCALE @ WHICH FULL THY & EFT ARE DEFINED TO AGREE. EFT & FULL THY WILL AGREE @ <u>LOWER</u> SCALES, BUT NOT HIGHER SCALES.

$\rightarrow$ THIS IS A SEPARATION OF $\underset{\uparrow c_i}{\text{UV}}$ & $\underset{\uparrow \mathcal{O}_i}{\text{IR}}$

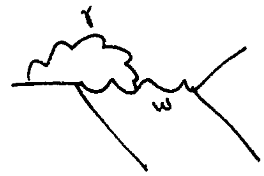
A HINT OF LOOP CORRECTIONS

EFT: Wilson taught us that EFTs ARE THEORIES w/ CUTOFFS
→ We need to specify Renormalization prescription to get finite & physical quantities.

@ $E \ll M_W$, WE CAN CONSIDER QED RG OF $\mu \to e \nu \bar{\nu}$

FULL THY:

BOX DIAGRAM: FINITE



vertex renormaliz.

FINITE AFTER WAVEFUNC REN.

EFT:

FIRST: THERE IS NO EFT DIAGRAM CAPTURING VERTEX RENORMALIZATION, THIS IS BECAUSE EFT HAS NO W BOSON!

THE DOMINANT CORRECTIONS TO EFT (from matching to full thy) COME FROM THIS TYPE OF DIAGRAM. ~~BO~~ MAIN CONTRIBUTION WHEN W IS ON SHELL, SO WE SEE THAT EFT IS UNRELIABLE ABOVE $E \sim \mathcal{O}(M_W)$.

⇒ CONVERSELY, WE EXPECT GOOD AGREEMENT FOR $E \ll \mathcal{O}(M_W)$

NEXT: THE FULL THY BOX DIAGRAM IS FINITE (eg, POWER COUNTING) BUT THE EFT IS DIVERGENT! (treat ⊗⊗ AS ONE VERTEX)

Makes sense: EFT is non-renormalizable. HAVE TO INTRODUCE A COUNTERTERM

WE WILL DO THIS EXPLICITLY IN OUR NEXT EXAMPLE FOR NOW WE'LL REVIEW QUALITATIVE FEATURES.

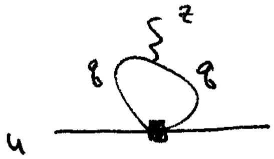
DEALING WITH DIVERGENCES

- MASS INDEPENDENT vs. MASS DEPENDENT SCHEME

IN THIS CASE THIS IS NOT A BIG DEAL, SINCE

$\sim \log \Lambda$ ... SO JUST ADD A COUNTER TERM $(1+\delta)\mathcal{O} = Z\mathcal{O}$

BUT SUPPOSE WE HAD AN EFT DIAGRAM THAT HAD A POWER LAW DIVERGENCE, eg.



using eg momentum sub.

$$\sim \frac{1}{M_W^2}\int d^4k \, \frac{1}{k^2} \sim \mathcal{O}(1)$$

PULLED OUT FROM G_F

up to overall factors eg $g^2/16\pi^2$

BECAUSE THE LOOP CUT OFF IS $\sim M_W$, THESE INTEGRALS ARE ALL $\mathcal{O}(1)$! NOT GOOD FOR PERTURBATIVITY, HAVE TO SUM ALL SUCH CONTRIBUTIONS

SOLUTION: MASS-INDEPENDENT SCHEME, eg $\overline{MS}$

IN THIS CASE WE HAVE AN 'ARBITRARY' DIMENSIONFUL PARAMETER μ THAT ONLY APPEARS IN LOGS. β FUNCTIONS & ANOMALOUS DIMENSIONS γ ARE INDEPENDENT OF MASS & INTEGRALS LOOK LIKE

$$\frac{1}{M_W^2}\int d^4k \, \frac{1}{k^2} \sim \frac{m^2}{M_W^2} \log \mu$$

we'll make a big deal about μ soon!

m IS SOME OTHER MASS SCALE IN THE PROBLEM, eg. m_μ.

INTEGRALS ARE NOW MUCH BETTER BEHAVED. LESSON: DIM RG + $\overline{MS}$

OF COURSE: PHYSICS IS SCHEME-INDEPENDENT, BUT AN INTELLIGENT CHOICE OF SCHEME MAKES LIFE MUCH EASIER.

THE DOWNSIDE: MASS-INDEPENDENT SCHEMES DO NOT AUTOMATICALLY DECOUPLE HEAVY PARTICLES; WE HAVE TO DO THIS BY HAND. SO WE GET A SEQUENCE OF EFTs AFTER INTEGRATING OUT EACH HEAVY PARTICLE SPECIES.

↳ for more see Manohar §7
Rothstein §1.11

- A REMARK ABOUT RESUMMATION OF LOGARITHMS (really the same remark as before)

WE CAN REMOVE POWER LAW 'DIVERGENCES' USING REGULARIZATION AND A SUBTRACTION SCHEME. $\mathcal{L}_{ren} = \mathcal{L}_{BARE}$ + counter terms

WHAT'S LEFT OVER: LOGARITHMS $\sim \log \mu/M_W$

Philosophy: POWER LAW DIVERGENCES ARE REALLY UV EFFECTS IN POSITION SPACE THESE ARE δ FUNCTIONS & DERIVATIVES OF δ FUNCTIONS.

HOWEVER, LOGARITHMIC DIVERGENCES ARE DIFFERENT WHILE POWER LAW DIVERGENCES HAVE MOST OF THEIR SUPPORT IN THE UV, LOG DIVERGENCES SAMPLE EACH 'DECADE' OF ENERGY SCALE EQUALLY — it is a "real" physical effect within the effective theory

↳ this is why logs are important in RG

PROBLEM: LOGS NEEDN'T BE SMALL!
e.g. Naive (very naive) standard Model $\Lambda \sim M_{Pl}$
USUALLY 1 LOOP $\sim g^2 \log \mu/\Lambda$ ... if this is $O(1)$, pert. thy lost!

BUT FORTUNATELY WE CAN RE-SUM THESE [POTENTIALLY] LARGE LOGS.

PERTURBATION THEORY: $\mathcal{L} = (1+\delta) c\mathcal{O} \equiv Z c \mathcal{O} \rightarrow Z^{-n/2} Z c \mathcal{O}$

this is already RENORMALIZED, never had to write a β function, etc.

canonical normalization BUT Z CAN HAVE LARGE LOGS!

"IMPROVED" PERTURBATION THEORY (historical name, now it's JUST PART OF THE RG PROGRAM)

THIS IS WHERE THE RENORMALIZATION GROUP COMES IN.

IDEA: WOULDN'T IT BE GREAT IF WE COULD GEOMETRICALLY SUM THESE LOG FACTORS?

$$1 + \frac{\alpha}{4\pi}\log + \left(\frac{\alpha}{4\pi}\log\right)^2 + \cdots = \frac{1}{1 - \frac{\alpha}{4\pi}\log}$$

ANSWER: YES, AND WE CAN DO THIS ... AND WE ALREADY KNOW HOW

→ Callan-Symanzik ("RG") Equation!

[continued: IMPROVED PERT THY & RESUMMATION OF LOGS]

TECHNICALLY, THE CALLAN-SYMANZIK EQUATION (which just expresses the μ-independence of physical quantities) GIVES A <u>CONSISTENCY CONDITION</u> THAT DETERMINES HIGHER COEFFICIENTS OF ~~[illegible]~~ $g^n \log^n$ RECURSIVELY IN TERMS OF LOWER POWERS. $g \log$

ie GIVEN 1 LOOP RESULT, YOU CAN DETERMINE ALL COEFFICIENTS OF $(g \log)^n$. GIVEN 2 LOOP RESULT, YOU CAN DETERMINE COEFFICIENTS $g^n \log^{n-1}$, etc.

(I've been sloppy w/ notation, but the main idea is correct.)

A NICE EXPLICIT DEMONSTRATION OF THIS IS McKeon, INT. J. TH. PHYS. 37 '98
SEE ALSO WEINBERG CH. 18.1, 18.2.

MORE INTUITIVELY, WE GAIN A LOT FROM DOING RG TRANSFORM <u>INFINITESIMALLY</u> WHERE THERE ARE NO LARGE LOGS. BY INTEGRATING THE CALLAN-SYMANZIK EQN, WE DO A SERIES OF INFINITESIMAL RG TRANSFORMS THAT NEVER SUFFER FROM BAD CONVERGENCE

↳ WE <u>AUTOMATICALLY</u> RESUM THE LOGS!

<u>Remark</u>: THIS IS INTIMATELY TIED TO OUR MASS-INDEPENDENT REN. SCHEME [this is from Weinberg §18.1]

CONSIDER A GREEN'S FUNCTION OF DIMENSION D

$$G(E, x, g, m) = E^D G\left(1, x, g, \frac{m}{E}\right)$$

overall Energy scale — E; dimless variables — x; couplings — g; masses of particles — m

logs of <u>this</u> ($\frac{m}{E}$) can be large & invalidate pert. thy!

BUT IN ~~[illegible]~~ A MASS-INDEP. SCHEME, INTRODUCE SCALE μ WHICH IS a priori UNRELATED TO ANY MASS IN THE PROBLEM.

$$G(E, x, g, m, \mu) = E^D G\left(1, x, \underline{\underline{g(\mu)}}, \frac{m}{E}, \underline{\underline{\frac{\mu}{E}}}\right)$$

- LOGS ARE NOW IN μ/E ($m \to 0$ limit is safe!); natural to pick $\mu = E$
- g IS NOW THE RUNNING COUPLING

CONSIDER φ^4 THEORY @ 1 LOOP

ASSUME: WE'VE ALREADY SUBTRACTED ALL DIVERGENCES BY INTRODUCING THE NECESSARY COUNTER TERMS.

ALL POWER LAW DIVERGENCES GO AWAY w/o A TRACE
... BUT LOG DIVERGENCES LEAVE FACTORS OF $\log \mu^2$
... technically $\log M^2/\mu^2$ where $M = \varphi$ MASS.

$$\text{def}: \quad \log \equiv \log M^2/\mu^2$$

$$\mathcal{L}_{1\text{-loop}} = \frac{1}{2} Z (\partial\varphi_0)^2 + \underbrace{\cancel{\frac{1}{2} Z_m m_0^2 \varphi_0^2}}_{\text{ignore for simplicity}} + \frac{1}{4!} Z_\lambda \lambda_0 \varphi_0^4 + \underbrace{\cancel{\dots}}_{\text{ignore for simplicity}}$$

λ_0, φ_0: <u>BARE</u> QUANTITIES

NOW: WRITE OUT THE Zs IN <u>TWO</u> EXPANSIONS (we'll see why later)

$$\mathcal{L}_{1\text{-loop}} = \frac{1}{2}\left(a_{ij}\lambda^i \log^j\right)(\partial\varphi_0)^2 + \frac{1}{4!}\left(b_{ij}\lambda^i \log^j\right)\varphi_0^4$$

"PER LOOP"

where $a_{00} = b_{00} = 1$; $i \le j$ (@ most one log per λ)

$$= \frac{1}{2}\left(A^i(\lambda) \log^i\right)(\partial\varphi_0)^2 + \frac{1}{4!}\left(B^i(\lambda)\log^i\right)\varphi_0^4$$

~~CALLAN SYMANZIK (RG) EQUATION~~

@ THIS POINT, WE CAN JUST <u>LITERALLY</u> RE-NORMALIZE:

$$\mathcal{L} \to \frac{1}{2}(\partial\varphi)^2 + \frac{1}{4!}\underbrace{\left(\frac{Z_\lambda}{Z^2}\right)\lambda_0}_{\equiv \lambda \text{, renormalized coupling}}\varphi^4$$

SO THIS IS A FINITE (w/ CUTOFF PRESCRIPTION) 1 LOOP EFFECTIVE $\mathcal{L}$.
PROBLEM: THIS STILL HAS LARGE LOGS IN λ! NOT VERY GOOD FOR PERTURBATION THEORY.

SO WE HAVE TO DO BETTER: NOT JUST ~~RE~~ RENORMALIZATION, BUT <u>RENORMALIZATION GROUP</u>.

CALLAN-SYMANZIK (RG) EQN

$$\mu \frac{d}{d\mu} \mathcal{L} = 0 = \left[\mu \frac{\partial}{\partial\mu} + \beta \frac{\partial}{\partial\lambda} - \gamma \varphi \frac{\partial}{\partial\varphi} \right] \mathcal{L}$$

$$\beta = \mu \frac{\partial\lambda}{\partial\mu} \qquad \frac{\partial \log \varphi}{\partial \log \mu} = \gamma_1 \lambda + \gamma_2 \lambda^2 + \cdots$$

$$= \beta_2 \lambda^2 + \beta_3 \lambda^3 + \cdots$$

from $\log = \log M^2/\mu^2$

$$\text{eg: } \beta = \frac{\partial}{\partial \log\mu} \left[\lambda_0 \frac{b_{ij} \lambda^i \log^j}{(a_{ij} \lambda_0^i \log^j)^2} \right] = (-2) \lambda_0^2 \underbrace{(b_{11} - 2a_{11})}_{\equiv \beta_2} + \cdots$$

$$\text{eg: } \gamma = -\lambda_0 \underbrace{a_{11}}_{\equiv \gamma_1} + \cdots$$

GREAT. WE WROTE ONE SET OF UNKNOWNS IN TERMS OF ANOTHER ☺
LET'S DO IT AGAIN W/ THE OTHER EXPANSION.

LET'S ONLY FOCUS ON THE INTERACTION TERM

$$\mu \frac{d}{d\mu} \left[\lambda_0 \cancel{\tfrac{1}{4!}} B^i(\lambda) \log^i \varphi_0^4 \right] = 0$$

$$0 = -2i \lambda_0 B_i \log^{i-1} \varphi_0^4 + \beta B_i \log^i \varphi_0^4 + i \beta \lambda_0 B_i' \log^i \varphi_0^4 - 4 \gamma \lambda_0 B_i \log^i \varphi_0^4$$

Pick only $\log^n$ terms:

$$0 = -2n \lambda_0 B_{n+1} + \frac{1}{\lambda_0} \beta B_n + n \beta B_n' - 4 \gamma B_n \qquad B_n = \sum_i b_{in} \lambda^i$$

$\beta \to \beta_2 \lambda_0^2$, $\gamma \to \gamma_1 \lambda_0$

PICK ONLY $\mathcal{O}(\lambda^{n+1})$ TERMS

$$\boxed{+ 2n\, b_{(n+1)(n+1)} = \beta_2 b_{nn} + \beta_2 n\, b_{nn} - 4 \gamma_1 b_{nn}}$$

→ RECURSIVE FORMULA FOR b_{nn} FROM RGE (CONSISTENCY.)

hence one can resum leading log w/ lo in λ
n/lo w/ n/lo in λ

•• WHEN WE REPLACE BARE COUPLINGS + FIELDS

$\downarrow$

RENORMALIZED COUPLINGS + FIELDS, DEF IN TERMS OF MATRIX ELEMENTS @ E. SCALE μ,

THE INTEGRALS OVER MOMENTA WILL BE EFFECTIVELY CUT OFF AT μ -W

- weinberg § 18.1

THIS IS VERY IMPORTANT!

μ TELLS US HOW WE SEPARATE UV & IR!

"integrated out" "active dof"

eg LOOP INTEGRALS:

$$\underbrace{\int_{-p^2}^{\Lambda^2} d^4k \frac{1}{k^4}}_{\ln(\Lambda^2/p^2)} = \Big(\underbrace{\int_{\mu^2}^{\Lambda^2}}_{\substack{\ln(\Lambda^2/\mu^2) \\ \text{GOES INTO } C_i}} + \underbrace{\int_{-p^2}^{\mu^2}}_{\substack{\ln(\mu^2/p^2) \\ \text{GOES INTO } \mathcal{O}_i}} \Big) d^4k \frac{1}{k^4}$$

WANT TO CALCULATE C_i (MATCHING) WHERE PERT. THY VALID (ie avoid large logs), SO PICK $\mu \sim \mathcal{O}(\Lambda)$
eg. $\Lambda = M_W$

RG EQUATION:

$$0 = \mu \frac{d}{d\mu} \langle f | \sum_j c_j \mathcal{O}_j | i \rangle$$

$$= \left(\mu \frac{d}{d\mu} C_i\right) \mathcal{O}_i + c_i \underbrace{\left(\mu \frac{d}{d\mu} \mathcal{O}_i\right)}_{\gamma_{ij} \mathcal{O}_j}$$

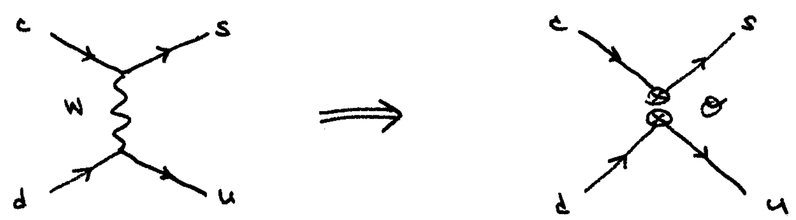
WE WILL SHOW THIS EXPLICITLY IN OUR MAIN EXAMPLE.

MID-LECTURE SUMMARY

1. EFT of a known fundamental theory matches full theory w/in domain of validity: ie what we define to be IR) @ arbitrary precision in powers of, eg., $1/M_W^2$.

2. KEY POINT IS SEPARATION OF UV INFO INTO C_i
IR INFO INTO $\mathcal{O}_i$

OUR PRIMARY EXAMPLE : $c \to u\bar{d}s$



STEP I : WRITE OUT $\mathcal{L}_{EFF}$

... unfortunately there's a lot of stupid historical convention...

$$\mathcal{L}_{EFF} = -\frac{G_F}{\sqrt{2}} V_{cs}^{*} V_{ud} \sum_i C_i \mathcal{O}_i$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

Why 8? $\left(\frac{g}{\sqrt{2}}\right)^2$ COUPLING & $\left[\frac{1}{2}(1-\gamma^5)\right]^2$ PROJECTION
Why $\sqrt{2}$? NORMALIZATION OF f_π vs. F_π

the tree-level operator is easy to write out
[note: BY STUPID CONVENTION WE'VE PULLED OUT A FACTOR OF $(\frac{1}{2})^2$ TO WRITE (V-A) CURRENTS.]

$$\mathcal{O}_2 = \left(\bar{s}_a \gamma^\mu (1-\gamma^5) c_a\right) \cdot \left(\bar{u}_b \gamma^\mu (1-\gamma^5) d_b\right)$$

a & b are color indices

historical name

$$= \bar{\chi}_s^a \bar{\sigma}^\mu \chi_c^a \cdot \bar{\chi}_u^b \bar{\sigma}_\mu \chi_d^b \times 4$$

← GROWN UP NOTATION

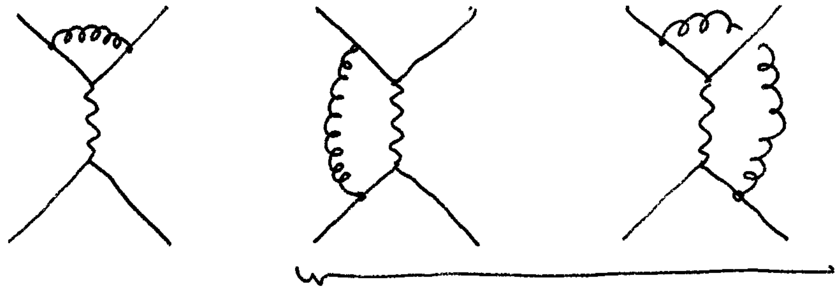
$$\equiv (\bar{s}_a c_a)_{V-A} \cdot (\bar{u}_b d_b)_{V-A}$$

w/ OUR CHOICE OF NORMALIZATION , $\boxed{C_2 = 1}$ (+ loop level)

CLEARLY @ LO IN THE WOLFENSTEIN PARAMETER λ
& @ TREE - LEVEL , THIS IS THE ONLY OPERATOR.

THINGS GET MORE INTERESTING @ LOOP LEVEL.
WHAT LOOPS DO WE CARE ABOUT? ONLY QCD - by far the dominant effect

examples :



RESHUFFLES THE COLOR CONTRACTIONS!
(eg use double line notation)

GENERATES ANOTHER OPERATOR

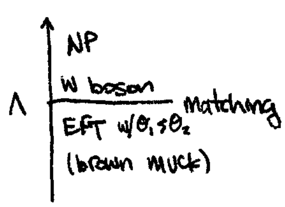
$$\mathcal{O}_1 = (\bar{s}_a c_b)_{V-A} (\bar{u}_b d_a)_{V-A}$$

$$C_1 = 0 + \mathcal{O}(\alpha_s)$$

using group theory completeness relation for SU(3)

$$\left(\tfrac{1}{2}\lambda^A\right)_{ab}\left(\tfrac{1}{2}\lambda^B\right)_{cd} = -\tfrac{1}{6}\delta_{ab}\delta_{cd} + \tfrac{1}{2}\delta_{ad}\delta_{bc}$$

STEP II : DETERMINE $C_i(\Lambda)$ @ THE MATCHING SCALE



WE ARE REALLY MATCHING AMPLITUDES w/ SOME GIVEN EXTERNAL STATES.

BUT C_i & $\mathcal{O}_i$ ARE INDEPENDENT OF EXT STATE! THUS WE ARE FREE TO CHOOSE THESE STATES CONVENIENTLY, even unphysically.

$$\langle f | \cdots | i \rangle = \langle u(p)\bar{d}(-p)s(p) \mid \cdots \mid c(p) \rangle$$

↳ note that we are sidestepping issues about HADRONIC matrix elements

TO DO OUR MATCHING, WE HAVE TO DO THE LOOP LEVEL CALC IN BOTH THEORIES!

↳ why is this more efficient?
OTHERS HAVE NAVIGATED THE BROWN MUCK @ MULTI-LOOP ORDER
WE CAN MAP ONTO THEIR WORK!

FULL THEORY CALCULATION

DEFINE: $A_{1,2} \equiv \langle u\bar{d}s | \mathcal{O}_{1,2} | c \rangle$

$$\mathcal{M}^{(0)}_{\text{FULL}} = \frac{G_F}{\sqrt{2}} V^*_{cs} V_{ud} \left[A_2 \left(1 + 2C_F \frac{\alpha_s}{4\pi} \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{-p^2} \right) + A_2 \frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{-p^2} - 3A_1 \frac{\alpha_s}{4\pi} \ln \frac{M_W^2}{-p^2} \right] \right.$$

$$\uparrow$$

$$C_F = \frac{N^2-1}{2N} \quad \text{s.t. } \mathrm{Tr}\left(\frac{\lambda^A}{2} \frac{\lambda^B}{2} \right) = C_F \delta^{AB}$$

OBSERVE: DIVERGENCE IN 1ST TERM FROM VERTEX CORRECTION: 

THAT'S OK! SM IS RENORMALIZABLE, DIVERGENCE GOES AWAY AFTER WAVEFUNCTION RENORMALIZATION OF QUARKS:

$$\psi^{(0)} = Z_\psi^{1/2} \psi \qquad \text{w/} \quad Z_\psi = 1 - \underbrace{C_F \frac{\alpha_s}{4\pi} \frac{1}{\epsilon}}$$

× 4 quarks, cancels div above ✓

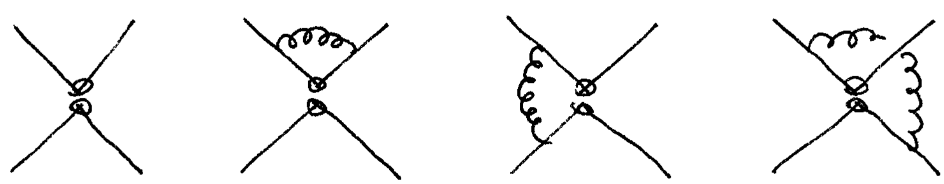
NO OTHER DIVERGENCES!  BOX DIAGRAMS ARE MANIFESTLY FINITE BY POWER COUNTING.

SUBTLE: ONE SHOULD ALSO BE CONCERNED ABOUT EXPLICIT FACTORS OF p^2 SINCE WILSON COEFFICIENTS DO NOT DEPEND ON EXT. STATES!

(Why "$-p^2$" in logs? We chose unphysical momenta)

EFT CALCULATION

$$\mathcal{M}_{\text{EFT}} = \frac{G_F}{\sqrt{2}} V^*_{cs} V_{ud} \left(C_1 \langle \mathcal{O}_1 \rangle + C_2 \langle \mathcal{O}_2 \rangle \right)$$

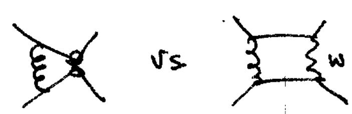


$$\langle \mathcal{O}_1 \rangle^{(0)} = A_1 \left(1 + 2C_F \frac{\alpha_s}{4\pi} \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{-p^2} \right) \right) + A_1 \frac{\alpha_s}{4\pi} \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{-p^2} \right) - 3A_2 \frac{\alpha_s}{4\pi} \left(\frac{1}{\epsilon} + \ln \frac{\mu^2}{-p^2} \right)$$

$\langle \mathcal{O}_2 \rangle^{(0)}$ = same w/ $A_1 \leftrightarrow A_2$

SAME DIVERGENCE

NEW DIVERGENCES!



(NON-RENORMALIZABLE THY!)

OKAY: WE'VE SEEN THIS ALREADY IN OUR TOY $\mu \to e\nu\bar{\nu}$ DIAGRAM.
WE JUST INTRODUCE A COUNTER TERM + SUBTRACTION SCHEME, right?

SOMETHING NEW: <u>OPERATOR MIXING</u>

$\langle \mathcal{O}_2 \rangle^{(0)}$ CONTAINS <u>BOTH</u> A_2 <u>AND</u> A_1 !

SO HAVE TO INTRODUCE A MATRIX OF COUNTERTERMS

$$\mathcal{O}_i^{(0)} = Z^{\mathcal{O}}_{ij}\, \mathcal{O}_j \qquad \text{w/} \quad Z^{\mathcal{O}} = \mathbb{1} + \frac{\alpha_s}{4\pi}\frac{1}{\epsilon}\underbrace{\begin{pmatrix} 1 & 3 \\ -3 & 1 \end{pmatrix}}_{\gamma_{ij}}$$

γ_{ij} ANOMALOUS DIM

IN PRINCIPLE: DIAGONALIZE γ TO GET $\mathcal{O}_\pm = \frac{1}{2}(\mathcal{O}_2 \pm \mathcal{O}_1)$
THESE DIAGONALIZE THE EFFECTIVE HAMILTONIAN

IN THIS CASE, NOT A BIG DEAL SINCE WE WOULD INCLUDE <u>BOTH</u> OPERATORS IN ANY HADRONIC MATRIX ELEMENT; BUT GENERALLY ALL OPS OF SAME DIMENSION MIX & WE ONLY CARE ABOUT A SUBSET @ LOW ENERGIES. [see next example]

<u>INTUITION</u>: WHEN WE DO WAVEFUNCTION RENORMALIZATION, WE DEFINE OUR "LOW E" RENORMALIZED FIELDS AS <u>INCLUDING</u> QUANTUM CORRECTIONS FROM THE UV

IN THE SAME WAY THE OPERATOR INCLUDES THE QUANTUM EFFECTS FROM UV, EVEN COMING FROM DIFFERENT OPS!

<u>REALLY</u> WHAT IS HAPPENING IS THAT AS WE SHIFT μ, WE PACKAGE "UV" & "IR" PHYSICS DIFFERENTLY. WE ARE PUTTING DIFFERENT EFFECTS IN THE WILSON COEFFICIENT VS. EFF OPERATOR; BUT <u>BOTH</u> RENORMALIZE:

$$0 = \mu \frac{d}{d\mu}\langle c_i \mathcal{O}_i \rangle = \left(\mu \frac{d}{d\mu} c_i\right)\mathcal{O}_i + c_i\left(\mu\frac{d}{d\mu}\mathcal{O}_i\right)$$

@ LOW E, MAYBE I ONLY WANT $\mathcal{O}_1$. SO I TAKE EFT & CALC $\langle \mathcal{O}_1 \rangle$. BUT THIS $\mathcal{O}_1$ IS MIXED W/ OTHER OPS IN THE UV. (We'll see this in $b \to s\gamma$)

$\to$ <u>Wilsonian</u> EFT in action

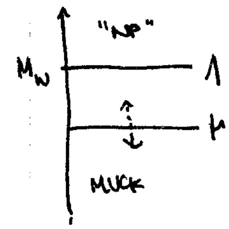
STEP III : DO MATCHING & WRITE WILSON COEFFICIENTS.

COMPARE $\mathcal{M}_{FULL}$ w/ $\mathcal{M}_{EFF} = -\frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \sum_i C_i \langle \mathcal{O}_i \rangle$

$$C_1 = -3\frac{\alpha_s}{4\pi}\left(\ln\frac{M_W^2}{-p^2} - \ln\frac{\mu^2}{-p^2}\right) = \boxed{-3\frac{\alpha_s}{4\pi}\ln\frac{M_W^2}{\mu^2}}$$

$$C_2 = \qquad = \boxed{1 + \frac{\alpha_s}{4\pi}\ln\frac{M_W^2}{\mu^2}}$$

Remarks



- INDEED INDEPENDENT OF EXTERNAL MOMENTA ✓
- DOES DEPEND ON μ AND M_W $(=\Lambda)$

EMPHASIZE ONCE MORE: μ TELLS US HOW WE SEPARATE UV & IR!

eg. $\mu = M_W$	$C_1 = 0$	$C_2 = 1$	ALL INFO IN OPERATORS
$\mu = m_c$	$C_1 =$ big log	$C_2 =$ big log	ALL INFO IN COEFF

FULL AMPLITUDE IS INDEPENDENT OF μ, SO IS EFT!

WANT $\mu \sim \mathcal{O}(M_W)$ TO REMOVE LARGE LOGS*
AND TO AVOID HAVING TO DO CALC IN THE MUCK; WANT TO DO CALCULATION WHERE QCD IS AS PERTURBATIVE AS POSSIBLE.

(* - actually, when there are multiple scales in the problem, these logs pop up in other places; eg in B decays, 'physics' occurs @ m_b, not M_W.)

- REMARK: EFT HERE IS AN EXPANSION IN $1/M_W$
  WE CONSTRUCT THE THEORY TO MATCH FULL THY @ SOME $\mathcal{O}$ IN $1/M_W$

  WE CAN WORK TO ARBITRARILY HIGH ORDERS OF $1/M_W$, @ SOME POINT EASIER TO JUST USE FULL THEORY, BUT IN PRINCIPLE YOU CAN MATCH FULL THY TO ARBITRARY PRECISION IN THE DOMAIN OF VALIDITY OF EFT $(E \ll M_W)$.

<u>ONE LAST EXAMPLE</u>: YUHSIN & I HAVE A FANTASTIC CALCULATION FOR $\mu \to e\gamma$ IN A WARPED EXTRA DIMENSION.

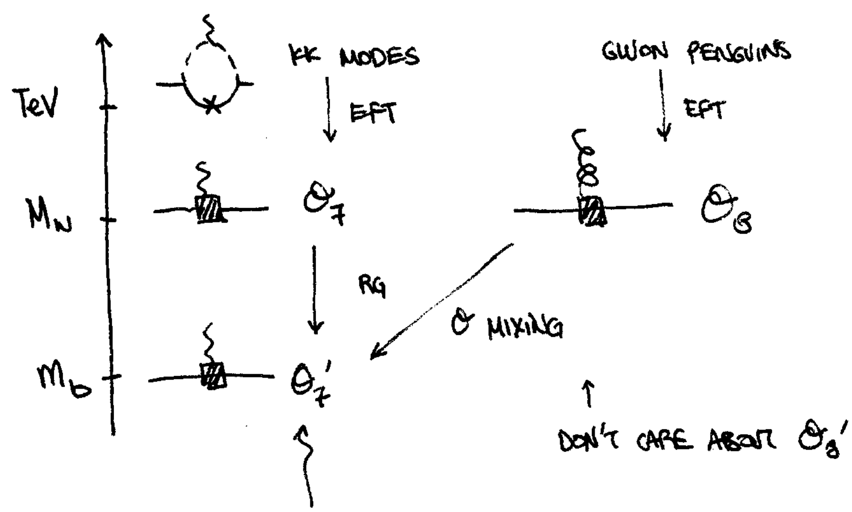
$\Rightarrow$ WANT TO CONVERT THIS INTO A CALC. FOR $b \to s\gamma$ $(B \to X_s \gamma)$

$\mu \to e\gamma$: $e_L \sigma^{\mu\nu} F_{\mu\nu} \mu_R$

$b \to s\gamma$: $s_L \sigma^{\mu\nu} F_{\mu\nu} b_R$

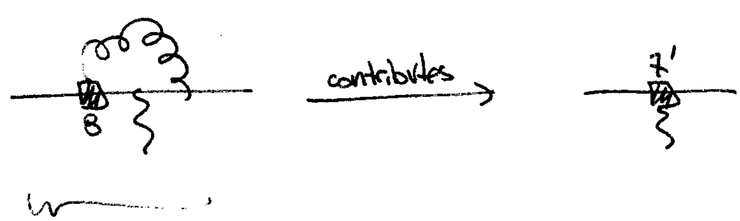
1-loop amplitude
very similar structure

THIS ALSO HAS A SILLY NAME: $\mathcal{O}_7$



<u>this</u> is WHAT WE COMPARE TO EXPERIMENT
"not even tree level" × form factors

<u>PHYSICALLY</u>:



this is <u>TWO-LOOP</u> IN FUNDAMENTAL THEORY!
THIS IS WHY WE MATCH @ A HIGH SCALE

CLEVER CALCULATORS (eg BURAS) HAVE ALREADY CALCULATED THE RG RUNNING OF EFT FROM $M_W \to m_b$, AT <u>MANY</u> LOOP ORDER (~ 3) !

WE CAN MAP TO THEIR RESULT & USE IT AS A BRIDGE TO CONNECT TO DATA.
THIS CAN BE DONE $\forall$ EXOTIC MODEL SINCE ALL OF THE "NEW" FEATURES ARE PACKAGED INTO THE WILSON COEFFICIENTS.

<u>THIS</u> is THE POWER OF EFT IN FLAVOR!