SUSY in $d=3$, $\mathcal{N}=2$

REVIEW: Strassler th/0309149
ORIGINAL: Aharony et al. th/9703110 + de Boer th/9612131

Motivation :

- ODD DIMENSIONAL SPACETIMES ARE DIFFERENT → no anomalies, for example | ↗ simpler (?) playground for $d=5$, eg CHERN-SIMONS TERMS
- $d=4$ $\mathcal{N}=1$ ⟶ $d=3$ $\mathcal{N}=2$; so learning about 4D theories indirectly
- NICE PLAYGROUND FOR GENERAL IDEAS IN QFT
  ↳ duality, solitons, phase structure, ...
- CONDENSED MATTER SYSTEMS? (eg QUANTUM HALL FLUIDS? see Zee)

DIMENSIONAL REDUCTION FROM $d=4$, $\mathcal{N}=1$ THEORY

SUSY GENERATORS $= \mathcal{N}_4 \times (\underset{Q_\alpha}{2} + \underset{\bar{Q}^{\dot\alpha}}{2})$ ($\mathbb{C}$ Weyl) $= \underset{\text{Dirac (Majorana)}}{4}$

COMPACTIFY $\mathbb{R}^{3,1} \to \mathbb{R}^{2,1} \times S^1$

SUSY GENERATORS $= \mathcal{N}_3 \times (\underset{\text{MAJORANA SPINOR}}{2})$ $\Rightarrow$ $\boxed{\mathcal{N}_3 = 2}$

number of supercharges must match.

REMARK: $\mathcal{N}=1$, $d=3$ thy: no CHIRAL SUPERSPACE (only θ, no $\bar\theta$)
→ no HOLOMORPHY TO GIVE PROTECTION VS. QUANTUM CORR.

REMARK: WE WILL FOCUS ON THE $\mathbb{R}^{2,1}$ THY: IGNORE DETAILED EFFECTS of S^1
eg. KK MODES. STICK TO ZERO MODES, DETAILS of COMPACTIFICATION IRRELEVANT.

REMARK: $$\{Q_a, \tilde{Q}_b\} = 2\sigma^\mu{}_{ab} \underset{\mu=0,1,2}{P_\mu} + 2i\varepsilon_{ab} \underset{\mathbb{R}\text{ CENTRAL CHARGE } \sim P_3}{Z}$$

a 3d-PLET HAS A FIXED $Z = P_3$ (eg 0-MODE, 1st KK, ... etc).

DIMENSIONAL ANALYSIS IN $d=3$

$$S = \int d^dx\,(\partial\phi)^2 + \cdots \quad \to [\partial\phi] = d/2 \to \boxed{[\Phi] = d/2 - 1 = 1/2} \quad ①$$

$$\Delta\mathcal{L} = \int d^2\theta\, W[\Phi] + h.c.$$ $d=4$ $\mathcal{N}=1$ notation $\leftrightarrow$ $d=3$ $\mathcal{N}=2$

dim θ? RECALL $Q \sim \partial/\partial\theta$ & $\{Q, \tilde{Q}\} \sim P$

$\Rightarrow [\theta] = -[\partial/\partial\theta] = -1/2$

$\int d^2\theta\, \theta^2 = 1 \Rightarrow [d\theta] = -[\theta] = 1/2$

② $\Rightarrow \boxed{[W] = 2}$ more generally, $[W] = d-1$

①+②: MARGINAL OPERATORS GO LIKE $\Delta W \sim \Phi^4$ ← cf. $\Delta W = \Phi^3$ in 4d

so, eg, marginal Wess-Zumino superpotential in $d=4$ is relevant in $d=3$!

eg. $W = \hat{y}X^3$ w/ $[\hat{y}] = 1/2$

YUKAWA COUPLING HAS CLASSICAL SCALING DIMENSION - maybe we can study RG fixed points where the quantum contributions to β cancels the classical?

USUALLY this is HARD: HAVE TO GO TO STRONG COUPLING REGIME ... LOSE PERTURBATIVITY.

BUT: we have susy! NON-RENORMALIZ THM $\Rightarrow$ only anomalous dimensions contribute to quantum β function.

↳ USES HOLOMORPHY → θ AND $\bar{\theta} \sim \theta^\dagger$
No SUCH $\mathbb{C}$ SPINOR IN $d=3$ $\mathcal{N}=1$ → we're really relying on $\mathcal{N}=2$ reproducing HOLOM. STRUCTURE of $d=4$ $\mathcal{N}=1$.

recall strategy: only had to make arguments about direction of flow. COULD USE THMS ABOUT CONFORMAL FIXED POINTS.

eg. physical coupling (dimensionless) ω,

$$|\omega|^2 = \frac{\hat{y}^\dagger \hat{y}}{\mu Z_\Phi^3} \quad w/ \quad \beta_\omega = \omega\left[-\tfrac{1}{2} + \tfrac{3}{2}\gamma_\Phi(\omega)\right]$$

UNITARITY @ FP : $\gamma \geq 0$
plausible cancellation

Sign is negative : FP NOT POSSIBLE
IN $d>4$!

in fact, most examples of non-trivial QFT fixed points come from $d=3$ (or @ least <4)

↳ WILSON-FISHER ~~(NON-SUSY VERSION)~~ $d=4-\varepsilon$
$O(N)$ model in $d=3$ $\longrightarrow$ $1/N$ expansion to control loops

(in $d=4$: BANKS-ZAKS, SEIBERG FP, HIGHER $\mathcal{N}$ DUALITIES)

IN FACT, for SUSY theories:

FACT: @ SCFT FP: $\dim(\text{χral op}) = \frac{d-1}{2} R$ ①

FACT: UNITARITY $\Rightarrow$ $\dim(\text{GAUGE INV } \mathcal{O}) \geq \frac{1}{2}(d-2)$ ②

then for $W \sim \Phi^3$ ↑ th/9712074

	①		②	
$[\Phi]$	$= 1$	$d=4$	≥ 1	← equality ⟺ noninteracting
	$= 2/3$	$d=3$	$\geq 1/2$	
	$= 1/3$	$d=2$	≥ 0	

so (up to gauge thy dualities!), only expect nontrivial SUSY FP for $d<4$.

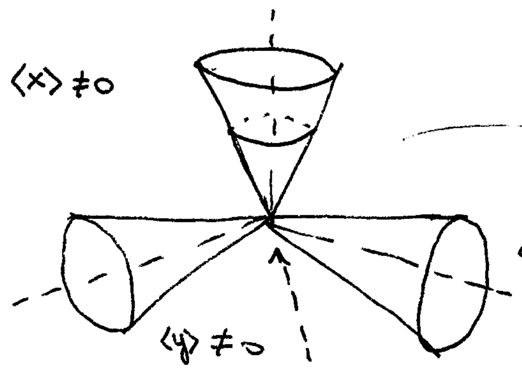
XYZ MODEL

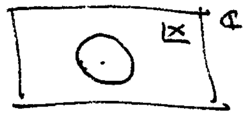
$$W = hXYZ \quad \rightarrow \quad V = |h|^2\left(|xy|^2 + |xz|^2 + |yz|^2\right) \geq 0$$

MODULI SPACE: when $V=0$, ie when any two scalars are zero.

← DRAW IN 3 $\mathbb{C}$ DIM SPACE

$\langle x\rangle \neq 0$





$\langle z\rangle \neq 0$

[$\mathbb{Z}_3$ sym.]

$\langle y\rangle \neq 0$

eg $z = r_z e^{i\theta_z}$ CONE

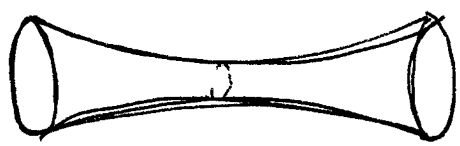
thy @ $\langle x\rangle = \langle y\rangle = \langle z\rangle = 0$ is a free thy. (CFT)

VORTEX SOLUTIONS

$$\Delta W = \xi X \quad \Rightarrow \quad \begin{aligned} W_X &= hYZ + \xi \\ W_Y &= hXZ \\ W_Z &= hXY \end{aligned} \quad \& \quad \begin{aligned} &X = 0 \\ &YZ = \xi/h \end{aligned}$$

← hyperbola ($\mathbb{C}$)

CONVERTS MODULI SPACE:



~~PROPOSE~~ CLASSICAL ~~STABLE~~ STATIONARY SOLUTION:

$$\left[\begin{aligned} Y &= \sqrt{\tfrac{\xi}{h}}\, f(r)\, e^{i\theta} \\ Z &= \sqrt{\tfrac{\xi}{h}}\, f(r)\, e^{-i\theta} \end{aligned}\right.$$

$f(r\to 0) = 0$
$f(r\to\infty) = 1$

↳ vortex solution
Y & Z wind around moduli space in opp. directions

cf KOSTERLITZ-THOULESS TRANSITION @ finite T get gas of vortex-antivortex pairs

REMARKS: single vortex sol. is log divergent (DERRICK'S THM)

but vortex-antivortex solution has finite energy & in fact behaves like e^+e^- in QED_3 ... log confined.

Vortex in $d=4$:

extend into a ring.

q: topo stability?

@ QUANTUM LEVEL

(skip)

write $\eta = h/\sqrt{\mu}$ s.t. $W = \sqrt{\mu}\,\eta XYZ$

$$\beta_\eta = \tfrac{1}{2}\eta\,[\,-1 + \gamma_x(\eta) + \gamma_y(\eta) + \gamma_z(\eta)\,]$$

classical

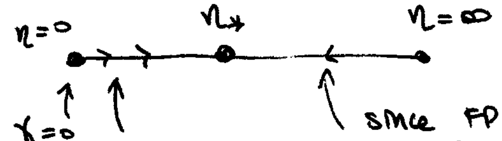
same coefficient by symmetry of $x \leftrightarrow y \leftrightarrow z$

fixed point if $\gamma(\eta_*) = 1/3$.

PLAUSIBILITY of SUCH A FIXED POINT BY APPEALING TO WESS-ZUMINO

↳ $W = \eta X^3$

$\beta = \tfrac{1}{2}\eta[-1 + 3\gamma]$



dom by classical flow

since FP is attractive on this axis (expect $\gamma > 1/3$ if $\eta > \eta_*$)

now let's look at a different 3D susy theory ... will be related

<u>$SQED_3$ $\mathcal{N}=2$</u>

now it pays to really think about compactified $d=4$ $\mathcal{N}=1$ thy.

$$A_\mu \quad 4\ \mathbb{R}\ \text{dof} \longrightarrow A_i + (\varphi = A_3) \leftarrow \mathbb{R}\ \text{SCALAR}$$

$$\lambda \quad 2\ \mathbb{C}\ \text{dof} \longrightarrow \lambda_1 + \lambda_2$$

(Majorana)

4 comp $\mathbb{R}$ maj SPINOR

2x (2 comp $\mathbb{R}$ maj spinor)

b/c $\mathcal{N}=2$:

$$A_i \xrightarrow{Q} \lambda_1 \xrightarrow{\tilde{Q}} \varphi \qquad A_i \xrightarrow{\tilde{Q}} \lambda_2 \xrightarrow{Q} \varphi$$

note: MODULI SPACE : $V[\varphi]=0$

BACKGROUND GAUGE FIELD IN $d=4$ PICTURE
CAN SAY WORDS LIKE FLUX.

looks like moduli space is $\mathbb{R}$. (classically, @ least)
KIND OF WEIRD ... DON'T EXPECT $\mathbb{R}$ MODULI SP IN 4D SUSY.

indeed: turns out that the <u>QUANTUM</u> MODULI SPACE IS DIFFERENT.

[... remark: from XYZ model: moduli space is important! can find very nontrivial behavior!]

Are there other directions in moduli space? (*) ~~TBD~~ SEE NEXT PAGE

$\updownarrow$

Are there other scalars in the theory?

yes.

APPARENT CLASSICAL MODULI SPACE

$\hookrightarrow \langle \varphi \rangle$ AND $\langle M \rangle = \langle Q \tilde{Q} \rangle$

vev of gauge-invariant operator formed out of selectrons

(LUTY-TAYLOR THEOREM)

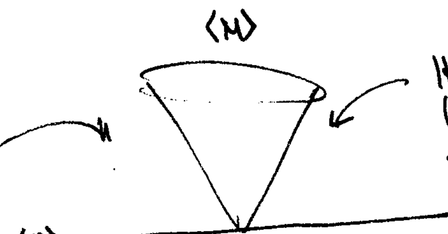
$$\underbrace{|D_3 Q|^2 + |D_3 \tilde{Q}|^2}_{\text{from 4D Kähler pot}} \xrightarrow{\text{RED.}} \boxed{\varphi^2 \left(|Q|^2 + |\tilde{Q}|^2\right)}$$

A_3

so moduli space:

either: $\langle \varphi \rangle \neq 0$ & $\langle Q \rangle, \langle \tilde{Q} \rangle = 0$
or: $\langle \varphi \rangle = 0$ & $\langle Q \tilde{Q} \rangle \neq 0$

DRAW:



HIGGS BRANCH
(breaks GAUGE SYM)
cf WINTERCAMP DISCUSSION

COULOMB BRANCH
SINCE U(1), φ is UNCHARGED
$\hookrightarrow$ does not break U(1)

same nomenclature as SQCD

REMARK: D-TERM CONDITION: $|Q|^2 - |\tilde{Q}|^2 = 0$
(from GAUGE invariance)
[which also gave $e^{i\theta} Q$, $e^{-i\theta} \tilde{Q}$ redundancy

$\rightarrow$ for MODULI SPACE: αQ, $\alpha \tilde{Q}$ 'GAUGE' REDUNDANCY
$\hookrightarrow$ complexified gauge redundancy
$\rightsquigarrow$ D flatness condition is an artifact of WZ gauge

See: th/9506098

2 # Xral-PLETS, BUT only need 1 TO DESCRIBE MODULI SPACE (M)
HIGGS: 1 XSF EATEN TO GIVE γ MASS

RECALL IN 4D: EM DUALITY $F \longleftrightarrow *F = \tilde{F}$

in 4D: 2-form $\mapsto$ 2-form, where we can see $\vec{E} \leftrightarrow \vec{B}$ (up to $\pm$)

in 3D: $*F$ is A 1-FORM.

IN THE ABSENCE OF CHARGES $\left\{ \begin{array}{l} dF = 0 \\ d{*F} = 0 \end{array} \right.$ BIANCHI $\Rightarrow$ maxwell eom

$\hookrightarrow$ $F = dA$; ~~$\tilde{F} = d\tilde{A}$, where $\tilde{A}$ a scalar in 3D~~

~~call it σ: $\tilde{A} \to \sigma$~~

$\tilde{F} = d\sigma$ $\boxed{\text{scalar}}$

aha! found a new scalar in the theory!

$\searrow$ does this make sense? why did we have to go to dual theory to find it?

IT WAS ALREADY THERE IN $\vec{A} = (A_0, A_1, A_2)$

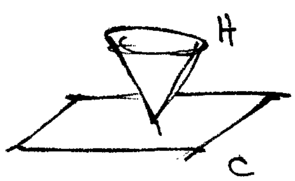
$\hookrightarrow$ 3 components (dof)
-1 MASSLESS CONDITION (no longitudinal mode)
-1 GAUGE REDUNDANCY (eg. Ward Identity)

$= 1$ dof. $\Rightarrow$ A_μ^{3D} is EFFECTIVELY A SCALAR dof

actual identification: $A_\mu = \partial_\mu \sigma$

FURTHER, NOTE $\sigma \to \sigma + c$ SHIFT SYMMETRY.
BROKEN IF $\langle \sigma \rangle \neq 0$ $\Rightarrow$ $\partial_\mu \sigma$ IS GOLDSTONE MODE

So SQED$_3$ HAS MODULI SPACE SPANNED BY $\varphi, \sigma \sim \mathbb{R}^2$?

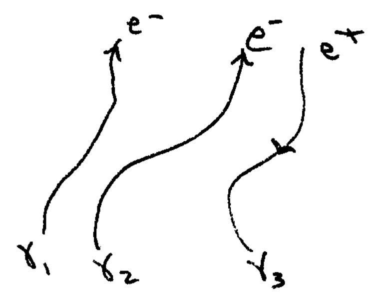


ACTUALLY, MODULI SPACE HAS A MORE NONTRIVIAL TOPOLOGY.

EOM: $d*F = j$ in the presence of electric sources

↑ CANNOT WRITE $\tilde{F} = d\sigma$ GLOBALLY

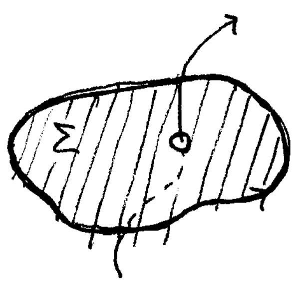
but the currents are just the worldlines of the electrons.



THESE ARE δ-functions in space.

$$j = \sum_i 2\pi q_i \, \delta^{(2)}(\vec{\gamma}_i - \vec{x}) \, dx^i \wedge dx^j$$

SO ANYWHERE AWAY FROM WORLDLINE, $\tilde{F} = d\sigma$ (by Poincaré)



$$d\tilde{F} = j$$

$$\int_\Sigma d\tilde{F} = \underbrace{\int_\Sigma j}$$

$q_i = \pm e$ (quantized)

$$= \sum_{\gamma_i \cap \Sigma} 2\pi q_i \in 2\pi e \times \mathbb{Z}$$

$$= \int_{\partial\Sigma} \tilde{F}$$

$$= \int_{\partial\Sigma} d\sigma$$ SINCE $d\tilde{F} = 0$ ALONG $\partial\Sigma$ (away from support of j)

$\sim \sigma(\theta=0) - \sigma(\theta=2\pi)$ ← should be zero but is only zero mod

$\Rightarrow$ $\boxed{\sigma \text{ is ONLY DEFINED mod } 2\pi}$

"In the presence of electric charges, holonomy of $\tilde{F}$ is U(1)-valued"

↪ magnetic dual is U(1) gauge thy, not $\mathbb{R}$

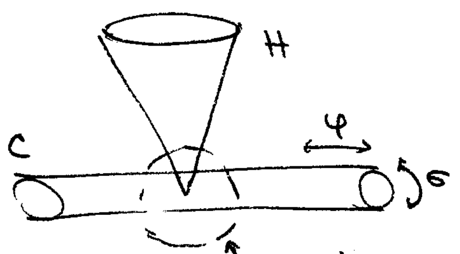
<u>MODULI SPACE</u>: $\underset{\varphi}{\mathbb{R}} \times \underset{\sigma}{U(1)}$ = CYLINDER

RADIUS of CYLINDER: $2\pi e$

$\subset [e] = \frac{1}{2}$ (eg $[\partial] = [eA] = [e\varphi]$ & $[\varphi] = \frac{1}{2}$)

Then when e is LARGE ($\leftrightarrow$ IR LIMIT), RADIUS $\to \infty$
MODULI SPACE $\to \mathbb{C}$
accidental SO(2) sym.

When e is SMALL ($\leftrightarrow$ UV), RADIUS $\to 0$
MODULI SPACE $\to \mathbb{R}$



To UNDERSTAND THE finite e BEHAVIOR (effect of σ), NEED TO UNDERSTAND RG OF e.

So we CALCULATE:

← not

(corrects mag. moment, but not e^2)

$$\mathcal{M} \sim \int d^3k \, \frac{1}{\not{k}} \gamma \frac{1}{\not{k}} \gamma \sim \Lambda$$ ... too naive.

PROTECTION AGAINST DIVERGENCE:
1. Gauge inv. (Ward)
2. Lorentz inv.

remark: same as $\mu \to e\gamma$ in $d=5$

fermion

$$\mathcal{M} \sim \bar{F} \int d^3k_E \frac{1}{(k^2+\Delta)^2} \Big[\underbrace{\cancel{\not{k}\gamma\not{k}\gamma}}_{\text{WARD}} + \underbrace{\cancel{\not{k}\gamma\not{p}\gamma}}_{\text{LORENTZ}} + \underbrace{\not{p}\gamma\not{p}\gamma}_{\text{finite!}} \Big]$$

$$\sim \frac{-F\, d^3k}{k^4} = \boxed{\frac{-e}{\mu}}$$ ← sign doesn't matter. we'll fix it later.

END UP w/ $$\frac{1}{e^2(\mu)} = \frac{1}{e_0^2} + c\frac{F}{\mu}$$

gives quantum running of $1/e^2$

SKIP

BUT WE WORK w/ <u>DIMENSIONLESS COUPLINGS</u> SO:

$$\xi \equiv \frac{e^2(\mu)}{\mu} = \frac{1}{\frac{\mu}{e_0^2} + cF}$$

gives classical scaling

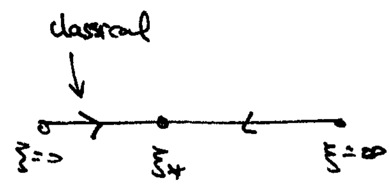
$$\beta_\xi = \mu\frac{d\xi}{d\mu} = \mu\frac{1/e_0^2}{\left(\frac{\mu}{e_0^2} + cF\right)^2} =$$

$$= \frac{\mu}{e_0^2}\xi^2 \leftarrow \frac{\mu}{e_0^2} = \frac{1}{\xi} - cF$$

$$= \xi - \xi^2 cF$$

$$= \xi\,\underbrace{(1 - cF\xi)}$$

FIXED POINT @ $\xi_* = 1/cF$
weakly coupled for large F



~~WHAT'S GOING ON HERE?~~

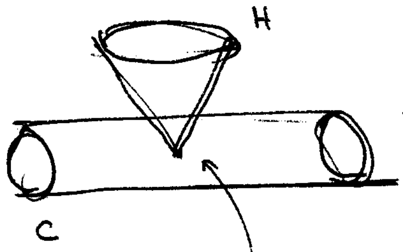
<u>BUT ACTUALLY</u>, WE'RE MISSING SOME PHYSICS

mass of ~~[illegible]~~ SELECTRONS set by

$$\varphi^2\left(|Q|^2+|\tilde{Q}|^2\right) + \varphi\left(\psi\bar{\psi} + \tilde{\psi}\bar{\tilde{\psi}}\right)$$

ie RG STOPS @ $r = \langle\varphi\rangle$

$$\Rightarrow \boxed{\frac{1}{e^2(0)} = \frac{1}{e_0^2} + \frac{1}{\langle\varphi\rangle}}$$

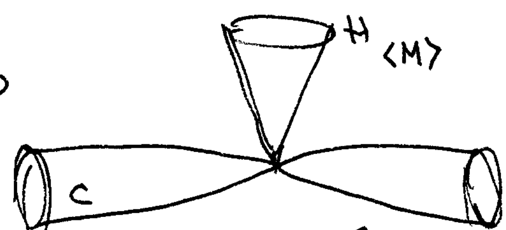


SO FOR LARGE $\langle\varphi\rangle$, SIZE OF CYLINDER GIVEN BY e_0

(my dimensions are wrong above ... $R \sim e_0^2$)

HERE, $\langle\varphi\rangle$ IS SMALL SO $1/\langle\varphi\rangle$ DOMINATES AND $\boxed{R \sim \langle\varphi\rangle}$

$\Rightarrow$



this looks like the moduli space of XYZ!

(in IR)

$$\left\langle e^{\pm(\varphi + i\sigma)}\right\rangle$$

note: $\mathbb{Z}_3$ sym of XYZ $\to$ ACCIDENTAL (IR) $\mathbb{Z}_3$ in $SQED_3$

CENTER IS CFT $\to$ same IR FP AS XYZ

XYZ (chiral)

X, Y, Z moduli

$\mathbb{Z}_3$ sym (exact)

$\langle X \rangle \neq 0$ BRANCH
$Y + Z$ massive **particles**

← PARTICLE - VORTEX DUALITY →

$\mathcal{N}=2$ SQED$_3$ (gauge)

M, $e^{\pm(\varphi + i\sigma)}$ moduli

$\mathbb{Z}_3$ accidental

$\langle M \rangle \neq 0$, HIGGS BRANCH
$M \sim Q\tilde{Q}$, ~~SOLITONIC~~ **VORTEX** SOLUTIONS WHERE $Q, \tilde{Q}$ PHASES WIND IN OPPOSITE DIRECTIONS ABOUT $|\langle M \rangle| = $ const.

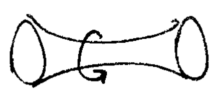
"duality" vortex of a vortex

$\Delta W = \xi X$

← VORTEX - PARTICLE DUALITY →

$\Delta W = M Q\tilde{Q}$

MASSIVE PARTICLES $Q, \tilde{Q}$
THESE ARE LOG CONFINED
eg. $V \sim \int d^{d-1}k \, e^{ikx} \frac{1}{k^2} \sim r^{d-3}$



Y, Z VORTICES

XYZ → IR ← SQED$_3$

REMARK :

$\mathcal{N}=2$ CFT

$\Delta W = \Phi Q\tilde{Q} \sqrt{2}$
DESTABILIZES FP → flows to $\mathcal{N}=4$ CFT
($\mathcal{N}=2$ in $d=4$)

EXACT

~~REMARK~~

$\Delta W = \Phi X$
mass term decouples Φ, X

leaves thy of Y, Z w/ $W=0$
→ **FREE**

← $\mathcal{N}=4$ SQED

"MIRROR" SYMMETRY