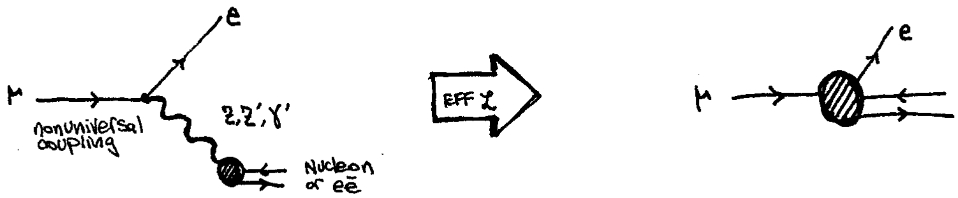


$$\mathcal{L}_{eff} = \frac{4G_F}{\sqrt{2}}\left[g_3\bar{e}_R\gamma\mu_R\cdot\bar{e}_R\gamma e_R + g_4\bar{e}_L\gamma^\mu\mu_L\cdot\bar{e}_L\gamma_\mu e_L + g_5\bar{e}_R\gamma^\mu\mu_R\cdot\bar{e}_L\gamma_\mu e_L + g_6\bar{e}_L\gamma^\mu\mu_L\cdot\bar{e}_R\gamma_\mu e_R\right]$$
$$+\sqrt{2}\,G_F\,\bar{e}\gamma^\mu(v-a\gamma^5)\mu\cdot\sum_q\bar{q}\gamma_\mu(v^q-a^q\gamma^5)q$$

+ TERMS THAT VANISH FOR RS MODELS

ONCE YOU KNOW THE EFFECTIVE COUPLINGS, YOU CAN PLUG INTO THE Br FORMULAE (hep-ph/0501161)

$$Br(\mu\to 3e) = 2(g_3^2+g_4^2)+g_5^2+g_6^2$$
$$Br(\mu\to e) = \frac{p_eE_eG_F^2F_p^2m_\mu^3\alpha^3Z_{eff}^4}{2\pi^2 Z\,\Gamma_{capt}}\,Q_N^2\cdot 2\cdot\underbrace{(v^2+a^2)}_{=\frac{1}{2}[(v+a)^2+(v-a)^2]}$$

FEINBERG-WEINBERG APPROXIMATION (1959)

$${}^{48}_{22}Ti\begin{cases}E_e\sim p_e\sim m_\mu\\ F_p\sim 0.55\\ Z_{eff}\sim 17.61\\ \Gamma_{capt}\sim 2.6\times10^6\ \text{1/sec}\end{cases}\qquad \begin{aligned}Q_N &= v^u(2Z+N)+v^d(2N+Z)\\ v^q &= T_3-2Qs_W^2\end{aligned}$$

DISCUSSION: COUPLING TO NUCLEI

- IN GOING FROM $\mathcal{L}_{eff}\to Br(\mu\to e)_{Ti}$ WE HAVE TO DRESS THE q CURRENT $\to$ NUCLEAR CURRENT. THIS IS DONE USING QCD, WHICH IS PARITY-CONSERVING $\Rightarrow$ PSEUDOSCALAR & AXIAL CURRENT VANISHES : $\langle N|\bar{q}\gamma^5 q|N\rangle = \langle N|\bar{q}\gamma^\mu\gamma^5 q|N\rangle = 0$.
- NOTE THE NORMALIZATION OF v^q ; ie v^q is THE VECTOR COUPLING TO FERMIONS : LH + RH. FOR EXAMPLE, CONSIDER THE Z COUPLING TO UP-TYPE QUARKS:

$$\frac{g}{c_W}\Big[\bar{u}\gamma^\mu\underbrace{(T^3-Qs_W^2)}_{(\frac{1}{2}-\frac{2}{3}s_W^2)}P_Lu+\bar{u}\gamma^\mu\underbrace{(-Qs_W^2)}_{(-\frac{2}{3}s_W^2)}P_Ru\Big]Z_\mu = \underbrace{\frac{g}{2c_W}}_{\text{note factor of }\frac{1}{2}}\left[v^u\bar{u}\gamma^\mu u+a^u\bar{u}\gamma^\mu\gamma^5u\right]$$

REMARK: IT IS MORE NATURAL TO WRITE $(\mathcal{L}_{eff})_{\mu\to e}$ IN TERMS OF CHIRAL CURRENTS

$$(\mathcal{L}_{eff})_{\mu\to e} = \sqrt{2}\,G_F\left[\bar{e}(v+a)\gamma^\mu P_L\mu+\bar{e}(v-a)\gamma^\mu P_R\mu\right]\cdot\sum_q\bar{q}\gamma_\mu v^q q$$

SAMPLE MATCHING CALCULATION : $(\mu\to e)_{Ti}$ VIA SM Z — NOT PHYSICALLY INTERESTING, JUST TO FIX CONVENTIONS, eg. FACTORS OF 2

$$= \frac{g\,g_L}{\cos\theta_W}\bar{e}\gamma^\mu P_L e\,\frac{1}{M_Z^2}\frac{1}{2}\sum_q\bar{q}v^q\gamma_\mu q = \sqrt{2}\,G_F(v+a)\,\bar{e}\gamma^\mu P_Le\cdot\sum_q\bar{q}\gamma_\mu v^q q$$

$g_L = (s_W^2-\frac{1}{2})$ THIS ALSO FIXES CONVENTION FOR Q_ℓ

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_Z^2c_W^2}$$

$$\Rightarrow (v+a) = 2g_L$$

NOW LET'S REVIEW THE PROPERTIES OF BULK FERMIONS & BOSONS IN RS

THE GENERAL SOLUTION FOR THE n^{th} KK MODE GAUGE BOSON PROFILE IS (see eg. hep-ph/0203034)

$$h^{(n)}(z) = N z\left[Y_0(M_{KK}^{(n)}R)\, J_1(M_{KK}^{(n)}z) - J_0(M_{KK}^{(n)}R)\, Y_1(M_{KK}^{(n)}z)\right]$$

Pf/ (HEURISTIC) WE KNOW THAT THE 5D EOM HAS A GENERAL SOLUTION

$$h^{(n)}(z) = a J_1(M_{KK}^{(n)} z) + b Y_1(M_{KK}^{(n)} z) \qquad (\because)$$

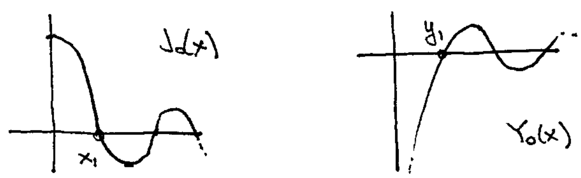
THE $M_{KK}^{(n)}$ FACTOR COMES FROM SOLVING THE n^{th} MODE EOM

SINCE THE Z BOSON HAS A ZERO MODE, IT MUST HAVE NEUMANN BC
BY INVOKING THE FORMULAE FOR DERIVATIVES OF BESSEL FUNCTIONS, WE FIND THAT THE $z=R'$ BC IS

$$Y_0(M_{KK}^{(n)}R)\, J_0(M_{KK}^{(n)}R') = J_0(M_{KK}^{(n)}R)\, Y_0(M_{KK}^{(n)}R') \qquad \forall \text{ reasonable } n$$

WE KNOW THAT $M_{KK}^{(n)} \sim n/R'$ & $R \lll R' \Rightarrow M_{KK}R \approx 0$
NOW RECALL TWO IMPORTANT PROPERTIES OF THE J_0 & Y_0 BESSEL FUNCTIONS

1. $J_0(0) = 1$ & $J_0(x>0)$ "UNDER CONTROL" $(|J_0(x)| < 1)$
2. $Y_0(0) = -\infty$ & $Y_0(x>y_1)$ "UNDER CONTROL" (y_1 is 1^{st} ZERO OF Y_0)



THUS THE LHS OF eq $(\because)$ IS VERY LARGE & NEGATIVE DUE TO $Y_0(M_{KK}R)$ WHILE THE RHS IS A PRODUCT OF "UNDER CONTROL" ($O(1)$ OR LESS) NUMBERS.
$\Rightarrow J_0(M_{KK}R') \approx 0 \Rightarrow M_{KK}R'$ IS A ZERO OF J_0.

THE FIRST KK MODE THUS SATISFIES $\quad M_{KK}^{(1)}R' = x_1 \approx 2.405$
MORE GENERALLY, THE SPACING OF THE KK TOWER FOLLOWS THE ZEROS OF J_0.
[see: hep-ph/0203034, hep-th/0108114, hep-ph/9911262]

<u>THE ZERO MODE Z</u> : goals
1. WRITE DOWN SM Z COUPLING IN TERMS OF 5D PARAMETERS
2. IDENTIFY THE NONUNIVERSAL (FCNC) COUPLING OF THE SM Z (EWSB → 5D ZERO MODE BECOMES SLIGHTLY NONUNIVERSAL & ∃ a NEW FLAVOR-VIOLATING COUPLING TO FERMIONS)

WE APPROXIMATE THE ZERO-MODE Z BOSON WAVEFUNCTION PROFILE BY EXPANDING THE BESSEL FUNCTIONS FOR SMALL ARGUMENT ($M_Z \ll M_Z^{(1)}$ or $M_Z R' \ll 1$)

$$h^{(0)}(z) = N\left[1 + \frac{M_Z^2}{4}\left(z^2 - 2z \log z/R\right) + \cdots\right]$$

TO FIX THE NORMALIZATION $\mathcal{N}$, WE CANONICALLY NORMALIZE THE 4D KINETIC TERM

$$\int d^4x \int_R^{R'} dz \left(\tfrac{R}{z}\right)^5 F^{(5)}_{MN} F^{(5)}_{PQ} g^{MP} g^{NQ} = \int d^4x \int_R^{R'} \tfrac{R}{z} F^{(0)}_{\mu\nu} F^{(0)\mu\nu} \left(h^{(0)}(z)\right)^2 + \cdots$$

REQUIRING THE $(F^{(0)})^2$ TO BE CANONICALLY NORMALIZED AFTER THE dz INTEGRAL,

$$h_Z^{(0)}(z) = \frac{1}{\sqrt{R \log R'/R}} \Big[1 - \underbrace{\frac{M_Z^2}{4}\left(z^2 - 2z^2 \log z/R\right)} \Big]$$

THIS TERM VANISHES FOR A MASSLESS ZERO MODE (eg $A^{(0)}$)
$\Rightarrow$ PROFILE FOR SUCH GAUGE BOSONS IS FLAT

NOW DETERMINE COUPLINGS TO FERMIONS. RECALL THE LH-FERMION ZERO-MODE PROFILE, cf hep-ph/0510275

$$\Psi_c^{(0)}(x,z) = \frac{1}{\sqrt{R'}} \left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^{-c} f_c P_L \underbrace{\psi_c^{(0)}(x)}$$

$\uparrow$ CANONICALLY NORMALIZED 4D FIELD

$$f_c = \sqrt{\frac{1-2c}{1-(R/R')^{1-2c}}}$$

THUS IN THE C-BASIS (5D CANONICAL BASIS) THE FERMION COUPLING IS (performing dz int)

$$g_{4D}^{Zff} Z_\mu^{(0)} \bar{\psi}_c^{(0)} \gamma^\mu \psi_c^{(0)} = \int dz \left(\frac{R}{z}\right)^5 g_5^{Zff} Z_M^{(0)}(x,z) \bar{\Psi}_c(x,z) \Gamma^M \Psi_c^{(0)}(x,z)$$

$$g_5^{Zff} = g_5 c_W T_3 - g_5' s_W Y \qquad \Gamma^M \equiv \frac{z}{R}\gamma^M$$

$$= g_5^{Zff} \int_R^{R'} dz \left(\frac{R}{z}\right)^5 \frac{z}{R} \left[\frac{1}{\sqrt{R'}}\left(\frac{z}{R}\right)^2 \left(\frac{z}{R'}\right)^{-c} f_c\right]^2 \frac{1}{\sqrt{R \log R'/R}} \left[1 + \frac{M_Z}{4}\left(z^2 - 2z^2 \log \frac{z}{R}\right)\right]$$

$$= g_5^{Zff} \int_R^{R'} dz \frac{1}{R'} \left(\frac{z}{R'}\right)^{-2c} f_c^2 \frac{1}{\sqrt{R \log R'/R}} \Big[\underset{\text{UNIVERSAL PART}}{1} + \underbrace{\frac{M_Z}{4}\left(z^2 - 2z^2 \log\frac{z}{R}\right)}_{\text{NON-UNIVERSAL}} \Big]$$

<u>THE SM COUPLING</u>: COMES FROM THE UNIVERSAL TERM

$$g_{SM}^{Zff} = g_5^{Zff} \int_R^{R'} dz \frac{f_c^2 (R')^{2c}}{R' \sqrt{R \log R'/R}} z^{-2c} = \frac{g_5^{Zff} f_c^2 (R')^{2c}}{R' \sqrt{R \log R'/R}} \frac{1}{1-2c}\left[(R')^{1-2c} - R^{1-2c}\right]$$

$$= \frac{g_5^{Zff} f_c^2}{R' \sqrt{R \log R'/R}} \frac{R'}{1-2c}\left[1 - \left(\frac{R}{R'}\right)^{1-2c}\right] = \frac{g_5^{Zff}}{\sqrt{R \log R'/R}}$$

note: still in C-BASIS!

<u>THE FCNC NON-UNIVERSAL PART</u>

CHANGE VARS: $y = z/R$

$$g_{FCNC}^{Zff} = \frac{g_5^{Zff} f_c^2}{R' \sqrt{R \log R'/R}} \frac{M_Z^2}{4} \int_R^{R'} dz \left(\frac{z}{R'}\right)^{-2c} z^2 \left(1 - 2\log\frac{z}{R}\right)$$

$$= B \cdot \left(\frac{R}{R'}\right)^{-2c} R^3 \underbrace{\int_1^{R'/R} dy\, y^{2-2c}\left(1 - 2\log y\right)}$$

EVALUATE BY CAREFUL INTEGRATION BY PARTS (SEE APPENDIX)

$$= \underbrace{\frac{5-2c}{3-2c} \cdot \frac{1}{3-2c}\left(\left(\frac{R'}{R}\right)^{3-2c} - 1\right)}_{\text{SUBLEADING, CAN DROP}} - \frac{2}{3-2c}\left(\frac{R'}{R}\right)^{3-2c} \log\frac{R'}{R}$$

(continued)

$$g^{Z ff}_{FCNC} = \frac{-g_5^{Z ff}}{\sqrt{R \log R'/R}} f_c^2 \frac{M_Z^2}{2(3-2c)} (R')^2 \log\frac{R'}{R} = \boxed{-g^{Z ff}_{SM} \frac{(M_Z R')^2 \log(R'/R)}{2(3-2c)} f_c^2}$$

THE FULL COUPLING IS $g^{Zff}_{4D} = g^{Zff}_{SM} + g^{Zff}_{FCNC}$. NOTE WE'RE STILL IN C-BASIS.

RS GIM MECHANISM : IN THIS 5D C-BASIS, THE NONUNIVERSAL COUPLINGS ARE DIAGONAL, BUT NOT PROPORTIONAL TO $\mathbb{1}$. WHEN WE ROTATE INTO THE PHYSICAL (KK) BASIS, WE GET FCNC.

FACT: THE ROTATION FROM C-BASIS FLAVOR $j \to$ KK BASIS FLAVOR i GO LIKE f_i/f_j
[FOR pf SEE 14 DEC NOTES.]

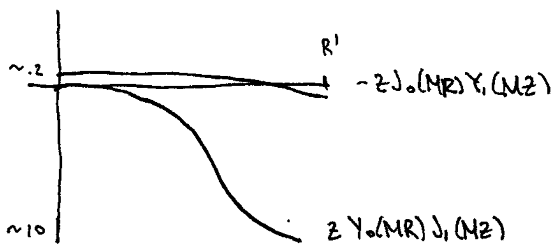
$$g^{Z\mu e}_{4D} = \left(U_L^\dagger g^{Zff}_{FCNC} U_L\right)_{\mu e} \sim \frac{f_e}{f_\mu}\left(\frac{f_\mu^2}{3-2c_\mu} - \underbrace{\frac{f_e^2}{3-2c_e}}_{f_e \ll f_\mu \text{, DROP.}}\right)(M_Z R')^2 \cdot \frac{1}{2}\log\frac{R'}{R}\, \underbrace{g^{Z e_L e_L}_{SM}}_{\text{FLAVOR UNIVERSAL}} (-)$$

$$= \boxed{-g^{Z e_L e_L}_{SM} \cdot \frac{(M_Z R')^2}{2(3-2c_\mu)} \log\frac{R'}{R} f_\mu f_e \equiv \Delta^{(0)}_{\mu e}\, g^{Z e_L e_L}_{SM}}$$

II THE KK Z : for now we will write $Z' = Z^{(1)}$ (will be more careful in custodial model)

RECALL : $h^{(1)}_Z(z) \propto z\left[Y_0(MR) J_1(Mz) - J_0(MR) Y_1(Mz)\right]$

REMARKS : THE SECOND TERM IS MUCH SMALLER THAN THE FIRST OVER MOST OF THE RANGE OF z.



BUT THIS SECOND TERM HAS AN IMPORTANT EFFECT: IT GIVES THE DOMINANT CONTRIBUTION TO THE FLAVOR-CONSERVING (UNIVERSAL) COUPLING TO FERMIONS. THERE ARE TWO HEURISTIC WAYS TO UNDERSTAND THIS.

① $Y_1(z) = -\frac{2}{\pi z} + \mathcal{O}[(\log z) z]$ (Taylor-like expansion)

THUS $zY_1(z)$ IS FLAT TO LEADING ORDER IN z. THIS CERTAINLY ISN'T A VALID APPROXIMATION AT LARGE z, BUT THE POINT ISN'T THAT $zY_1(z) \approx$ CONST $\forall z$. THE POINT IS THAT IN THE EXPANSION OF $zY_1(z)$ $\exists$ A UNIVERSAL PART. THIS GIVES A FLAVOR-CONSERVING COUPLING AS WE SAW FOR THE ZERO MODE Z SINCE THE ORTHOGONALITY (& NORMALITY) OF THE FERMION WAVEFUNCTIONS REMOVES ANY C-DEPENDENCE. THERE ARE FLAVOR VIOLATING TERMS IN THE REST OF THE EXPANSION FOR $zY_1(z)$, BUT AS SEEN IN THE PLOT, THESE ARE NEGLIGIBLE COMPARED TO THE FLAVOR-VIOLATING PROFILE OF THE $zJ_1(z)$ TERM.

SANITY CHECK : $J_1(z) = \frac{1}{2}z + \mathcal{O}(z^2)$; ie $zJ_1(z)$ DOES NOT CONTAIN A FLAT PIECE IN ITS EXPANSION. THUS THE UNIVERSAL PART OF $zY_1(z)$ IS INDEED THE ONLY* SOURCE OF FLAVOR-CONSERVING COUPLINGS. [* - THE NON-FLAT TERMS ALSO GIVE A FLAVOR-CONSERVING PIECE, BUT WE WILL SHORTLY SEE THAT THIS IS SUPPRESSED BY THE FERMION f_c FUNCTIONS.]

② ANOTHER HEURISTIC WAY TO UNDERSTAND THE CONTRIBUTION OF THE $zY_1(z)$ TERM IS TO APPEAL TO THE AdS/CFT DICTIONARY. IN THE CFT THE UV BRANE ~ ELEMENTARY STATES WHILE IR BRANE ~ COMPOSITE STATES. NAIVELY WE EXPECT OUR LIGHT (eg zero-mode) FIELDS TO BE ELEMENTARY. HOWEVER, THE FLAT (ish) GAUGE BOSON ZERO MODE PROBES BOTH BRANES & IS THUS A MIXTURE OF ELEMENTARY WITH SOME COMPOSITE. MORE PRECICELY, THE ZERO MODE IS A ROTATION OF ELEMENTARY w/ SOME COMPOSITE. THIS MEANS THAT THE KK MODES, WHICH ARE NAIVELY COMPOSITE, MUST ALSO CONTAIN SOME ELEMENTARY STATE. IT IS THIS "ELEMENTARY STATE COMPONENT" OF THE KK Z THAT WE ARE CONSIDERING IN THE LEADING FLAVOR-UNIVERSAL TERM COMING FROM THE $zY_1(z)$ TERM.

FIRST WE NEED THE NORMALIZATION OF $h^{(1)}_z(z)$. RECALL THAT THIS COMES FROM REQUIRING THE 4D KINETIC TERM (ie KK decompose then do $\int dz$) TO BE CANONICALLY NORMALIZED; cf. p.2 FOR THE ZERO MODE.

$$h^{(1)}_z(z) = \mathcal{N}\, z\,[Y_0(MR)J_1(Mz) - J_0(MR)Y_1(Mz)]$$

LET US REDEFINE $\mathcal{N}$ TO ABSORB A FACTOR OF $Y_0(MR)$

$$= \mathcal{N}\, z\,[J_1(Mz) - A\,Y_1(Mz)]$$

$$\text{where:}\quad A = \frac{J_0(MR)}{Y_0(MR)} \quad \& \quad M = M^{(1)}_{KK} = \frac{x_1}{R'}$$

WE KNOW THAT THE $AY_1(Mz)$ TERM IS SMALL COMPARED TO THE FIRST TERM. THUS LET US SIMPLIFY OUR JOB BY NEGLECTING IT IN OUR DETERMINATION OF $\mathcal{N}$. THE ERROR WILL BE SMALL SINCE THE $AY_1(Mz)$ TERM IS ROUGHLY A FEW % OF THE LEADING TERM OVER MOST OF THE dz INTEGRAL.

OUR NORMALIZATION CONDITION IS $\int_R^{R'} dz\, \frac{R}{z}\left(h^{(n)}(z)\right)^2 = 1$.
THIS INTEGRAL IS STRAIGHTFORWARD IF ONE USES THE ORTHOGONALITY OF BESSEL FUNCTIONS OF THE FIRST KIND, namely:

$$\int_0^a J_\nu\left(\alpha_{\nu m}\frac{\rho}{a}\right)J_\nu\left(\alpha_{\nu n}\frac{\rho}{a}\right)\rho\, d\rho = \frac{1}{2}a^2\left[J_{\nu+1}(\alpha_{\nu m})\right]^2\delta_{mn}$$

ALTERNATELY, ONE MAY USE

$$J_\nu(z) = \frac{z}{2\nu}\left(J_{\nu-1}(z) + J_{\nu+1}(z)\right) \qquad J'_\nu(z) = \frac{1}{2}\left(J_{\nu-1}(z) - J_{\nu+1}(z)\right)$$

ONE FINDS THAT THE CORRECTLY-NORMALIZED APPROXIMATION FOR $h^{(1)}_z(z)$ (neglecting the $AY_1(Mz)$ term) IS:

$$\boxed{h^{(1)}_z(z) \approx \underbrace{\sqrt{\frac{2}{R}}\,\frac{1}{J_1(x_1)R'}}_{\mathcal{N}} \cdot z J_1\Big(\underbrace{x_1\frac{z}{R'}}_{Mz}\Big)}$$

WE ALREADY MADE THE CASE THAT THE $\Delta Y_1(Mz)$ TERM GIVES THE LEADING UNIVERSAL CONTRIBUTION, SO WE CANNOT COMPLETELY NEGLECT IT. WE WILL ASSUME THE NORMALIZATION $\mathcal{N}$ FROM THE PREVIOUS $J_1(Mz)$ TERM APPROXIMATION:

$$h_2^{(1)}(z) \approx \sqrt{\frac{2}{R}} \frac{1}{J_1(x_1)R'} \left(z J_1\left(x_1 \frac{z}{R'}\right) - \frac{J_0(x_1 R/R')}{Y_0(x_1 R/R')} z Y_1\left(x_1 \frac{z}{R'}\right) \right)$$

WHERE WE WILL ONLY CONSIDER THE SECOND TERM FOR THE UNIVERSAL COUPLING. WE NOW PROCEED ANALOGOUSLY TO WHAT WE DID FOR $h_z^{(0)}$ ON P.3

UNIVERSAL KK Z COUPLING

FOR THIS WE ONLY NEED TO CONSIDER THE SECOND TERM. LET'S MAKE SOME APPROXIMATIONS:

$$J_0(x_1 R/R') \approx J_0(0) = 1$$

$$Y_0\left(x_1 \frac{R}{R'}\right) \approx \frac{2(\gamma + \log(x_1/2) + \log(R/R'))}{\pi} \approx -\frac{2}{\pi}\left(\log\frac{R'}{R}\right)$$

up to $\mathcal{O}(R/R')$

γ = EULER GAMMA $\sim \mathcal{O}(0.1)$, $\log\left(\frac{x_1}{2}\right) \sim \mathcal{O}(0.2)$ } small vs $\log(R/R')$ → drop.

NEXT WE PULL OUT THE UNIVERSAL PART OF $z Y_1(x_1 z/R')$:

$$Y_1\left(x_1 \frac{z}{R'}\right) = \underbrace{-\frac{2}{\pi}\cdot\left(\frac{R'}{x_1 z}\right)} + \mathcal{O}(z)$$

so that $zY_1(x_1 z/R')$ gives universal term $(\mathcal{O}(z^0))$

RECALL: WE ARE NOT APPROXIMATING $Y_1(x_1 z/R')$, THIS WOULD BE A BAD APPROX! THIS IS IDENTIFYING AND ISOLATING THE UNIVERSAL PART OF $h_z^{(1)}$. [IT IS EASY TO SEE THAT $zJ_1(x_1 z/R')$ DOES NOT HAVE A UNIVERSAL PART.]

NOW WE FOLLOW EXACTLY THE SAME PROCEDURE AS ON PAGE 3.

$$h_z^{(1)}(z)\Big|_{\text{UNIVERSAL}} \approx \sqrt{\frac{2}{R}} \frac{1}{J_1(x_1)R'} \left(-\left[\frac{-\pi}{2\log(R'/R)}\right] \times z\left(-\frac{2}{\pi}\cdot\frac{R'}{x_1 z}\right)\right)$$

$$\approx \underbrace{-\sqrt{\frac{2}{R}}\cdot\frac{1}{x_1 J_1(x_1)}}_{\approx -1.13 \to \sim 1} \cdot \frac{1}{\log(R'/R)} \approx \frac{-1}{\log(R'/R)}$$

THEN FOLLOWING THE ANALYSIS OF $g_{SM}^{Z\text{eff}}$ ON P.3 WE OBTAIN

$$\boxed{g_{4D}^{Z'\text{eff}} = \underbrace{\frac{g_5^{Z\text{eff}}}{\sqrt{R}}}_{\text{DIMENSIONLESS}} \frac{1}{\log R'/R}} = \boxed{\frac{g_{SM}^{Z\text{eff}}}{\sqrt{\log R'/R}}}$$

NON-UNIVERSAL (FCNC) COUPLING

OK, NOW THAT WE'RE DONE WITH THE UNIVERSAL PART, WE CAN FORGET THE $zY_1(M_Z)$ TERM ALTOGETHER. ITS CONTRIBUTION TO THE FCNC PART IS NEGLIGIBLE SINCE ITS INTEGRAL IS SO SMALL. THUS WE'RE BACK TO

$$h_Z^{(1)}(z) = \sqrt{\frac{2}{R}}\,\frac{z}{J_1(x_1)\,R'}\,J_1\!\left(x_1\frac{z}{R'}\right).$$

NOW WE PERFORM THE OVERLAP INTEGRAL WITH FERMIONS TO GET THE 4D EFFECTIVE NON UNIVERSAL COUPLING

$$\begin{aligned}
g^{Z'ff}_{4D,\,FCNC} &= g_5^{Zff}\int_R^{R'} dz \left(\frac{R}{z}\right)^5\left(\frac{z}{R}\right)\left[\frac{1}{\sqrt{R'}}\left(\frac{z}{R}\right)^2\left(\frac{z}{R'}\right)^{-c}f_c\right]^2\sqrt{\frac{2}{R}}\,\frac{z}{J_1(x_1)\,R'}\,J_1\!\left(x_1\frac{z}{R'}\right)\\
&= g_5^{Zff}\int_0^1 R'\,dx\,\frac{1}{R'}\,x^{1-2c}\,\frac{J_1(x_1 x)}{J_1(x_1)}\sqrt{\frac{2}{R}}\,f_c^2\\
&= g_5^{Zff}\,\frac{f_c^2}{\sqrt{R}}\,\underbrace{\frac{\sqrt{2}}{J_1(x_1)}\int_0^1 dx\;x^{1-2c}\,J_1(x_1 x)}\\
&\qquad\equiv \gamma_c \approx \frac{\sqrt{2}}{J_1(x_1)}\,\frac{0.7}{2(3-2c)}\left(1+e^{c/2}\right) \approx \frac{\sqrt{2}}{J_1(x_1)}\,\frac{0.7}{2(3-2c)}\,x_1
\end{aligned}$$

THIS IS A WEAK FUNCTION OF c

NOW ROTATE TO THE KK MASS BASIS

$$\boxed{g^{Z'e\mu}_{4D} = g^{Zff}_{SM}\sqrt{\log\frac{R'}{R}}\;\gamma_c\,f_e f_\mu}$$

REMARKS: RECALL THAT THE WHOLE POINT OF THE UNIVERSAL PIECE WAS THAT THE UNIVERSALITY PREVENTS ANY FCNC EFFECTS EVEN AFTER ROTATING INTO THE KK BASIS.

HOWEVER, THE NON-UNIVERSAL PART DOES CONTRIBUTE TO THE FLAVOR-CONSERVING COUPLING,

$$g^{Z'f_if_i}_{4D,\,\text{non-universal}} = g^{Zff}_{SM}\sqrt{\log\frac{R'}{R}}\;\gamma_c\,f_i^2$$

WE CAN SEE, HOWEVER, THAT FOR ZERO MODE FERMIONS $f_i \ll 1$ (especially for light fermions in the anarchic scenario) SO THAT THIS IS SUPPRESSED RELATIVE TO g^{Zff}_{4D} ON P.6.

III MATCHING TO THE EFFECTIVE LFV $\mathcal{L}$ (see p.1)

LET US REMIND OURSELVES OF OUR NOTATION (cf Peskin p. 709)

$$\Delta\mathcal{L}_{Zff} = gZ_\mu J_Z^\mu = \frac{g}{c_W} Z_\mu \Big[\bar{e}_L \gamma^\mu \underbrace{(s_W^2 - \tfrac{1}{2})}_{\equiv g_L} e_L + \bar{e}_R \gamma^\mu \underbrace{(s_W^2)}_{\equiv g_R} e_R + \cdots\Big]$$

IMPORTANT DEF. OF SM COUPLINGS

$$\Delta\mathcal{L}^{4\text{-fermi}}_{Zff} = \frac{4G_F}{\sqrt{2}} \Big(\sum_f \bar{f}\gamma(T^3 - s_W^2 Q) f\Big)^2$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{g^2}{8c_W^2 M_Z^2}$$

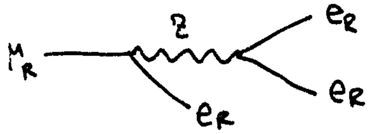
ie $g_{L,R}$ are defined via:

COUPLING OF Z to e_L's $= \frac{g}{c_W} g_L$

COUPLING OF Z to e_R's $= \frac{g}{c_W} g_R$

NOW CONSIDER $\Delta\mathcal{L}_{eff} = \frac{4}{\sqrt{2}} G_F\, g_3 (\bar{e}_R \gamma \mu_R)(\bar{e}_R \gamma e_R)$

LET US IGNORE THE KK CONTRIBUTIONS FOR NOW. LET US MATCH THIS EFFECTIVE OPERATOR TO THE Z-EXCHANGE DIAGRAM.



$$= g_{4D}^{Ze_R\mu_R} \frac{1}{M_Z^2} g_{4D}^{Ze_Re_R} (\bar{e}_R \gamma \mu_R)(\bar{e}_R \gamma e_R)$$

FLAVOR-CONSERVING

FCNC COUPLING:

$$g_{4D}^{Ze_R\mu_R} = g_{SM}^{Ze_Re_R} \Delta_{\mu e} = g_{SM}^{Ze_Re_R} \frac{(M_Z R')^2}{2(3-2c_\mu)} \log\frac{R'}{R} f_\mu f_e$$

WHERE WE'VE DEFINED THE IMPORTANT FLAVOR-CHANGING FACTOR

$$\boxed{\Delta_{e\mu} = \frac{(M_Z R')^2}{2(3-2c_\mu)} \log\frac{R'}{R} f_\mu f_e}$$

$$f_c = \sqrt{\frac{1-2c}{1-(R/R')^{1-2c}}} \xrightarrow[\text{APPROXIMATION}]{\text{MINIMAL MODEL}} f_\mu \sim \sqrt{\frac{\lambda_\mu}{Y_*}}$$

$$f_{\mu_L} = f_{\mu_R}$$

NOW DOING THE MATCHING:

$$\frac{4}{\sqrt{2}} G_F g_3 = \frac{g^2 g_3}{2c_W^2 M_Z^2} = \big(g_{SM}^{Ze_Re_R}\big)^2 \frac{1}{M_Z^2} \Delta_{e\mu} = \Big[\frac{g}{c_W} g_R\Big]^2 \frac{1}{M_Z^2}\Delta_{e\mu}$$

FROM WHICH WE DEDUCE: $\Rightarrow \boxed{g_3 = 2g_R^2 \Delta_{e\mu}}$

SIMILARLY: $\frac{g^2}{2c_W^2 M_Z^2} g_4 = \Big[\frac{g}{c_W} g_L\Big]^2 \frac{1}{M_Z^2} \Delta_{e\mu} \Rightarrow \boxed{g_4 = 2g_L^2 \Delta_{e\mu}}$

$$\frac{g^2}{2c_W^2 M_Z^2} g_{5,6} = \Big(\frac{g}{c_W}\Big)^2 g_L g_R \frac{1}{M_Z^2} \Delta_{e\mu} \Rightarrow \boxed{g_{5,6} = 2g_L g_R \Delta_{e\mu}}$$

NOW CONSIDER THE $\mu \to e$ EFFECTIVE $\mathcal{L}$

$$\sqrt{2}\, G_F\, \bar{e}\, \gamma (v \pm a) P_{L,R}\, \mu \cdot \sum_q \bar{q}\, v^q \gamma q = \left(\frac{g}{c_W}\right)^2 g_{L,R}\, \bar{e}\, \gamma P_{L,R}\, \mu\, \frac{\Delta}{M_Z^2} \sum_q \bar{q}\, \tfrac{1}{2} V^q \gamma q$$

$$\Rightarrow \boxed{v \pm a = 2 g_{L,R}\, \Delta^{(0)}_{e\mu}}$$

$$\Delta^{(0)}_{ij} = \frac{-(M_Z R')^2 \log R'/R}{2(3-2c)} f_i f_j$$

NOW INCLUDE A KK Z TO THE MINIMAL MODEL

WE INTRODUCE A HANDY NOTATION

cf $g^{Zf_if_j}_{4D} = g^{Zff}_{SM} (\delta^{ij} + \Delta^{(0)}_{ij})$

$$g^{Z' f_i f_j} = g^{Zff}_{SM} (g_{KK}\, \delta^{ij} + \Delta^{KK}_{ij})$$

$$g_{KK} = \frac{1}{\sqrt{\log R'/R}} \qquad \Delta^{KK}_{ij} = \sqrt{\log \frac{R'}{R}}\, \gamma_c\, f_i f_j$$

NOW WE CAN WRITE THE MODIFIED EFFECTIVE COUPLINGS HEURISTICALLY : (effective coupling) $= (g^{Zff}_{SM})^2 \left([\text{zero mode}] + g_{KK} \frac{M_Z^2}{M_{Z'}^2} \Delta^{KK} \right)$

THUS :

$$\boxed{\begin{aligned} g_{3,4} &= 2 (g_{R,L})^2 \left[\Delta^{(0)}_{e\mu} + g_{KK} \frac{M_Z^2}{M_{Z'}^2} \Delta^{KK}_{\mu e} \right] \\ g_{5,6} &= 2 g_L g_R \left[\Delta^{(0)}_{e\mu} + g_{KK} \frac{M_Z^2}{M_{Z'}^2} \Delta^{KK}_{\mu e} \right] \\ v \pm a &= 2 g_{L,R} \left[\Delta^{(0)}_{e\mu} + g_{KK} \frac{M_Z^2}{M_{Z'}^2} \Delta^{KK}_{\mu e} \right] \end{aligned}}$$

NOW INCLUDE THE KK PHOTON

$$e_{SM} = \frac{e_5}{\sqrt{R \log R'/R}} = g s_W \quad \& \quad e^{A' f_i f_j} = e_{SM} (g_{KK}\, \delta^{ij} + \Delta^{KK}_{ij})$$

$$\boxed{\begin{aligned} g_{3,4} &= 2 (g_{R,L})^2 \left[\Delta^{(0)}_{e\mu} + g_{KK} \frac{M_Z^2}{M_{Z'}^2} \Delta^{KK}_{e\mu} \right] - 2 s_W^2 c_W^2\, g_{KK} \frac{M_Z^2}{M_{A'}^2} \Delta^{KK}_{e\mu} \\ g_{5,6} &= 2 g_L g_R \left[\Delta^{(0)}_{e\mu} + g_{KK} \frac{M_Z^2}{M_{Z'}^2} \Delta^{KK}_{e\mu} \right] - 2 s_W^2 c_W^2\, g_{KK} \frac{M_Z^2}{M_{A'}^2} \Delta^{KK}_{e\mu} \end{aligned}}$$

we will assume $M_{A'} = M_{Z'} = x_1/R'$ ie ignore small splitting from EWSB

FOR THE $\mu \to e$ AMPLITUDE WE WILL WRITE THE COUPLING AS : (this defines Q_N^γ)

$$e_{SM}\, \tilde{Q}_N^\gamma = \frac{g}{\cos\theta_W} Q_N^\gamma \quad \Leftarrow \left[Q_N^\gamma \equiv s_W c_W\, \tilde{Q}_N^\gamma \right]$$

$\tilde{Q}_N^\gamma = v_\gamma^u (2Z+N) + v_\gamma^d (2N+Z)$, ELECTRIC CHARGE OF NUCLEUS

SIMILARLY : $e_{SM}\, Q_\ell = \dfrac{g}{\cos\theta_W}\, g_L^\gamma \quad \leftarrow g^\gamma_{L,R} = s_W c_W (+1)$ note: convention for lepton charge fixed by convention for, eg, g_L

NOW IT IS EASY TO MATCH COEFFICIENTS :

this term converts weak charge to electric

$$\boxed{(v \pm a) = 2 g_{L,R} \left[\Delta^{(0)}_{e\mu} + g_{KK} \frac{M_Z^2}{M_{Z'}^2} \Delta^{KK}_{e\mu} \right] - 2 g^\gamma_{L,R}\, g_{KK} \frac{M_Z^2}{M_{Z'}^2} \frac{Q_N^\gamma}{Q_N} \Delta^{KK}_{e\mu}}$$

VII THE CUSTODIALLY PROTECTED MODEL

DETAILS OF THE CUSTODIALLY-PROTECTED RS1 MODEL CAN BE FOUND IN MONIKA'S THESIS & 0903.2415. WE WILL ONLY SUMMARIZE THE RELEVANT RESULTS.

- RS MODELS w/ BULK FIELDS SUFFER FROM A LARGE T-PARAMETER. ONE WAY TO SOLVE THIS IS TO EXPAND THE BULK GAUGE SYMMETRY TO $SU(3)_c \times SU(2)_L \times \underbrace{SU(2)_R}_{\to \text{ BREAKS TO } U(1)_Y \text{ ON UV BRANE}} \times U(1)_X$ (hep-ph/0308036, 0308038) ($U(1)_X$ nec. to get correct $U(1)_Y$ CHARGES)

- ONE CAN IMPOSE A DISCRETE $P_{LR}: SU(2)_L \leftrightarrow SU(2)_R$ SYMMETRY. THIS IS EQUIVALENT TO GAUGING CUSTODIAL SYMMETRY. THIS PROTECTS THE EXPERIMENTALLY-CONSTRAINING $Zb_L\bar{b}_L$ COUPLING (hep-ph/0605341) AND CAN BE USED TO PROTECT AGAINST TREE-LEVEL FCNCs.

  HOW THIS WORKS: $SU(2)_L \times SU(2)_R \to SU(2)_V \supset U(1)_V$, generated by $T^3_L \oplus T^3_R$. P_{LR} IMPOSES $T^3_L = T^3_R$ SO THAT THE EFFECT OF 'NEW PHYSICS' MUST OBEY $\delta Q^3_L = \delta Q^3_R$. ON THE OTHER HAND, $Q_V = Q^3_L + Q^3_R$ IS CONSERVED. THUS $\delta Q^3_L = -\delta Q^3_R \Rightarrow \delta Q^3_L = 0$, FROM THE BSM SECTOR. RECALL THE Z COUPLING IS $\propto [Q^3_L - Q_{EM} s_W^2]$. SINCE BOTH TERMS ARE CONSERVED, NEW PHYSICS CANNOT GIVE AN ANOMALOUS $Zb\bar{b}$ COUPLING.

- THE LOW ENERGY GAUGE EIGENSTATE SPECTRUM INCLUDES A $Z^{(0)}, Z^{(1)}, Z^{(1)}_X$ WHICH MIX INTO MASS EIGENSTATES Z, Z', Z_H. [note: previously we wrote $Z' = Z^{(1)}$.]

- THE Z & Z' FCNC COUPLING TO LH FERMIONS IS PROTECTED ($=0$), BUT THE RH COUPLING IS UNCONSTRAINED. THE LEADING LH FCNC COMES FROM THE $Z^{(1)}$ COMPONENT OF THE Z_H:

$$Z_H \approx \underbrace{\boxed{\cos\xi \; Z^{(1)}}}_{\text{NON-UNIVERSAL}} + \underbrace{\sin\xi \; Z_X^{(1)}}_{\substack{\text{no coupling to leptons} \\ \text{(no X charge)} \to \text{BUT COUPLES TO } q\text{'s}}} + \underbrace{\beta Z^{(0)}}_{\text{UNIVERSAL} \to \text{no FCNC}}$$

- OUR STRATEGY: INSTEAD OF THE MINIMAL MODEL ($f_L = f_R$), WE WILL TRY TO PUSH ALL THE FCNC INTO THE LH SECTOR WHERE CUSTODIAL PROTECTION TAKES CARE OF [MOST OF] IT. THIS MEANS PUSHING THE LH FERMIONS TOWARD THE IR BRANE & THE RH FERMIONS TO THE UV BRANE.

- P_{LR} IS BROKEN ON THE UV BRANE, BUT WE WILL IGNORE THIS SMALL EFFECT.

WE WILL HAVE TO TREAT THE LH & RH SECTORS SEPARATELY. THE LH SECTOR WILL HAVE FCNC ONLY FROM THE Z_H & $\gamma^{(1)}$. (IN PARTICULAR, ONLY THE $Z^{(1)} \subset Z_H$ GIVES LEPTON FLAVOR VIOL.) THE RH SECTOR WILL HAVE THE SAME FCNC STRUCTURE AS IN THE MINIMAL MODEL. WE WILL WANT TO MINIMIZE $Br(\mu\to e)$ OVER $f_{e_{L,R}}$ & $f_{\mu_{L,R}}$ VALUES SUBJECT TO THE SM MASS SPECTRUM

A NICE SHORTCUT: $Br(\mu\to e) \sim [A f_{e_L}^2 f_{\mu_L}^2 + B f_{e_R}^2 f_{\mu_R}^2]$

then use: $(a-b)^2 = a^2 - 2ab + b^2 \Rightarrow A+B \geq 2\sqrt{AB}$

$$\Rightarrow Br(\mu\to e) \gtrsim 2\sqrt{AB}\, f_{e_L} f_{e_R} f_{\mu_L} f_{\mu_R} = 2\sqrt{AB}\,\frac{m_e m_\mu}{Y_*^2 m_t^2}$$

ANARCHIC ASSUMPTION $f_R = \frac{m}{Y_* m_t f_L}$

SINCE $Br(\mu\to e)$ IS THE STRONGEST BOUND, WE WILL ONLY FOCUS ON THIS.

SOME USEFUL CONVERSIONS: LIMIT OF UNBROKEN P_{LR} $\Rightarrow \begin{cases} \cos\xi = \frac{1}{\sqrt{2}}\cos\phi \\ g' = g\sin\phi = g_x\cos\phi \end{cases}$

$$\Rightarrow \frac{g'}{g} = \tan\theta_W = \sin\phi$$

$$\Rightarrow \cos^2\xi = \tfrac{1}{2}\cos^2\phi = \tfrac{1}{2}(1-\sin^2\phi) = \tfrac{1}{2}(1-\tan^2\theta_W) = \frac{1}{2}\frac{c_W^2 - s_W^2}{c_W^2} = \frac{\frac{1}{2}-s_W^2}{c_W^2}$$

$\hookrightarrow \cos\xi \approx 0.60$

$$g_x = \frac{g'}{\cos\phi} = \frac{\tan\theta_W}{\cos\phi}\, g = \frac{\tan\theta_W}{\sqrt{2}\cos\xi}\, g$$

CUSTODIAL EFFECTIVE COUPLINGS FOR $\mu\to 3e$

$$\boxed{\begin{aligned}
g_3 &= 2g_R^2\left[\Delta^{R(0)}_{e\mu} + g_{KK}\frac{M_Z^2}{M_{Z'}^2}\Delta^{R\,KK}_{e\mu}\right] - 2(g_R^\gamma)^2 g_{KK}\frac{M_Z^2}{M_{\gamma'}^2}\Delta^{R\,KK}_{e\mu} \\
g_4 &= 2g_L^2 g_{KK}\frac{M_Z^2}{M_{Z'}^2}\Delta^{L\,KK}_{e\mu}\cos^2\xi - 2(g_L^\gamma)^2 g_{KK}\frac{M_Z^2}{M_{\gamma'}^2}\Delta^{L\,KK}_{e\mu} \\
g_5 &= 2g_L g_R\left[\Delta^{R(0)}_{e\mu} + g_{KK}\frac{M_Z^2}{M_{Z'}^2}\Delta^{R\,KK}_{e\mu}\right] - 2(g_L^\gamma)(g_R^\gamma) g_{KK}\frac{M_Z^2}{M_{\gamma'}^2}\Delta^{R\,KK}_{e\mu} \\
g_6 &= 2g_L g_R g_{KK}\frac{M_Z^2}{M_{Z'}^2}\Delta^{L\,KK}_{e\mu}\cos^2\xi - 2(g_L^\gamma)(g_R^\gamma) g_{KK}\frac{M_Z^2}{M_{\gamma'}^2}\Delta^{L\,KK}_{e\mu}
\end{aligned}}$$

WHERE $\Delta^{L,R}$ IS WRITTEN W/ $f^{L,R}$s ONLY.

CUSTODIAL EFFECTIVE COUPLINGS FOR $\mu \to e$

THIS REQUIRES MORE WORK

LEPTONS HAVE NO X CHARGE, ONLY COUPLES TO $Z^{(1)} \subset Z_H$

QUARKS COUPLE TO BOTH $Z^{(1)}$ & Z_X (Z COMPONENT IS NEGLIGIBLE)

$$Z_H = \cos\xi \, Z^{(1)} + \sin\xi \, Z_X^{(1)} + \beta Z^{(0)}$$

THE Z_X IS A NEW GAUGE BOSON. LET'S WORK OUT ITS COUPLINGS.
[see, eg. MONIKA'S THESIS]

CUSTODIAL MODEL HAS: $\begin{matrix} W^a_R & SU(2)_R \\ X & U(1)_X \end{matrix} \Rightarrow \begin{matrix} Z_X = \cos\phi \, W^3_R - \sin\phi \, X \\ B = \sin\phi \, W^3_R + \cos\phi \, X \end{matrix}$

BREAKING TO $U(1)_Y$

$P_{LR} \Rightarrow g^{SU(2)_L} = g^{SU(2)_R}$

ANALOGY: USUAL EWSB: $\begin{matrix} Z = c_W W^3_L - s_W B \\ A = s_W W^3_L + c_W B \end{matrix} \Bigg\} \; c_W = \dfrac{g}{\sqrt{g^2+g'^2}} \Rightarrow \cos\phi = \dfrac{g}{\sqrt{g^2+g_X^2}}$

$$\Rightarrow g^{Z_X ff} = g\cos\phi \, T^3_R - g_X \sin\phi \, T_X$$

FOR THE QUARKS

	$SU(2)_L$	$SU(2)_R$	$U(1)_X$
Q_L	$\square$	$\square$	2/3
u_R	1	1	2/3
d_ℓ	$\square\!\square\!\square$	1	2/3
d_r	1	$\square\!\square\!\square$	2/3

$\Big\} \; d_R = d_\ell \oplus d_r$

$$Q_L = \overbrace{\begin{pmatrix} \chi^u & q^u \\ \chi^d & q^d \end{pmatrix}}^{SU(2)_R}_{2/3} \Big\} SU(2)_L$$

$$d_R = \begin{pmatrix} \psi_\ell \\ u_\ell \\ d_\ell \end{pmatrix}_{2/3} \oplus (\psi_r \, u_r \, d_r)_{2/3}$$

BY CHOICE OF BC, THE ONLY FIELDS W/ ZERO MODES ARE: q^u, q^d, u_R, d_R

	q^u	q^d	u_R	d_R
T^3_R	-1/2	-1/2	0	-1
T_X	2/3	2/3	2/3	2/3

$$\Rightarrow g^{Z_X NN} = g\cos\phi\Big[\underset{\#u}{(2Z+N)}\underset{\Sigma_u T^3_R}{\left(-\tfrac{1}{2}\right)} + \underset{\#d}{(2N+Z)}\underset{\Sigma_d T^3_R}{\left(-\tfrac{3}{2}\right)}\Big] - g_X \sin\phi \, (3Z+3N)\left(\tfrac{2}{3}\right)\cdot \underset{LH+RH}{2}$$

$$\equiv \frac{g}{\cos\theta_W} Q_N^{Z_X} \; ; \qquad Q_N^{Z_X} \equiv \cos\theta_W\left[(2Z+N)\left(-\tfrac{1}{2}\right) + (2N+Z)\left(-\tfrac{3}{2}\right)\right] - \frac{g_X}{g}\sin\phi\cos\theta_W \cdot 4(Z+N)$$

$$\boxed{\begin{aligned} (V-a) &= 2g_R\left[\Delta^{R(0)}_{e\mu} + g_{KK}\frac{M_Z^2}{M_{Z'}^2}\Delta^{R\,KK}_{e\mu}\right] - 2g^\gamma_R g_{KK}\frac{M_Z^2}{M_{Z'}^2}\frac{Q_N^\gamma}{Q_N}\Delta^{R\,KK}_{e\mu} \\ (V+a) &= 2g_L g_{KK}\frac{M_Z^2}{M_{Z_H}^2}\Delta^{LKK}_{e\mu}\cos^2\xi + 2g_L g_{KK}\frac{M_Z^2}{M_{Z_H}^2}\frac{Q_N^{Z_X}}{Q_N}\Delta^{LKK}_{e\mu}\cos\xi\sin\xi - 2g^\gamma_L g_{KK}\frac{M_Z^2}{M_{\gamma'}^2}\frac{Q_N^\gamma}{Q_N}\Delta^{LKK}_{e\mu} \end{aligned}}$$

OLD NOTES: INTEGRATION OF $Z^{(1)}$ NON UNIVERSAL PIECE

NON-UNIVERSAL PART : change vars to $y \equiv z/R$

$$\underbrace{\frac{g_5^{2ff}\, f_c^2}{R' \sqrt{R \log R'/R}}} \cdot \frac{M_Z^2}{4} \int_R^{R'} dz \left(\frac{z}{R'}\right)^{-2c} z^2 \left(1 - 2\log\frac{z}{R}\right)$$

$$= B \cdot \frac{M_Z^2}{4} \left(\frac{R}{R'}\right)^{-2c} \int_1^{R'/R} R\, dy\; y^{-2c} (Ry)^2 (1 - 2\log y)$$

$$= B \frac{M_Z^2}{4} \left(\frac{R}{R'}\right)^{-2c} R^3 \underbrace{\int_1^{R'/R} dy\; y^{2-2c} (1 - 2\log y)}$$

$$\underbrace{\int_1^{R'/R} dy\; y^{2-2c}}_{= \frac{1}{3-2c}\left[y^{3-2c}\right]_1^{R'/R}} \; \underbrace{- 2\int_1^{R'/R} dy\; y^{2-2c} \log y}_{\equiv (\ddot\smile)}$$

$$= B \frac{M_Z^2}{4} \left(\frac{R}{R'}\right)^{-2c} R^3 \left\{ \frac{1}{3-2c}\left[y^{3-2c}\right]_1^{R'/R} + (\ddot\smile) \right\}$$

$$(\ddot\smile) = -2 \int_1^{R'/R} dy\; y^{2-2c} \log y$$

TRICK: INTEGRATE BY PARTS

$$\int dy\; y^a \log y = \frac{1}{a+1} y^{a+1} \log y - \int dy \frac{1}{a+1} y^a$$

$$(\ddot\smile) = -2\left[\frac{1}{3-2c} y^{3-2c} \log y\right]_1^{R'/R} + \underbrace{2\int dy\, \frac{1}{3-2c}\, y^{2-2c}}_{= \frac{2}{3-2c} \cdot \frac{1}{3-2c}\left[y^{3-2c}\right]_1^{R'/R}}$$

no more integrals. just algebra.

$$\left\{\frac{1}{3-2c}\left[y^{3-2c}\right]_1^{R'/R} + (\ddot\smile)\right\} = \left(1 + \frac{1}{3-2c}\right)\frac{1}{3-2c}\left[y^{3-2c}\right]_1^{R'/R} - 2\left[\frac{1}{3-2c} y^{3-2c} \log y\right]_1^{R'/R}$$

$$= \underbrace{\frac{5-2c}{3-2c} \cdot \frac{1}{3-2c}\left(\left(\frac{R'}{R}\right)^{3-2c} - 1\right)}_{\text{SUBLEADING! CAN DROP.}} - \underbrace{\frac{2}{3-2c}\left(\frac{R'}{R}\right)^{3-2c} \log\frac{R'}{R}}_{\text{LEADING TERM IN } R'/R}$$

APPENDIX (that's right, these notes have an appendix!)

NOW WE PROVE SOME USEFUL FACTS ABOUT THE ANARCHIC YUKAWA MATRICES.

IN THE C-BASIS, THE SM YUKAWAS LOOK LIKE

$$\begin{pmatrix} f_1 c_{11} f_1 & f_1 c_{12} f_2 & f_1 c_{13} f_3 \\ f_2 c_{21} f_1 & f_2 c_{22} f_2 & f_2 c_{23} f_3 \\ f_3 c_{31} f_1 & f_3 c_{32} f_2 & f_3 c_{33} f_3 \end{pmatrix}$$

WHERE ALL THE c_{ij} ARE $\mathcal{O}(1)$ (or $\mathcal{O}(Y_*)$) W/ NO HIERARCHIES. THE f's INTRODUCE THE OBSERVED MASS HIERARCHIES. TO MAKE THIS MANIFEST, LET US DEFINE

$$\begin{aligned} \delta_1^2 &= f_1/f_3 \qquad \text{s.t. } \delta_1 \sim \delta_2 \ll 1 \\ \delta_2 &= f_2/f_3 \end{aligned}$$

THEN THE YUKAWAS TAKE THE FORM

$$f_3^2 \begin{pmatrix} \delta_1^4 c_{11} & \delta_1^2\delta_2 c_{12} & \delta_1^2 c_{13} \\ \delta_2\delta_1^2 c_{21} & \delta_2^2 c_{22} & \delta_2 c_{23} \\ \delta_1^2 c_{31} & \delta_2 c_{32} & c_{33} \end{pmatrix}$$

CLAIM: UPON DIAGONALIZATION, $\lambda \sim \text{diag}(f_1^2, f_2^2, f_3^2)$
ie we get a realistic hierarchy from generic c_{ij}'s.
THIS IS IMPORTANT BECAUSE WE WOULD THEN KNOW THAT THE ROTATION MATRIX WILL BE SOMETHING WITH δs ON THE OFF-DIAGONAL ELEMENTS.

Pf/ USE PERTURBATION THEORY & THE HIERARCHIES IN THE δ's. THE EIGENVALUES ARE GIVEN BY SOLUTIONS TO

$$\det(\lambda - \lambda_i) = 0$$

$$= (1-\lambda_i)(\delta_2^2 - \lambda_i)(\delta_1^4 - \lambda_i) + \#\,\delta_1^4\delta_2^2 = 0$$

CONSIDER THE LARGEST EIGENVALUE, λ_3. WE MAY WRITE

$$(1-\lambda_3) = \frac{-\#\,\delta_1^4\delta_2^2}{(\delta_2^2-\lambda_3)(\delta_1^4-\lambda_3)} \longleftarrow \mathcal{O}(\delta^6)$$

SINCE λ_3 IS LARGEST EIG ($\lambda_3 \sim 1$)
THE DENOMINATOR IS $\mathcal{O}(1)$

RHS is $\mathcal{O}(\delta^6)$, $\Rightarrow$ λ_3 IS INDEED ~ 1.
LHS is $\mathcal{O}(1)$

FOR THE SMALLER EIGENVALUES, eg.

$$(\delta_2^2 - \lambda_2) = \frac{-\# \delta_1^4 \delta_2^2}{\underset{\uparrow\ \mathcal{O}(1)}{(1-\lambda_2)} \underset{\uparrow\ \mathcal{O}(\lambda_2) \sim \mathcal{O}(\delta^2)}{(\delta_1^4 - \lambda_2)}} \leftarrow \mathcal{O}(\delta^6)$$

again gives $\lambda_2 \sim \mathcal{O}(\delta^2)$.

THUS: $\hat{\lambda} \sim f_3^2 \begin{pmatrix} \delta_1^4 & & \\ & \delta_2^2 & \\ & & 1 \end{pmatrix} \sim \begin{pmatrix} f_1^2 & & \\ & f_2^2 & \\ & & f_3^2 \end{pmatrix}$

COROLLARY: IF $U^+ \lambda U = \hat{\lambda}$
THEN THE OFF-DIAG ELEMENTS OF U GO LIKE THESE δ's.

eg: $\begin{pmatrix} f_1 f_1 & f_1 f_2 \\ f_2 f_1 & f_2 f_2 \end{pmatrix} = f_1^2 \begin{pmatrix} 1 & \theta \\ \theta & \theta^2 \end{pmatrix}$

DIAG. VIA

$$\begin{pmatrix} 1 & \theta \\ -\theta & 1 \end{pmatrix} \begin{pmatrix} 1 & \theta \\ \theta & \theta^2 \end{pmatrix} \begin{pmatrix} 1 & -\theta \\ \theta & 1 \end{pmatrix} = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix} + \mathcal{O}(\theta^2)$$