National Brand 42-381 50 SHEETS EYE EASE 5 SQUARE 42-382 100 SHEETS EYE EASE 5 SQUARE 42-389 200 SHEETS EYE EASE 5 SQUARE

7.5 STANDARD CANDLE ⇒ LUMINOSITY KNOWN
STANDARD YARDSTICK ⇒ PROPER LENGTH KNOWN

i.e. (7.28) $d_L = S_k(r)(1+z)$
(7.36) $d_A = S_k(r)(1+z)^{-1}$

⇒ (7.37) $d_A = d_L(1+z)^{-2}$

DEFINITIONS

(7.21) $d_L \equiv \left(\frac{L}{4\pi f}\right)^{1/2}$

(7.33) $d_A \equiv \frac{\ell}{\delta\theta}$

$$\Sigma \sim \frac{f}{(\delta\theta)^2} = \left(\frac{L}{4\pi d_L^2}\right)\left(\frac{d_A}{\ell}\right)^2$$
$$= \frac{L}{4\pi d_L^2}\cdot\frac{d_L^2}{\ell^2(1+z)^4}$$
$$= \boxed{\frac{L}{4\pi \ell^2(1+z)^4}}$$

7.6 from (3.44) & (3.45) $\delta t_e = \delta t_0(1+z)^{-1}$

⇒ $\boxed{\delta t_e = 12 \text{ hrs}}$

$\boxed{R_{max} = c\delta t_e = 87 \text{ AU}}$

(7.33) $d_A = \frac{R_{max}}{\delta\theta} = \frac{d_p(t_0)}{1+z}$ (7.38)

(7.13) $d_p(t_0) = c\int_{t_e}^{t_0}\frac{dt}{a(t)} = c\int_{a(t_e)}^{a(t_0)}\left(\frac{dt}{da}\right)\frac{da}{a(t)}$
$$= c\int_{1/6}^{1}\frac{da}{a\dot{a}}$$

SIMPLIFY USING FRIEDMANN EQUATION FOR BENCHMARK MODEL

$$\frac{H(t)^2}{H_0^2} = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0}$$

$$\dot{a} = H_0\sqrt{\Omega_{r,0}a^{-2} + \Omega_{m,0}a^{-1} + \Omega_{\Lambda,0}a^2}$$

thus $d_p(t_0) = c\int_{1/6}^{1} \frac{da}{H_0 \sqrt{\Omega_{r,0} + \Omega_{m,0} a + \Omega_{\Lambda,0} a^4}}$

$\Omega_{r,0} \sim 0$, $\Omega_{m,0} = .3$, $\Omega_{\Lambda,0} = .7$

$d_p(t_0) \approx 7800$ Mpc

$d_A = d_p / 1+z = 1300$ Mpc

$$\boxed{\delta\theta = \frac{R_{max}}{d_A} = 3.2 \times 10^{-13} \text{ rad}}$$

8.2 $L = (1.8 \pm 0.8) \times 10^5 L_\odot$

SPHERE OF RADIUS $r_h = 120 \pm 12$ pc CONTAINS $\frac{1}{2}L$

(8.38) $\sigma_r = \langle (v_r - \langle v_r \rangle)^2 \rangle^{1/2}$

ISOTROPIC $\Rightarrow \langle v^2 \rangle = 3\sigma_r^2$

(8.34) $M = \frac{\langle v^2 \rangle r_h}{\alpha G} = \boxed{23 \times 10^7 \; M_\odot}$

$\alpha \approx .4$ (P 135)

$$\boxed{\frac{M}{L} = 128 \; M_\odot / L_\odot}$$

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8.3 (8.48) $$\alpha = \frac{4GM}{c^2 b}$$

EARTH :	$\alpha = 2.8 \times 10^{-9}$ RAD	=	5.7×10^{-4} ARCSEC
W.DWARF :	$\alpha = 3.95 \times 10^{-4}$ RAD	=	$81.5''$
N. STAR :	$\alpha = 0.78$ RAD	=	$42.5°$

8.5

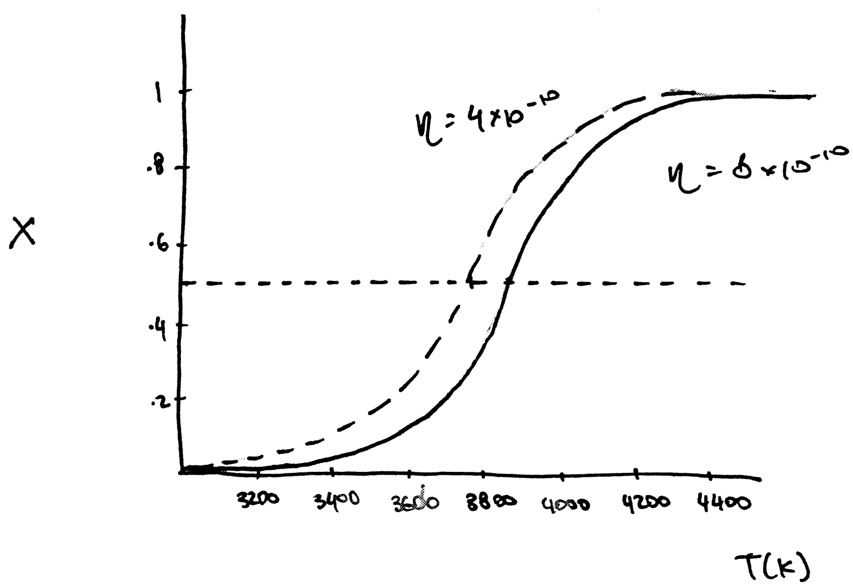
$$n = \frac{N}{\frac{4}{3}\pi r_h^3} = \boxed{70.74 \ \text{gal}/\text{Mpc}^3}$$

$$l = \frac{1}{n\Sigma} = \boxed{14.14 \ \text{Mpc}}$$

$$\tau = \frac{l}{v} = \frac{l}{\sqrt{3}\sigma} = \boxed{9 \ \text{Gyr}}$$

9.1 (9.30) $$X = \frac{-1 + \sqrt{1+4S}}{2S}$$

(9.31) $$S(T,\eta) = 3.84\eta \left(\frac{kT}{m_e c^2}\right)^{3/2} e^{Q/kT}$$



9.2 (2.25)
$$\mathcal{E}(\nu)\,d\nu = \frac{8\pi h}{c^3}\frac{\nu^3\,d\nu}{\exp(h\nu/kT)-1}$$

$$n(\nu)\,d\nu = \frac{1}{h\nu}\mathcal{E}(\nu) = \frac{8\pi}{c^3}\frac{\nu^2\,d\nu}{\exp(h\nu/kT)-1}$$

for $kT \ll Q$

this is a <u>very</u> important integral in physics!

$$n_{ionize} = \int_Q^\infty n(\nu)\,d\nu \approx \frac{8\pi}{c^3}\int_Q^\infty \frac{\nu^2\,d\nu}{\exp(h\nu/kT)}$$
$$= \frac{8\pi}{c^3}\int_Q^\infty \nu^2 \exp(-h\nu/kT)\,d\nu$$
$$= \frac{8\pi}{c^3}\left(\frac{kT}{h}\right)^3 \int_{hQ/kT}^\infty u^2 e^{-u}\,du$$
$$= 8\pi\left(\frac{kT}{ch}\right)^3 \left\{ -u^2 e^{-u}\Big|_{hQ/kT}^\infty + \int 2u e^{-u}\,du \right\}$$

$$2\left\{ -u e^{-u}\Big|_{hQ/kT}^\infty + \int_{hQ/kT}^\infty e^{-u}\,du \right\}$$

$$= 8\pi\left(\frac{kT}{ch}\right)^3 \left\{ \left(\frac{hQ}{kT}\right)^2 e^{-hQ/kT} + 2\left(\frac{hQ}{kT}\right) e^{-hQ/kT} + e^{-hQ/kT} \right\}$$

$$n_{tot} = \int_0^\infty n(\nu)\,d\nu = \beta T^3 \qquad (2.28)$$

$$\beta = \frac{2.404}{\pi^2}\frac{k^3}{\hbar^3 c^3} = 2.03 \times 10^7\ m^{-3} K^{-3} \qquad (2.29)$$

$$\boxed{f = \frac{n_{ionize}}{n_{tot}}}$$

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9.4 $z_{LS} = 1100$

USING THE SAME MANIPULATIONS AS IN QUESTION 7.6

$$d_p(t_0) = c\int_{1/1101}^{1}\left(H_0\sqrt{\underset{\sim 0}{\Omega_{r,0}} + \underset{\sim .3}{\Omega_{m,0}a} + \underset{\sim .7}{\Omega_{\Lambda,0}a^4}}\right)^{-1} da$$

$$d_c(t_0) = (1+z)\, d_p(t_0)$$
$$= 1101 \cdot d_p(t_0)$$

9.5 (9.61) $\tau = \int_{t_*}^{t_0} n_e(t)\sigma_e c\, dt$

$$n_e(t) = \frac{n_{bary,0}}{a^3}$$

$$= \int_{t_*}^{t_0} a(t)^{-3}\, \underbrace{n_{bary,0}\sigma_e c}_{4.4\times 10^{-21}\ s^{-1}}\, dt \qquad \text{from } (9.34)$$

FOR A FLAT MATTER-ONLY UNIVERSE
(5.59) $t_0 = \frac{2}{3H_0}$ $\quad a(t) = (t/t_0)^{2/3}$

$$\Rightarrow \tau = n_{bary,0}\sigma_e c t_0^2\left(\frac{1}{t_*} - \frac{1}{t_0}\right)$$

$$1 = (4.4\times 10^{-21}\, s^{-1})\left(\frac{2}{3H_0}\right)^2\left(\frac{1}{t_*} - \left(\frac{2}{3H_0}\right)^{-1}\right)$$

$$\Rightarrow \boxed{\begin{aligned} t_* &= 3.9\times 10^{14}\, s \\ &= 13\ Myr \end{aligned}}$$

$$1 + z_* = a_*^{-1} = (t_0/t_*)^{2/3}$$

$$\boxed{z_* \approx 83}$$