Cutoff matching for 5D dipoles

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based on work with M. Blanke, C. Csáki, Y. Grossman, and B. Shakya

1 Leading behavior of charged Higgs dipole?

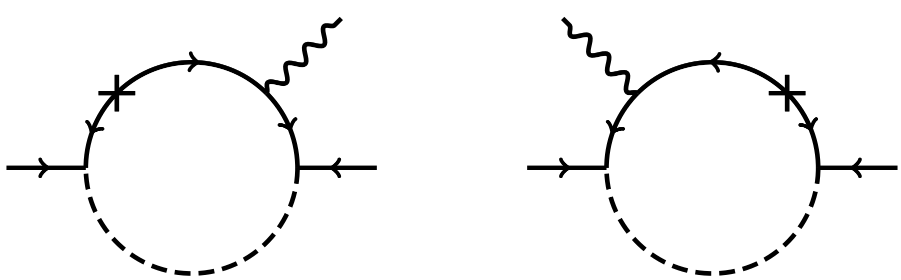
We agree upon the following statements (see, e.g., the dimensional analysi in Ref. [1] or the document NaivePowerCounting.pdf):

- The loop-level dipole operators in 5D models are finite and go like $1/M_{\text{KK}}^2$.
- The neutral Higgs and neutral Goldstone diagrams cancel so that these carry an additional suppression of $(m_h^2 - M_Z^2)/M_{\text{KK}}^2$.
- The diagram the photon emitted from a brane-localized charged Higgs is suppressed by a factor of M_W^2/M_{KK}^2 due to an algebraic cancellation in the gauge-invariant piece of the scalar propagators (applying the Ward identity)

What remains to be discussed is the power counting for the charged Higgs diagram where the photon is emitted from the internal fermion. Our claim is that the charged Higgs diagrams indeed go like $1/M_{\text{KK}}^2$, but we point out that a 4D calculation can misleadingly appear to imply a $1/M_{\text{KK}}^4$ dependence. We explain this effect below. Note also that this behavior was found explicitly through a 5D calculation in $b \to s\gamma$ in Ref. [2].

2 Calculation summary

In the document 4Dcalculation.pdf we present a calculation of the following diagrams[1]:



Note the following remarks about the calculation:

- We work in the gauge basis which is easiest to compare to a 5D calculation.

[1]Note that in the write up we refer to the fermions as μ and e, though technically these diagrams do not appear when the scalar is a charged Higgs.

- A trick to simplify the calculation is to identify the gauge invariant piece before integrating. This amounts to simplifying the Dirac structure as much as possible and identifying the coefficient of the $(p+p')^\mu$ term [3]. This is a manifestation of the Ward identity.

The result of the calculation is summarized on the last page of `4Dcalculation.pdf`, and is then fed into the *Mathematica* notebook `4Dcalculation_matching.nb`, where we explore the behavior of the $1/M_{\text{KK}}^2$ term.

3 When the 4D calculation is misleading & and how to get it to match the 5D calculation

Our claim is the following:

> Even though the loop diagram is manifestly finite, one should not take the loop integral to infinity. Rather, one should cutoff the loop at the EFT cutoff, the mass of the heaviest KK mode.

The heuristic reason for this is 5D Lorentz invariance: one wants the sum over KK modes and the 4D loop integral to integrate over spheres in [Euclidean] 5D momentum space.

3.1 What goes wrong

It seems strange that one needs to be careful with the finite loop cutoff. In particular, it seems sensible to do the loop calculation for a specific pair of KK modes—taking the cutoff to infinity—and then summing the result over the independent KK towers.

The reason why this fails is that by taking the cutoff Λ to infinity, one ends up dropping the leading order contribution, which goes like

$$\frac{1}{M_{\text{KK}}^2}\left[\left(\frac{n_f M_{\text{KK}}}{\Lambda}\right)^2 + \mathcal{O}\left(\frac{v^2}{M_{\text{KK}}^2}\right)\right],$$

where n_f is the highest KK number and $n_f M_{\text{KK}}$ is roughly the heaviest KK scale. Taking the cutoff to infinity right away kills the first term and leaves only the term that goes like v^2/M_{KK}^4. Trying to restore powers of M_{KK} in the numerator coming from summing over KK modes will not help since one would be applying this analysis to a sub-leading term—the leading term was removed by taking $\Lambda \to \infty$ too soon.

3.2 Is finite Λ consistent?

It may seem strange to require that Λ be finite (one can take $\Lambda \to \infty$ if one also takes $n_f \to \infty$). This is consistent with EFT: since the KK reduced theory breaks down after the scale at which we stop including KK modes, it is sensible truncate the loop at this scale.

What we observe here for this finite process is that taking the cutoff too far from the heaviest KK scale artificially kills the leading order term. It doesn't matter what Λ or n_f is—one could consider taking only a few KK modes—so long as Λ and $n_f M_{\text{KK}}$ are roughly matched. (This is demonstrated t the bottom of `4Dcalculation_matching.nb`.)

Ultimately this 'matching' is a reflection of 5D Lorentz invariance, which one expects to appear in the UV limit of the theory. One cannot consistently take large momentum space in the Minkowski directions without also including the high KK number states (oscillations in the extra dimension).

3.3 That sounds silly, part 1: I don't see the $(n_f M_{\mathbf{KK}}/\Lambda)^2$ term

One concern that was brought up in our discussions was that one doesn't necessarily see the $(n_f M_{\text{KK}}/\Lambda)^2$ when doing the 4D loop calculation. This is probably because one has assumed $\Lambda \to \infty$ from the beginning so that this term never had a chance to show up. In `4Dcalculation_matching.nb` we integrated the Wick-rotated amplitude and identify the leading order term by taking the $M_{\text{KK}} \to \infty$ limit of $(M_{\text{KK}}^2 \mathcal{M})$.

In terms of dimensionless numbers, the leading term goes like

$$-\frac{n_f^2 \lambda^2}{2(n_1^2 + n_f^2\lambda^2)(n_2^2 + n_f^2\lambda^2)}, \tag{1}$$

where n_f is the maximum KK number, $n_{1,2}$ are KK numbers to be summed over, and

$$\lambda = \frac{\Lambda}{n_f M_{\text{KK}}}$$

is a dimensionless parameter which quantifies the difference between the highest KK mode and the loop cutoff so that the crux of our claim is that it is important to keep $\lambda \sim 1$.

Observe that (1) does not behave like $(n_f M_{\text{KK}}/\Lambda)^2$ at low energies, i.e. for $\lambda \ll n_{1,2}$. The $(n_f M_{\text{KK}}/\Lambda)^2 \sim \lambda^{-2}$ behavior only manifests itself when $\lambda \gg n_{1,2}$, i.e. when $\Lambda \gg n_f M_{\text{KK}}$.

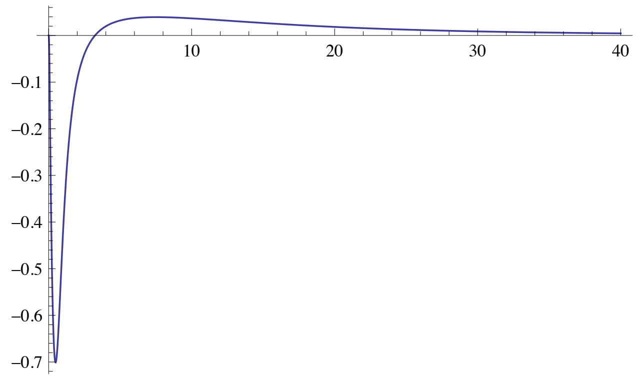
3.4 That sounds silly, part 2: the integrand changes sign?

One point that Lisa pointed out was that there's another reason why this story seems fishy. We are claiming that when you take the loop integral to infinity, the result is *smaller* than when you cut off the loop integral at a finite value $n_f M_{\text{KK}}$. Since

$$\int_0^\infty dk \text{ Int} = \int_0^{n_f M_{\text{KK}}} dk \text{ Int} + \int_{n_f M_{\text{KK}}}^\infty dk \text{ Int},$$

what we are saying is that the integral over $(n_f M_{\text{KK}}, \infty)$ contributes with a sign opposite that of the integral over $(0, n_f M_{\text{KK}})$. This may seem odd, but this is indeed the behavior demonstrated by the integrand as shown in `4Dcalculation_matching.nb`.

We reproduce a sample plot of the integrand as a function of the loop momentum below, where we take $n_f = 20$ and consider the behavior as the loop momentum goes beyond this:



One can see that the initial contribution is large and negative for small loop momentum, with some cancellation as the loop momentum approaches the cutoff $n_f M_{\text{KK}} = 20$. As one continues the loop integration beyond this cutoff, the positive contributions will asymptotically cancel the initial negative contribution from low momenta/light modes.

3.5 That sounds silly, part 3: UV sensitivity?

Later on in `4Dcalculation_matching.nb` we explicitly show that fixing $\lambda = 1$ (i.e. keeping the loop cutoff and KK cutoff matched) converges on a value for the integral. Note that the largest contributions to the loop are not necessarily coming from the heaviest states—this is not necessarily a UV sensitivity. In particular, one is in the ballpark of the full 5D integral value even with just a few KK modes so long as the cutoff is matched to the heaviest of these few modes.

Other questions of UV sensitivity (e.g. the strong coupling limit and NDA) are discussed in Appendix C of [2][2].

4 Conclusion

Our main conclusion is that the 4D calculation for the charged Higgs diagram appears to go like $1/M_{\text{KK}}^4$, but if one cuts off the finite loop at the scale where the KK theory breaks down ($n_f M_{\text{KK}}$), the result actually goes like $1/M_{\text{KK}}^2$). This is indeed the same scaling observed from a full 5D calculation, though this is perhaps tautologically so since this cutoff procedure maintains 5D Lorentz invariance.

5 Further topics

There are a few topics that were brought up in our conversation with Lisa that we do not address here.

[2]We acknowledge Kaustubh Agashe for prolonged discussions on this topic.

1. **External mass insertion diagrams**[3]. Lisa mentioned contributions from diagrams with an external mass insertion. We did not have a chance to talk about these contributions in much detail, but we included similar diagrams (where the mass insertion mixes the external zero mode with a KK mode) with gauge boson loops in Ref. [2].

2. **Validity of the 5D calculation**. In this discussion we have appealed to our 5D calculation as a consistency check[4]. Lisa mentioned the subtleties associated with 5D loops in curved space. The formalism we use for our loops, however, manifestly incorporates the 'effective brane' regularization presented in Ref. [4]. See Refs. [1] and [2] for some discussion.

References

[1] C. Csaki, Y. Grossman, P. Tanedo and Y. Tsai, Phys. Rev. D **83**, 073002 (2011) [arXiv:1004.2037 [hep-ph]].

[2] M. Blanke, B. Shakya, P. Tanedo and Y. Tsai, arXiv:1203.6650 [hep-ph].

[3] L. Lavoura, Eur. Phys. J. C **29**, 191 (2003) [hep-ph/0302221].

[4] L. Randall and M. D. Schwartz, JHEP **0111**, 003 (2001) [hep-th/0108114].

[3]We acknowledge Martin Beneke for discussions regarding these diagrams.

[4]Though originally the 4D calculation was a consistency check of our 5D calculation.