BIG PICTURE (for the younger students)

$\mathcal{M}(W'W' \to W'W') \sim s$

MASSIVE W, Z ⟶ BREAKDOWN OF PERTURBATIVE UNITARITY

⇒ LEADS US TO BELIEVE IN HIGGS-LIKE STATE (only consistent way to have g'm thy of massive gauge bosons)

NLΣM HAS AN NDA CUTOFF $\Lambda \approx 4\pi f$

HIGGS GIVES LONGITUDINAL POLARIZATIONS TO W, Z
THESE PGB STATES COME FROM A NLΣM
WHERE WE IGNORE THE MASSIVE STATE, h

NLΣM BREAKS DOWN IF WE EXTRAPOLATE TO HIGH E.
NEED h TO "UV COMPLETE" THE THEORY.
↳ LINEARIZE THE NLΣM

this is one of the important stories in HEP pheno
→ PION (? LIGHT MESON) χ, Xral P.T.
→ LITTLE HIGGS MODELS
→ DIMENSIONAL DECONSTRUCTION
→ models of strong coupling

NOW THAT WE HAVE A SCALAR HIGGS ⟶ QUADRATIC MASS DIVERGENCE
Hierarchy Problem

PROPOSED SOLUTIONS
- weak coupling → SUSY
- tuning → none
- strong coupling → XD / COMPOSITENESS (AdS/CFT) → MANY VARIANTS: RS, HIGGSLESS, TECHNICOLOR, GAUGE-HIGGS, LITTLE HIGGS, ...

XD → NON-RENORMALIZABLE. eg. GAUGE COUPLING HAS DIMENSIONS! NEEDS UV COMPLETION, WORRY ABOUT SENSITIVITY of PREDICTIONS TO UV PHYSICS.

→ string theory : GENERICALLY PREDICTS EXTRA STRUCTURE @ LOW E.

→ DECONSTRUCTION: 5^{th} DIM AS NLΣM IN WEIRD FOUR DIM SPACETIME.

<u>DECONSTRUCTION</u> → cf BIBHUSHAN'S fall '11 JOURNAL CLUB

- DISCRETIZE THE EXTRA DIMENSION
- TREAT LATTICE POINTS AS NODES IN "THEORY SPACE"
  - GAUGE GROUP G IN 5D → G ON EACH NODE
  - THEN CONNECT NODES IN A WAY WHICH MIMICS XD
    - ↳ NONLINEAR SIGMA MODEL ~~[illegible]~~ : $G^N \to G$
      PROVIDES LINK FIELDS OF DISCRETIZED 5D THY
      ALLOWS GAUGE FIELDS TO HOP BETWEEN NODES

SEEMS TRIVIAL: JUST A REINTERPRETATION OF A HARD MOMENTUM CUTOFF?

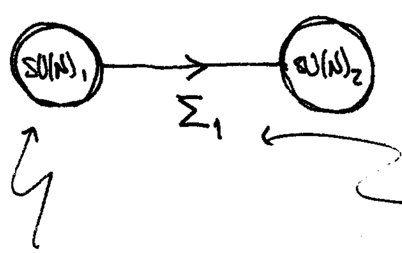
↳ yes, but: DECONSTRUCTED THY HAS CUTOFF $\Lambda \sim 4\pi f$
DUE TO — for eg — NEGLECTING RADIAL MODE
↳ ie EASY TO UV COMPLETE w/ LINEAR SIGMA MODEL

<u>FOR</u> DETAILS: hep-th/0104005, ARAM FALKOWSKI'S MSc THESIS

DESCRIPTION OF DECONSTRUCTED THEORY: "<u>MOOSE</u>" DIAGRAMS
↓
called "quiver" diagrams by string theorists

REVIEW ARTICLE: GEORGI LES HOUCHES 1985 LECTURES

MAIN IDEA:



ARROW: BIFUNDAMENTAL NONLINEAR FIELD

$$\Sigma_j = e^{i\pi^a_j(x)T^a_j/f_j} \to U_j^\dagger \Sigma_j U_{j+1}$$

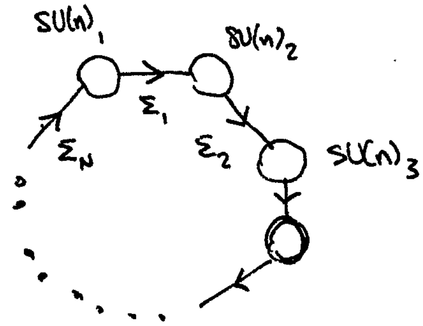
<u>NODE</u>: COPY OF GAUGE GROUP

vev: f s.t. $\langle \Sigma \rangle = \mathbb{1}$ (linear: $(f + r(x))\,\Sigma_j(x)$)

CAN DO FANCIER THINGS — gauge & global groups, write Σ as composite of fermions — WE'LL ONLY NEED THIS VERY BASIC PICTURE.

Jesse Thaler IS AN EXPERT AT DRAWING MORE SOPHISTICATED DIAGRAMS.
[COMPARE MAGNETIC HIGGS VS. PARTIALLY COMPOSITE MSSM]

CYCLIC N-MOOSE FOR SU(n)



MIMICS EXTRA DIMENSION COMPACTIFIED w/ TRIVIAL BC.

(ORBIFOLD: O—O— ⋯ —O)

remark: RS is adequately reproduced w/ 3-MOOSE ; warping from diff gauge couplings.

$$\mathcal{L} = \underbrace{-\frac{1}{2}\sum_{i=1}^{N} \mathrm{Tr}\, F_i^2}_{\text{GAUGE KIN.}} + \underbrace{\sum_{j=1}^{N} f_j^2 \mathrm{Tr}\, |D_\mu \Sigma_j|^2}_{\text{NLSM "KINETIC" TERM}}$$

odd normaliz.

(note: PULLED OUT f_j)

$$D_\mu \Sigma_j = \partial_\mu \Sigma_j \underbrace{- i g_j A_\mu^j \Sigma_j + i g_{j+1} \Sigma_j A_\mu^{j+1}}$$

NOTE SIGNS, ORDER of MULTIP! (as req. by indices of <u>different</u> gauge groups)

FOR A FLAT XD: ALL g_i DEGENERATE
" f_i "

GAUGE BOSON MASS MATRIX

$$M^2 = g^2 f^2 \begin{pmatrix} 2 & -1 & & & -1 \\ -1 & 2 & -1 & 0 & \\ & -1 & 2 & -1 & \\ & 0 & -1 & \ddots & \\ -1 & & & & 2 \end{pmatrix}$$

2's: $f_i^2 + f_{i+1}^2$
-1's: $-f_i^2$

<u>SOLUTION</u>: COUPLED H.O.

EIGENVECTOR w/ $\lambda = 0$: $A^{(0)} = \underbrace{(A_1 + A_2 \cdots)}/\sqrt{N}$

non local, but not the nonlocality that we'll care about

⇒ this is just the XD ZERO MODE, constant in the XD $(\cos(0x))$

5D INTERPRETATION

$$F_{MN}{}^a = \partial_{[M} A^a_{N]} + g_5 f^{abc} A^a_M A^c_N$$

(different from $F_{\mu\nu}$ & A_μ)

$$S_{5D} = -\frac{1}{2} \int d^5x \, \mathrm{Tr}\, F^a_{MN} F^{MNa}$$

DECONSTRUCTION DICTIONARY

LATTICE SPACE : $a = \frac{1}{gf}$

RADIUS OF XD : $R = Na = N/gf$

5D GAUGE COUPLING : $g_5 = \sqrt{\frac{g}{f}}$ ← $[g_5] = -1/2$ AS EXPECTED

4D KK PREDICTION : $g_4 = \frac{g}{\sqrt{N}}$ ← 4D EFF THY of 5D EFF THY of 4D DECONST.
g_4 g_5 g

gauge coupling of zero modes

from usual relation: $g_4^2 = g_5^2/R$

BUT ALSO NONTRIVIAL : CONSIDER $SU(n)^N \to SU(n)$ BREAKING
PIONS EATEN BY $(N-1)$ MASSIVE GAUGE BOSONS
UNBROKEN $SU(n) \subset SU(n)^N$ IS A LIN. COMB. of A_i
w/ EFF GAUGE COUPLING

$$\underbrace{\frac{1}{g^2_{SU(n)}}}_{\text{IDENTIFY w/ } g_4 \ \checkmark} = \sum_{i=1}^{N} \frac{1}{g_i^2}$$

OTHER NONTRIVIAL CHECK :

solving coupled HO

DECONSTRUCTED MASS^2 EIGENVALUES: $M_k^2 = 4g^2f^2 \sin^2(\pi k/N)$

KALUZA-KLEIN DECOMPOSITION : $M_{KK}^2 = 4\pi^2 k^2/R^2$

just large N/k limit of M_k^2

WHAT ABOUT SPECTRUM?

DECONSTRUCTION : 1 MASSLESS GAUGE BOSON
1 MASSLESS PION
(N-1) MASSIVE GAUGE BOSONS (VIA HIGGSING)
↳ = MASSLESS GAUGE BOSON EATING PION

5D THEORY : 1 MASSLESS 5D GAUGE BOSON

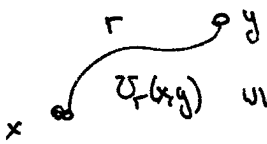
4D KK THY : 1 MASSLESS ZERO MODE GAUGE BOSON
(N-1) [TOWER] OF MASSIVE GAUGE BOSONS
↳ MASSLESS GAUGE BOSON EATING A^5
1 MASSLESS SCALAR ⟶ typically model-built away
EG BY ORBIFOLD B/C

~~WHAT ABOUT LAGRANGIAN?~~

~~4D PART WEEDS OUT~~

I'M A BIT CONFUSED BY THIS.
CAN ALSO INTERPRET THE MASSLESS PION IN THE DECONSTRUCTION AS A WILSON LOOP AROUND XD.

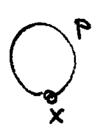
Remark: this is intuitive GEOMETRICALLY



$U_\Gamma(x,y)$ WILSON LINE

$$U_\Gamma(x,y) \to U(x)\, U_\Gamma(x,y)\, U(y)^\dagger$$

$$U_\Gamma(x,x+dx)\,\phi(x+dx) = \phi + D_\mu\phi(x)\,dx^\mu$$



$$\left[\ \rightsquigarrow\ U_P(x) = 1 + iF_{\mu\nu}(x)\,dx^\mu \wedge dx^\nu \right]$$

MEANING OF NLEM FIELDS

$$\Sigma_j(x) = P\, e^{-ig_5 \int_{(j-1)a}^{j} dy\, A_5(x,y)}$$

$$A_\mu^j(x) = \sqrt{a}\, A_\mu(x, aj)$$

NON LOCAL TERMS

<u>MAIN Q</u>: WHAT HAPPENS WHEN WE ADD OPERATORS IN DECONSTRUCTION THAT ARE <u>NON-LOCAL</u> WRT MOOSE (theory SPACE)

<u>IDEA</u>: ADD GAUGE INV'T NONLOCAL OP: $\varepsilon f^2 \mathcal{O}$ — which f?

ε SMALL, arb sign

~~[illegible]~~: THEN STUDY THE EFFECT ON THE CUTOFF $\Lambda \sim 4\pi f$

<u>INTERESTING RESULT</u>: $\Lambda_{\text{non-local}} \sim \Lambda_{\text{local}} (1 - \varepsilon^2)$

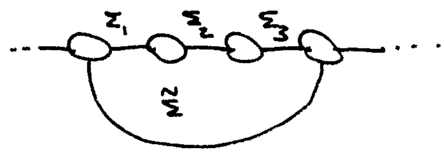
$\Rightarrow$ LOCALITY IN MOOSE SPACE $\leftrightarrow$ MAXIMIZING DECONSTR. CUTOFF.

<u>Disclaimer</u>: I DON'T CARE ABOUT THE DETAILS OF THE DERIVATION, BUT LET'S HIGHLIGHT THE STEPS.

<u>DEFINE A GOOD NONLOCAL OPERATOR</u>

<u>note</u>: nonlocal 5D → KK ops don't count → they're 5D local (eg. FCNC coupling of Z' in RS)

TRY:



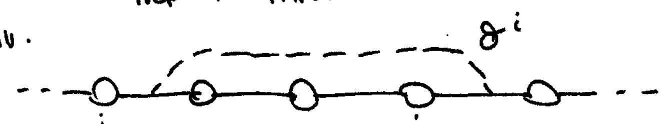
<u>BUT</u>: INTRODUCES NEW 4D DOF
PATHOLOGICAL LOCAL LIMIT: $\tilde{f} \to 0 \Rightarrow \Lambda \sim 4\pi f_? \to 0$

INSTEAD:

$$\mathcal{O}_h^{(i)} \equiv 2\,\mathrm{Tr}\left[(D_\mu \Sigma_i)(\Sigma_{i+1} \cdots \Sigma_j)(D^\mu \Sigma_i)^\dagger (\Sigma_{j-1}^\dagger \cdots \Sigma_j^\dagger) \right]$$

"N_{hop}" IN PAPER

by itself: breaks transl inv.



→ RESTORE TRANSLATION INVARIANCE IN MOOSE SPACE

$$\mathcal{O}_h \equiv \varepsilon f^2 \sum_{i=1}^{N} \mathcal{O}_h^{(i)}$$

CHARACTERISTIC NONLOCAL SCALE: $l \equiv ah$

INTERPRETATION IN 5D

$$\Theta_{NL} \to 2\varepsilon \int dy\, Tr\left[\underset{\sim D\Sigma}{\overset{(x,y)}{F_{\mu 5}}}\ \underset{P e^{-ig_5\int_y^{y+\ell} dy' A_5(x,y')}}{W(y,y+\ell)}\ \overset{(x,y+\ell)}{F^{\mu 5}}\ W^{\dagger}(y,y+\ell)\right]$$

NON-LOC. FIELD STRENGTHS CONNECTED BY WILSON LINES

OTHER OPS: NO SIMPLE INTERP of 5D OP
REDUNDANT w/ KIN TERM (after shifting fields)

WE RESTRICT TO 2 DERIVATIVES
- min req for nonlocality
- same Θ as $|D\Sigma|^2$ NLEM terms
- WANT TO STUDY EFFECT ON NL ON Λ
  NOT OTHER EFFECTS THAT ARE NON REN & AFFECT Λ DIRECTLY

<u>NEXT</u>: HOW TO DIAGNOSE EFFECT ON CUTOFF?

- MUST TAKE A RATIO of Λ TO ANOTHER SCALE

  ↳ CAN TRIVIALLY RESCALE EVERYTHING
  IN OTHER WORDS, THINGS APPEARING w/ Λ GO LIKE M/Λ
  (as Nic EMPHASIZED RECENTLY IN HIS TALK)

$$\mathcal{R} = \Lambda / \underset{\text{WHAT MASS TO USE?}}{\bar{m}}$$

<u>$\bar{m} = f$?</u>
- no well def continuum lim
- not well def in generic moose w/ $f_i \neq f_j$
- not well def in terms of phys obs!
  → usually 4π AMP, BUT NO LONGER UNAMBIG. W/ Θ_h

<u>MASS from M^2?</u> CAREFUL: $M^2 \sim g^2$
$g \to 0$ LIM $\Rightarrow$ $R \to \infty$ w/o ANY INPUT from Λ
SO WE MUST NORMALIZE BY g

↳ IN FACT, NORMALIZE BY g_4 (WELL DEF if $g_i \neq g_j$)

$\bar{m} = m_{\text{LIGHT}}$? ← LIGHTEST EIG. of M^2

↳ already "nonlocal" even in local thy

USE : $\bar{m} = \dfrac{\sqrt{\operatorname{Tr} M^2}}{N g_4}$ ← "average mass"

NICE CONTINUUM LIM:

$$\left.\begin{array}{l} f \to \alpha f \\ g \to \alpha g \\ N \to \alpha^2 N \end{array}\right\} \quad \begin{array}{l} g_5, R, g_4 \text{ PRESERVED} \\ \Lambda \to \alpha \Lambda \end{array}$$

⇒ this is a cutoff rescaling

WANT $\bar{m} \sim \alpha$ s.t. R is INDEP of α ~~TIMESCALES~~

PLUGGING IN : $\bar{m} = \dfrac{2fg\sqrt{N}}{Ng_4} \sim \alpha$ ✓

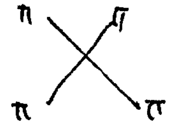
NOW: HOW TO STUDY EFFECT of Θ_{NL} ON Λ?

↗ UNITARITY (SCATTERING)

Disclaimer: this is where I didn't work out the details

↳ MAYBE WORTH DISCUSSING CONFUSING POINTS

SCATTERING: $\pi^a\pi^b \to \pi^c\pi^d$ ← GOLDSTONE EQUIVALENCE thm.



↳ analog of $W^LW^L \to W^LW^L$

turns out that strongest bound comes from <u>COUPLED CHANNEL</u> ANALYSIS of S-WAVE

eg. for EW thy (Lee, Quigg, Thacker 77)

$$W^LW^L \to \left.\begin{matrix} W^LW^L \\ Z^LZ^L \\ HH \end{matrix}\right\}$$ AMPLITUDE MATRIX

WANT LARGEST EIGENVALUE (LARGEST AMP) TO SET [perturbative] UNITARITY BOUND.

<u>Some formalism</u>

PARTIAL WAVE:

$$a^{(J)}_{\underbrace{\alpha\beta}_{\text{CHANNELS}}} = \frac{1}{32\pi}\int_{-1}^{1}\langle\alpha|T|\beta\rangle\, P_J(\cos\theta)\, d\cos\theta$$

$J=0$ IS LARGEST CHANNEL

I DON'T UNDERSTAND: S-WAVE UNITARITY $\Rightarrow$ $\boxed{\left|\mathrm{Re}\, a^{(0)}\right| < \frac{1}{2}}$

↳ SEE ph/0311177 eq (8)
ph/0612070 eq (6.1)

WE KNOW THAT FOR THE NNG-MODES (GOLDSTONES) $\lambda \sim s = s\lambda'(s)$

So UNITARITY GIVES: $|\lambda(s)| < 8\pi$

$$\Rightarrow \underset{\substack{\uparrow \\ S_{MAX}}}{\Lambda} = \sqrt{\frac{8\pi}{\lambda'_{max}}}$$

NOW ONE CAN ACTUALLY DO THE <u>COUPLED CHANNEL CALCULATION</u>. START W/ LOCAL THY.

WE WORK IN HI-E GOLDSTONE EQ. LIMIT, WORK IN GLOBAL SYM BASIS
not the GAUGE BOSON MASS BASIS.

↳ LOCAL INTERACTIONS IN MOOSE SPACE (WILL ADD $\mathcal{O}_L$ LATER)

$$\mathcal{L} > \frac{1}{6f^2} \sum_{j=1}^{N} \left[(\partial_\mu \pi^a) \pi^b (\partial^\mu \pi^c) \pi^d - (\partial_\mu \pi^a)(\partial^\mu \pi^b) \pi^c \pi^d \right]_j$$
$$\times \mathrm{Tr}(T^a T^b T^c T^d)$$

LINK FIELDS DECOUPLE

NICE EXPLICIT FORMULAE FOR SU(2) CASE

2 PARTICLE SU(2) PION SCATTERING: $\underline{3} \times \underline{3} = 1 + \underline{3} + \underline{5}$ (ISOSPIN)

$(I=1) \longrightarrow I = 0, \cancel{1}, 2$

antisym × bose sym = 0

"GAUGE SINGLET" STATE

$$|s\rangle = \sum_{a=1}^{3} \frac{1}{\sqrt{3}} \frac{1}{\sqrt{2}} |\pi^a \pi^a\rangle$$

$$\left.\begin{aligned} \lambda_{I=0} &= \frac{s}{4f^2} \\ \lambda_{I=2} &= \frac{-s}{8f^2} \end{aligned}\right\}$$

PLUGGING IN: $\lambda_{MAX} = \lambda_{I=2} \Rightarrow \Lambda_{LOCAL} = 4\sqrt{2\pi}\, f$

NON WOUDE NONLOCALITY

$\mathcal{O}_h$: CHANGES $\bar{m}$ & Λ FROM LOCAL THY
$\bar{m} \downarrow \bar{m}_{NL}$, $\Lambda \downarrow \Lambda_{NL}$

FOCUS ON LEADING BEHAVIOR IN $N_{hop} = N\ell/R$ w/ $N \to \infty$

RESULT : $\bar{m}$ NOT AFFECTED $\rightarrow$ ALL DEP IN Λ (AS WE WANT)

further, EIG VALS of SCATTERING MATRIX DO NOT CHANGE TO $\mathcal{O}(\varepsilon)$!
$\hookrightarrow$ WILL GIVE MAIN RESULT

VIGNETTES (not "details"):

$$\Delta M^2 = \begin{pmatrix} 0 & & & \\ & \ddots & & \\ & & \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} & \\ & & & \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \\ & & & & \ddots & 0 \end{pmatrix} \times \varepsilon g^2$$

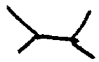
TRACE IS UNCHANGED $\rightarrow$ $\mathrm{Tr}\, M^2$ UNCHANGED IN $\bar{m}$ DEF. ✓

HOW TO CHECK EFFECT ON SCATTERING AMPLITUDES?
$\uparrow$
T MATRIX

DO USUAL QM PERTURBATION THEORY IN $\varepsilon f^2 \mathcal{O}_h$
$\hookrightarrow$ w/ "HAMILTONIAN" T
$\hookrightarrow$ GIVES λ IN EXPANSION IN ε

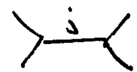
PUNCHLINE : $\lambda = \lambda^{(0)} + \underbrace{\varepsilon\lambda^{(1)}}_{=0!} + \varepsilon^2\lambda^{(2)}$ ← INDEP of $\mathrm{sgn}(\varepsilon)$ (s.t. tells us about EXTREMUM of Λ)

$\rightarrow$ HAS TO DO W/ TRACE-RELATED CANCELLATION of $I = 0$ STATE

SOMETHING TO UNDERSTAND BETTER: ORIGIN OF CANCELATION 

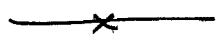
<u>SOME NOTES</u> : • Θ_h BREAKS $\pi \to -\pi$ sym → new 3-PT VERTICES (sounds like WZW term)

• $\Theta_h^{(i)}$ CONTAINS ALL Σ_j w $j \in [i, i+h]$

↳ MANY CHANNELS : 

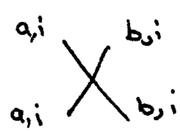
↳ DOMINANT ε^2 CONTRIBUTION

• Θ_h CHANGES KIN TERMS, MUST RE-CANONICALLY-NORMALIZE

↳  $Z^{-1/2}_{ij} = \delta_{ij} - \frac{1}{2}\varepsilon\left(\delta_{i,j+h} + \delta_{i,j-h}\right)\cdots$

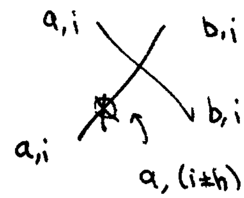
↑ treat field redef as mass ins.

<u>FIRST Θ SHIFT VANISHES</u>

LOCAL 4-PT TERMS :  EVEN IN π_i

FIELD REDEF TERMS ODD IN π_i

↳ no contrib. w/o Θ_{NL} INSERT.



Θ_{NL} INSERTION TERMS:

↳ claim: $\varepsilon(\partial\pi^a_i\, \pi^a_i)(\partial\pi^b_{i\pm h}\, \pi^b_{i\pm h})\, \mathrm{Tr}\left(T^a T^a T^b T^b\right)$

↳ ends up canceling w/ other term of opp sign eq (5.13) 2ND LINE

→ GEN PROPERTY of SU(N)

I WON'T SWEAT THE DETAILS ABOUT OBTAINING 2ND O TERM $\lambda^{(2)}$

<u>THE POINT</u>: $$\mathcal{R}_{NL} = \mathcal{R}_{loc}\left(1 - 4\varepsilon^2 N^2 \frac{l^2}{R^2}\right)$$

$\rightarrow$ MAX $\mathcal{R}$ (HIGHEST Λ) $\Leftrightarrow$ "LOCAL-EST" THY

Remark on PENGUIN REACTION.