CROUCHING SEIBERG, HIDDEN GAUGE GROUP

PAPER: Komargodski, "Vector Mesons & an Interpretation of Seiberg Duality"
arXiv: 1010.4105

USEFUL REFERENCES FOR BACKGROUND READING

CHIRAL PERTURBATION THEORY (χPT) (aka NLΣM)
- CSABA'S P7646 LECTURES
- GEORGI *WEAK INTERACTIONS*

HIDDEN GAUGE GROUP IN THE CHIRAL LAGRANGIAN
- BIRSE hep-ph/9603251

SEIBERG DUALITY
- INTRILIGATOR & SEIBERG hep-th/9509066

THIS TALK: focus PRIMARILY ON THE EMERGENCE + CONSEQUENCES OF THE HIDDEN GAUGE GROUP IN A LOW ENERGY EFT. SEIBERG DUALITY ONLY AS A MOTIVATION & APPLICATION.

BROAD MOTIVATION:

"FUCKING MAGNETS, HOW DO THEY WORK?"

↳ 2010 INTERNET MEME, see http://knowyourmeme.com
based on "Miracles", song by Insane Clown Posse (2009)

CLEARLY Insane Clown Posse is REFERRING TO THE STRUCTURE OF THE SEIBERG MAGNETIC PHASE IN SUSY QCD (SQCD)

ELECTRIC

	SU(N)	SU(F)	SU(F)
Q	$\square$	$\square$	1
$\tilde{Q}$	$\bar{\square}$	1	$\square$

ASYMP. FREE
BUT STRONGLY COUPLED IN THE IR

$\Updownarrow$

MAGNETIC

	SU(F-N)	SU(F)	SU(F)
M	1	$\square$	$\square$
q	$\square$	$\bar{\square}$	1
$\tilde{q}$	$\bar{\square}$	1	$\bar{\square}$

IR FREE
BUT LANDAU POLE IN UV

$W = \frac{1}{\mu} M q \tilde{q}$

IN WORDS, this is just the EFT paradigm in action

"FUNDAMENTAL" THEORY BECOMES STRONGLY COUPLED.
THIS IS NOT A PROBLEM W/ THE THEORY, IT JUST MEANS THAT OUR PERTURBATIVE TECHNIQUES FAIL.

BUT: "STRONGLY COUPLED NONABELIAN GAUGE THEORY" USUALLY MEANS CONFINEMENT. THE STRONG COUPLING MAKES PARTICLES (which are charged under this group) WANT TO STICK TOGETHER.

AT ENERGIES WELL BELOW THE CONFINEMENT SCALE Λ, IT IS HARD TO EXCITE QUANTA OF THESE "FUNDAMENTAL" FIELDS ; all that is left are the "stuck together" composite degrees of freedom.

↳ MORE PRECISELY: @ VERY LOW ENERGIES ALL THAT IS LEFT ARE THE GOLDSTONE PARTICLES FROM THE BREAKING OF GLOBAL SYMMETRIES (eg by the STRONGLY COUPLED SECTOR)

SO FOR $E \ll \Lambda$, WE EXPECT TO BE ABLE TO WRITE AN EFFECTIVE THEORY OF THESE LOW ENERGY DOF.

↳ IN THE "BOTTOM-UP" SENSE, NOT "TOP-DOWN" SENSE.

TOP-DOWN ↓ integrate out known (or assumed) high scale physics

BOTTOM-UP ↓ just say: "this is the shit that I have @ low energies ... can I write a Lagrangian?"

EXPECTED PROPERTIES OF SUCH AN EFT

- NO REMNANT OF SU(N) GAUGE THY
- PERHAPS SOME LEFTOVER FLAVOR SYMMETRIES ("flavor" = "global")
- PARTICLES ARE ALL LIGHT: massless up to sym breaking effects (pGB)
  - ↳ experimentally ∃ tower of resonances, BUT EFT valid only for light stuff
    - these are KK modes in the holographic picture
- NONLINEAR REALIZATION (NLSM)
- ⋮ (other properties... eg see Dean's talk on compositeness)

★ ALL FIELDS ARE GAUGE SINGLETS, UP TO WEAKLY COUPLED GAUGE GROUPS THAT EXISTED EVEN IN THE UV THEORY.

CERTAINLY THESE ARE ALL SATISFIED IN QCD. (? COMPOSITE HIGGS, I THINK)
[tautologically — QCD is where we built this intuition!]

THE LOW ENERGY EFFECTIVE THEORY IS CALLED χPT (OR THE NLΣM)

IDEA: BELOW Λ_{QCD}, QCD IS ESSENTIALLY A THEORY OF PIONS. WE NOW KNOW (from YUVAL) THAT PIONS ARE VERY WELL DESCRIBED BY $SU(3)_F$.

this is really the $SU(3)_V \subset SU(3)_L \times SU(3)_R$ which is preserved by the QCD vacuum.

SO: $SU(3)_A \subset SU(3)_L \times SU(3)_R$ IS BROKEN
EXPECT LIGHT (pseudo) GOLDSTONE BOSONS $\longrightarrow$ PIONS!

PROPERTIES:
- $q\bar{q}$ PAIR (b/c $SU(3)_A$ BROKEN BY $\langle q_L \bar{q}_R \rangle$)
- PSEUDOSCALAR, diff sign for LH vs. RH transf.
- OCTET UNDER $SU(3)_V = SU(3)_F$

$q\bar{q} = (3 \otimes \bar{3})_V \oplus (3 \otimes \bar{3})_A$; $3 \otimes \bar{3} = 8 \oplus 1$

HOW TO WRITE A THEORY OF GOLDSTONE BOSONS (NLΣM); $G \to H$

1. PICK A VACUUM STATE ϕ_0
2. TRANSFORM IT BY AN ELEMENT $g \in G/H$ (a broken generator)
3. PROMOTE THE TRANSFORMATION TO A FIELD
4. CALL IT A GOLDSTONE BOSON.

(cf THE $H^\pm$, H^0 IN THE SM)

IN QCD: $SU(3)_L \times SU(3)_R \to SU(3)_V$

PARAMETERIZE THIS BY $U = (3, \bar{3})$ OF $SU(3)_L \times SU(3)_R$

VACUUM:
$$\langle U(x) \rangle = f_\pi \cdot \mathbb{1}_{3\times3}$$

FOR AXIAL, $U_R^\dagger = U_L$

TRANSFORM:
$$U(x) \longrightarrow g_L\, U(x)\, g_R^\dagger \quad \text{w/ } g_{L,R} = e^{i\epsilon^a_{L,R} T^a}$$

PROMOTE:
$$U(x) = e^{\frac{i}{f_\pi}\pi^a T^a} f_\pi \mathbb{1} \cdot e^{\frac{i}{f_\pi}\pi^a T^a}$$
$$= \boxed{e^{\frac{2i}{f_\pi}\pi^a(x) T^a} \cdot f_\pi \mathbb{1}}$$

THEN WE CAN GO ON TO WRITE OUT A LAGRANGIAN & WORK OUT ππ SCATTERING. WE'LL DO A LITTLE OF THIS ... IN JUST A MOMENT.

$U(x)$ IS CLEARLY NONLINEAR IN THE LOW ENERGY EFFECTIVE DOF, $\pi^a(x)$, THIS IS KIND OF UGLY ... HAVE TO EXPAND EXPONENTIAL. LET US BE CLEAR ABOUT HOW WE GOT HERE:

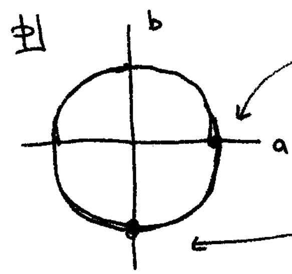
1. WE STARTED W/ THE BREAKING OF A GLOBAL SYMMETRY s.t. ∃ A NONTRIVIAL VACUUM MANIFOLD IN FIELD SPACE.

   PHYSICALLY WE KNOW THAT OUR GOLDSTONE FIELD(S) RUN ALONG THIS VACUUM MANIFOLD, A VALLEY IN THE POTENTIAL. IN ORDER TO IMPOSE THE GEOMETRY OF THE NONTRIVIAL VACUUM ONTO THE LOW ε DOF, WE HAD TO PACKAGE THE LOW ε DOF IN SUCH A WAY THAT THEY ALWAYS SATISFY A CONSTRAINT THAT FIXES THEM TO THE VACUUM MANIFOLD.

   eg. $\mathbb{C}$ SCALAR $\phi(x) = a(x) + i b(x)$ w/ $V = (\phi^\dagger\phi - 1)^2$

   write: $\phi(x) = r(x)\, e^{i\frac{1}{f}\theta(x)}$ ← LIGHT DOF

   ↑ HEAVY DOF, GETS VEV $r_0 = 1$

   if i pick $\phi_0 = 1$, then locally the goldstone direction is $\mathrm{Im}(\phi)$

   BUT @ $\phi_0 = -i$, then the goldstone is $\mathrm{Re}(\phi)$

   We can impose our low ε DOF to ALWAYS POINT IN THE GOLDSTONE DIRECTION, BUT THE COST IS TO PACKAGE IT INTO THE EXPONENTIAL: $G(x) = \exp(i\theta(x)/f)$

2. THERE ARE MANY WAYS TO DO THIS, ie MANY REPS OF THE NLΣM. [explicit examples: see Donoghue et al. *Dynamics of the SM*]

   A VERY ELEGANT (but intuitively obvious) RESULT IN FIELD THEORY IS THAT THE LOW ε PHYSICS IS INDEPENDENT OF HOW WE CHOOSE TO REPRESENT THE NLΣM.

   - Haag Phys Rev 112, 669 ('58)
   - Coleman et al., Phys Rev 117, 2239 & 2247 ('69)

ALL THIS IS VERY NICE, BUT LET'S GO BACK TO OUR ORIGINAL MOTIVATION: SEIBERG DUAL (MAGNETIC) THEORY

SOMETHING VERY SUSPICIOUS:

ELECTRIC — MAGNETIC

$SU(N) \xrightarrow[\text{DUALITY}]{\text{SEIBERG}} SU(F-N)$ ← WTF

LOW E THY HAS SOME NEW GAUGE SYMMETRY!!

"Who ordered that?" MOMENT.

$\boxed{\text{NO ANALOG IN QCD!}}$

... or is there?

BACK IN THE OLD DAYS (when QCD was still very young) PEOPLE WOULD WONDER WHAT TO MAKE OF THE HIGHER ~~???~~ RESONANCES IN THE HADRONIC SPECTRUM: PSEUDOSCALARS ARE GOLDSTONES ... WTF ARE VECTORS? (eg ρ)

MODERN VIEWPOINT: JUST QCD RESONANCES w/ ORBITAL ANGULAR MOMENTUM ... NOTHING SPECIAL. IN FACT, IN THE MODERN POV, WE KNOW THAT @ SOME POINT IT IS WRONG TO EXPECT OUR LOW E EFT TO BE ABLE TO SAY ANYTHING INTELLIGENT ABOUT HIGHER MASS STATES!!

... BUT WHEN ALL YOU HAVE IS χPT, YOU MILK IT FOR EVERY LAST DROP.

CLAIM: VECTORS ARE GAUGE BOSON OF A HIDDEN GAUGE sym!

GIVEN WHAT WAS KNOWN ABOUT FIELD THEORY, THIS IS THE OBVIOUS GUESS. WHAT ELSE COULD A VECTOR PARTICLE BE?

A GAUGE SYMMETRY is AN UNPHYSICAL REDUNDANCY OF OUR DESCRIPTION OF A THEORY.

NIMA: "THERE'S NO SUCH THING AS GAUGE SYMMETRY.
THERE'S CERTAINLY NO SUCH THING AS A SPONTANEOUSLY BROKEN GAUGE SYMMETRY!"

SO REALLY WHAT WE'RE CLAIMING IS THAT SUCH A REDUNDANCY EXISTS IN THE NLΣM.

GO BACK TO EXPONENTIAL REP:

$$U(x) = e^{i\pi^a(x)T^a}$$

SOMETIMES A FACTOR OF 2 HERE.

ZOHAR'S CONVENTION: $[\pi^a(x)] = 0$, not canonically normalized. OUR PREVIOUS π^a is

$$\pi^a_{\text{can}} = f_\pi \, \pi^a_{\text{ZOHAR}}$$

$$\mathcal{L}_{\chi PT} = \frac{1}{4} f_\pi^2 \, \mathrm{Tr}\left(\partial U \cdot (\partial U)^\dagger\right) + \text{HIGHER DERIVATIVE}$$

$$= \frac{1}{4} f_\pi^2 \Big[\underbrace{|\partial\pi|^2}_{\text{KINETIC TERM}} - \underbrace{\tfrac{1}{2}\pi^2 |\partial\pi|^2}_{\text{LEADING INTERACTION}} + \cdots \Big]$$

WE CAN CHOOSE A RELATED, BUT SLIGHTLY DIFFERENT, REPRESENTATION TO MAKE THE HIDDEN GAUGE SYMMETRY MORE MANIFEST:

$$U(x) = \xi_L(x)\, \xi_R^\dagger(x)$$

where $\xi_{L,R}$ are unitary matrices

$$\xi_{L,R} = e^{i\pi^a_{L,R} T^a}$$

CHARGED UNDER SU(2) GAUGE AND EITHER $SU(2)_{L,R}$ FLAVOR

$$\Big\downarrow \; \text{GAUGE TRANSF } h(x) \in SU(2)$$

$$\big(\xi_L(x)\, h(x)\big)\big(\xi_R(x)\, h(x)\big)^\dagger$$

this is the HIDDEN GAUGE SYM, not visible in usual exp. rep.

	$SU(2)$	$SU(2)_L$	$SU(2)_R$
ξ_L	$\square$	$\bar{\square}$	
$\xi_R^\dagger$	$\bar{\square}$		$\bar{\square}$

$$SU(2)_L \xrightarrow{\ \xi_L\ } SU(2) \xrightarrow{\ \xi_R^\dagger\ } SU(2)_R$$

$$\mathcal{L}_0 = \frac{1}{4} f_\pi^2 \, \mathrm{Tr}\left(\partial_\mu U(x) \cdot \partial^\mu U^\dagger(x) \right)$$

$$= \frac{1}{4} f_\pi^2 \, \mathrm{Tr}\left[\left(\partial \xi_L \cdot \xi_R^\dagger + \xi_L \partial \xi_R^\dagger \right)\left(\partial \xi_R \cdot \xi_L^\dagger + \xi_R \cdot \partial \xi_L^\dagger \right) \right]$$

$$= \frac{1}{4} f_\pi^2 \, \mathrm{Tr}\left[\xi_L^\dagger \partial \xi_L \cdot \xi_R^\dagger \partial \xi_R + \partial \xi_L \cdot \partial \xi_L^\dagger + \partial \xi_R^\dagger \cdot \partial \xi_R + \partial \xi_L^\dagger \cdot \xi_L \cdot \partial \xi_R^\dagger \xi_R \right]$$

$$\boxed{\mathcal{L}_0 = \frac{1}{4} f_\pi^2 \, \mathrm{Tr}\left[\left| \xi_L^\dagger \partial \xi_L - \xi_R^\dagger \partial \xi_R \right|^2 \right]}$$

<u>VACUA</u>: CONSTANT ξ_L & ξ_R UP TO <u>GAUGE REDUNDANCY</u>
↓
USE THIS TO FIX $\langle \xi_L \rangle = \mathbb{1}$

<u>GLOBAL SYMMETRY BREAKING PATTERN</u> ($SU(2)$ = global part of gauge sym)

$$SU(2)_L \times SU(2) \xrightarrow{\langle \xi_L \rangle} SU(2)_{LG} \qquad \text{(diagonal subgroup)}$$

$$SU(2)_{LG} \times SU(2)_R \xrightarrow{\langle \xi_R \rangle} SU(2)_D \qquad \leftarrow \text{THIS IS JUST } \underline{\text{ISOSPIN}}.$$

note: gauge sym has been broken by $\langle \xi_{L,R} \rangle$!

SO FAR WE'VE JUST INTRODUCED A SILLY REDUNDANCY BY HAND. NOW LET'S "GENUINELY" GAUGE THIS REDUNDANCY BY INTRODUCING A CONNECTION. INTRODUCE A GAUGE FIELD WHICH WE WILL IDENTIFY WITH THE ρ^a MESON.

↳ Bando, Kugo, Yamawaki, Yanagida; PRL <u>54</u>, 1215 ('85)

<u>ORIGINAL MOTIVATION</u>: EXPLAIN SOME 'COINCIDENCES' IN THE ρ MESON INTERACTIONS. [we'll point these out when we get to them]

BY '85 PEOPLE ALREADY KNEW ABOUT QCD. SOMETIMES THIS IDEA IS CALLED THE "COMPOSITE VECTOR BOSON" BECAUSE WE 'KNOW' THAT THE ρ IS REALLY A $q\bar{q}$ BOUND STATE W/ ORBITAL ANGULAR MOMENTUM. THIS REALLY ECHOES THE SPIRIT (& CONTENT!!) OF VERY MODERN IDEAS: AdS / CFT.

INTRODUCE: ρ_μ^a s.t. $\rho_\mu^a T^a \to h^\dagger \rho_\mu^a T^a h + i h^\dagger \partial_\mu h$

PROMOTE ORDINARY DERIVATIVE TO GAUGE COVARIANT DERIVATIVE

$$\xi_L^\dagger \partial_\mu \xi_L \longrightarrow \xi_L^\dagger (\partial_\mu - i\rho_\mu)\xi_L = \boxed{i\,\xi_L^\dagger(\rho_\mu - i\partial_\mu)\xi_L \equiv i\rho_\mu^L}$$

not canonically normalized
(GAUGE COUPLING ABSORBED INTO GAUGE FIELD)

MINOR REMARK: THIS DEFINITION OF ρ_μ^L (& SIMILARLY FOR ρ_μ^R) DIFFERS FROM eq (2.7) OF THE PAPER. WE AGREE WHEN WE GO TO UNITARY GAUGE ($\Pi_L = -\Pi_R$).

WE CAN WRITE OUR PREVIOUS LAGRANGIAN IN TERMS OF THE ρ_μ^L & ρ_μ^R

$$\mathcal{L}_0 = \frac{1}{4} f_\pi^2 \,\mathrm{Tr}\left[\,|\rho_\mu^L - \rho_\mu^R|^2\,\right]$$

TO THIS WE SHOULD ALSO ADD THE USUAL GAUGE KINETIC TERM

$$\mathcal{L}_\rho = -\frac{1}{g^2}\left(F^a_{\mu\nu}\right)^2$$

[WE ASSUME SUCH A TERM IS GENERATED DYNAMICALLY ALONG W/ THE EMERGENCE OF THE HIDDEN SU(2)]

NOW WE SHOULD STOP & THINK: ARE THERE ANY TERMS THAT WE'VE MISSED? WE'VE WRITTEN DOWN $\mathcal{L}_0$, WHICH REPRODUCES THE ORIGINAL EXPONENTIAL REPRESENTATION OF THE NLSM. WE'VE ADDED THE KINETIC TERM NECESSARY TO DESCRIBE THE ρ PERTURBATIVELY. WHAT ELSE?

WE'VE MISSED SOMETHING! CAN SEE IT IN THE FORM OF $\mathcal{L}_0$: curious minus sign. WHAT IS THAT MINUS SIGN DOING? ABSOLUTELY NOTHING. THE $\rho^{L,R}$ TERMS TRANSFORM HOMOGENEOUSLY, $\rho^{L,R} \to h^\dagger(x)\rho^{L,R}h(x)$, UNDER THE GAUGE SYM. SO THAT SIGN MIGHT AS WELL BE FLIPPED.
THIS GIVES AN <u>INDEPENDENT TERM</u> IN THE EFFECTIVE LAGRANGIAN.

(YET) UNDETERMINED PREFACTOR, a:

$$\mathcal{L}_a = \frac{a}{4} f_\pi^2 \,\mathrm{Tr}\left[\,|\rho_\mu^L + \rho_\mu^R|^2\,\right]$$

THE DETERMINATION OF a IS ONE OF OUR KEY GOALS.

@ this $\mathcal{O}$ (2 DERIVATIVES) these are the only $SU(2)_L \times SU(2)_R \times SU(2) \times P_{LR}$ invariants

implicit assumption of perturbativity ($g \ll 1$)
NOT VALID IN QCD, BUT RESULTS TURN OUT TO BE FAIRLY ROBUST!

OK. NOW THAT WE HAVE A $\mathcal{L}$ (most general to L.O. in g)
WE CAN START ASKING INTELLIGENT QUESTIONS.

MOTIVATED BY OBSERVED CURIOSITIES IN THE INTERACTIONS OF ρ MESONS. HERE WE'LL JUST ASK THE QUESTIONS W/ NO *a priori* MOTIVATION & SHOW THAT THEY HAPPEN TO DESCRIBE LOW E QCD QUITE WELL.

1. WE KNOW THAT THE HIDDEN GAUGE SYM IS BROKEN
[& we know what this means: NLΣM]
→ ρ IS NOT MASSLESS. WHAT IS M_ρ?

ANSWER: EXPAND $\xi = \mathbb{1} + i\pi^a T^a$; PICK UNITARY GAUGE $\pi_L = -\pi_R \equiv \pi$

$$\mathcal{L} = -\frac{1}{g^2}F^2 + \frac{1}{4}f_\pi^2 \mathrm{Tr}\left[|\partial\pi_L - \partial\pi_R|^2\right] + \frac{a}{4}f_\pi^2 \mathrm{Tr}\left[|2\rho + (\partial\pi_L - \partial\pi_R)|^2\right]$$

$$4\rho^a_\mu \rho^{a\mu} T^2 = 4\rho^a_\mu\rho^{a\mu} \underbrace{T^a_{ij}T^a_{ji}}_{1/2}$$

$$\Rightarrow \frac{1}{2}(af_\pi^2)\rho^2$$

$$\Rightarrow \frac{1}{2}a(f_\pi^2 g^2)\rho^2_{can}$$

canonically normalized

$$\boxed{m_\rho^2 = ag^2 f_\pi^2}$$

THIS IS NOT YET USEFUL B/C RHS ISN'T SOMETHING EASY TO RELATE TO THE MEASURED ρ PROPERTIES.

OBSERVE: ρ COUPLING TO PIONS ONLY THROUGH a TERM.

2. WHAT IS THE COUPLING OF ρ TO PIONS? ASSUME UNITARY GAUGE: $\pi_L = -\pi_R$

~~HERE'S~~ ANSWER: $\mathcal{L} \supset \frac{1}{2} a g \underbrace{\rho^a_{can} (\vec{\pi} \times \partial \vec{\pi})^a_{can}}_{\text{canonically normalized fields}}$

$$\boxed{g_{\rho\pi\pi} = \frac{1}{2} a g}$$

still not yet useful, g is arbitrary

PROOF (stupid way)

$$\mathcal{L} \supset \frac{a}{4} f_\pi^2 \,\mathrm{Tr}\left[\left| \xi_L^\dagger (\rho - i\partial)\xi_L + \xi_R^\dagger(\rho - i\partial)\xi_R \right|^2 \right]$$

$\pi_L = -\pi_R \equiv \pi$ s.t. $\xi_L = \xi_R^\dagger \equiv \xi$

$$\underbrace{\left| \xi^\dagger (\rho - i\partial)\xi + \xi(\rho - i\partial)\xi^\dagger \right|^2}_{2\rho \,+\, \xi^\dagger \partial\pi \xi \,-\, \xi \partial\pi \xi^\dagger \quad + \,\mathcal{O}(2\rho\pi^2)}$$

$$= 2\rho \quad \begin{matrix} -i\pi\partial\pi \\ +i\partial\pi\pi \end{matrix} \quad \begin{matrix} -i\pi\partial\pi \\ +i\partial\pi\pi \end{matrix}$$

$$= 2\left(\rho + i[\partial\pi, \pi]\right)$$
$$= 2\left(\rho^a T^a - \partial\pi^b \pi^c \varepsilon^{bca} T^a\right)$$

$$\mathcal{L} \supset \frac{a}{4} f_\pi^2 \cdot 4 \cdot (\rho^a - (\partial\pi \times \pi)^a)(\text{---})^b \underbrace{\mathrm{Tr}(T^a T^b)}_{1/2}$$

$$= \frac{1}{2} a f_\pi^2 \left(\rho^a - (\partial\pi \times \pi)^a\right)^2$$

$$\rho^a = g\rho^a_{can} \qquad \pi^a = \pi^a_{can}/f_\pi$$

$$= \frac{1}{2} \underbrace{a (f_\pi g)^2}_{m_\rho^2} \rho_{can}^2 + a g \rho^a_{can} (\vec{\pi} \times \partial\vec{\pi})^a_{can} + \mathcal{O}(\pi^4)$$

OOPS! THERE SHOULD BE A FACTOR OF 1/2 HERE...

WELL... NEVER MIND MY FACTOR OF 2 DELINQUENCIES...

REMARK: THE WAY THIS IS USUALLY PRESENTED IS AS FOLLOWS:

THE ρ KINETIC TERM IS GENERATED QUANTUM MECHANICALLY SO WE CAN TALK ABOUT THE THEORY "BEFORE" INCLUDING THIS TERM. THEN ρ IS AN AUXILIARY FIELD AND THE a TERM IS ESSENTIALLY A LAGRANGE MULTIPLIER.

ONE CAN THEN USE THE 'CLASSICAL' EOM FOR ρ ← VALID BELOW m_ρ
TO OBTAIN $\rho^a_\mu \sim (\vec{\pi}\times\partial_\mu\vec{\pi})^a$

FROM THIS WE MAY RE-SUBSTITUTE INTO $\mathcal{L}$
s.t. ρ^2 TERM $\longrightarrow$ $\rho^a(\pi\times\partial_\mu\pi)^a$

THUS: $\underbrace{\tfrac{1}{2} a f_\pi^2}_{\text{mass term before canonical normaliz.}} \rho^2 \longrightarrow \tfrac{1}{2} a f_\pi^2 \rho(\pi\times\partial\pi)$

$$\big\downarrow \quad \rho = g\rho_{can}, \quad \pi = 1/f_\pi \, \pi_{can}$$

$$\boxed{\tfrac{1}{2} a g}\, \rho_{can}(\pi\times\partial\pi)$$

this is now a measurable quantity... $\longrightarrow$ $g_{\rho\pi\pi}$ ✓

COMBINING W/ OUR EXPRESSION FOR m_ρ^2:

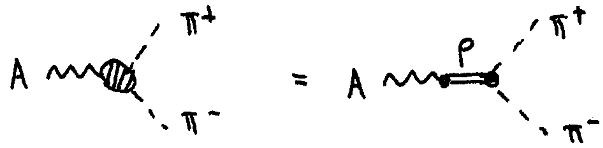
$$\boxed{\frac{4}{a}\, g_{\rho\pi\pi}^{\,2} f_\pi^2 = m_\rho^2}$$

so given a value for a, we have a nontrivial relation between naively independent experimentally observable quantities

NOW TO DETERMINE a. THE KEY IS AN OBSERVATION IN HADRONIC PHYSICS:

<u>VECTOR MESON DOMINANCE</u> (VMD)

A VIRTUAL PHOTON "CONVERTS INTO A NEUTRAL VECTOR MESON" BEFORE INTERACTING W/ A HADRONIC STATE.

ie 

SO THE THIRD QUESTION IS:

3. <u>HOW DO PHOTONS INTERACT WITH ρ & π</u> ?

a clever way to ask this: WHAT IS THE EM CURRENT?

$$j^{EM}_\mu = j^3_\mu + j^Y_\mu$$

j^3_μ: FROM T^3 OF SU(2)

recall $SU(2)_I$ is what gets weakly gauged

j^Y_μ: $Y[\xi_{L,R}] = 0$

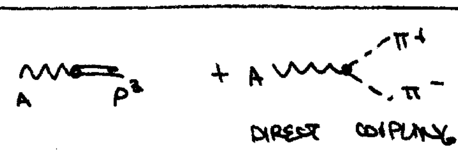
RECALL: $$j^a_\mu = -i \frac{\delta \mathcal{L}}{\delta(\partial^\mu \phi_i)} t^a_{ij} \phi_j$$

$$\mathcal{L} = -\frac{1}{g^2}F^2 + \frac{1}{4}f_\pi^2 \mathrm{Tr}\left[|\rho^L - \rho^R|^2\right] + \frac{a}{4}f_\pi^2 \mathrm{Tr}\left[|\rho^L + \rho^R|^2\right]$$

$$\left.\begin{aligned} \text{w/}\quad \rho^L - \rho^R &= \xi^\dagger i\partial\xi - \xi i\partial\xi^\dagger \\ \rho^L + \rho^R &= 2\rho + \xi^\dagger i\partial\xi + \xi i\partial\xi \end{aligned}\right\} \text{UNITARY GAUGE}$$

<u>ANSWER</u>:

$$\boxed{j^a = 2af_\pi^2 \rho^a + 2f_\pi^2(a-2)\varepsilon^{abc}\pi^b\partial\pi^c + (3+\text{ PARTICLES})}$$

$eA^\mu j^3_\mu =$  $+ \cdots$

KEY OBSERVATION: $a = 2$ GIVES VECTOR MESON DOMINANCE BY KILLING $\pi^+\pi^- A$ INTERACTION.

BEFORE MOTIVATING $a=2$, LET'S REVIEW WHAT HAPPENS IF IT IS TRUE:

$\boxed{m_\rho^2 = 2 g_{\rho\pi\pi}^2 f_\pi^2}$ from 1 & 2 ; $\boxed{\text{VMD}}$ from 3.

↑ OBSERVED IN THE REAL WORLD, 5% ERROR

$$g_{\rho\gamma} = g a f_\pi^2 \implies \boxed{g_{\rho\gamma} = 2 g_{\rho\pi\pi} f_\pi^2 = m_\rho^2 / g_{\rho\pi\pi}}$$

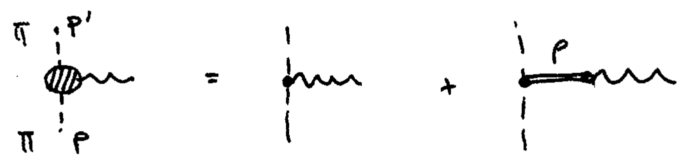
↑ OBSERVED @ 10% LEVEL!

BY THE WAY: $g_{\rho\gamma}$ IS BASICALLY A MASS TERM THAT MIXES ρ^3 AND γ. THIS IS JUST LIKE THE MIXING OF B AND W^3

↳ CHIRAL CONDENSATE BREAKS $SU(2)_{\text{HIDDEN}} \times U(1)_Q \to U(1)_{EM}$
PHYSICAL ρ MESON IS THE HEAVY GUY, PHYSICAL γ IS MASSLESS

4. <u>WHY IS $a=2$</u> ?

CONSIDER EM <u>FORM FACTOR</u> OF THE PION
└ DESCRIBES PION SUBSTRUCTURE



contact term $\qquad$ ρ EXCHANGE (HADRONIC SUBSTRUCTURE)

$$\langle \pi(p) | j_\mu^{EM}(q) | \pi(p') \rangle = (p+p')_\mu F(q^2)$$

← $(p-p')$ term $=0$ BY GAUGE INVARIANCE

$$F(q^2) = (\text{contact}) + \frac{g_{\rho\gamma}\, g_{\rho\pi\pi}}{m_\rho^2 - q^2}$$

$$= (\text{contact}) + \frac{(g a f_\pi^2)\cdot \frac{1}{2} a g}{m_\rho^2 - q^2}$$

$$= (\text{contact}) + \frac{\frac{1}{2} a\, m_\rho^2}{m_\rho^2 - q^2}$$

IN THE LIMIT $q^2 \to 0$, PHOTON WAVELENGTH GETS LARGE & WE'RE NOT PROBING THE PION SUBSTRUCTURE. IN THIS LIMIT WE KNOW THE PION CHARGE,

$$F(0) = Q_\pi = 1 .$$

$$\Rightarrow \boxed{F(q^2) = \left(1 - \frac{a}{2}\right) + \frac{\frac{1}{2} a M_\rho^2}{M_\rho^2 - q^2}}$$

ASYMPTOTIC FREEDOM (BJORKEN SCALING) TELLS US THAT

$$\lim_{q^2 \to -\infty} F(q^2) \sim \frac{1}{q^2}$$

(DEEP EUCLIDEAN)

NOW WE CAN BE TRICKY:

$$\int_\gamma d(q^2) \frac{F(q^2)}{q^2} \sim \int d(q^2) \frac{1}{q^4} = 0$$

WHERE WE ARE IGNORING 'HEAVY' RESONANCES

FOR LARGE CONTOUR γ.
ASSUME γ ENCLOSES ONLY THE ORIGIN + ρ MESON POLE

$$F(0) = 1$$

$$\Rightarrow \text{Res} \frac{F}{q^2} = 1$$

$$\text{Res} \frac{1}{q^2} \frac{\frac{1}{2} a M_\rho^2}{M_\rho^2 - q^2} \bigg|_{q^2 = M_\rho^2} = -\frac{1}{2} a$$

$$\Rightarrow \int_\gamma d(q^2) \frac{F(q^2)}{q^2} = 1 - \frac{1}{2} a = 0$$

$$\Rightarrow \boxed{a = 2}$$

SO: THE PUNCHLINE IS THAT IT IS SENSIBLE TO IMAGINE THE EMERGENCE OF A HIDDEN GAUGE GROUP IN THE LOW ENERGY DYNAMICS OF A STRONGLY COUPLED THEORY.

INDEED, SUCH A PROPOSAL SEEMS TO 'ACCURATELY' DESCRIBE THE INTERACTION OF THE ρ MESON WITH PHOTONS & PIONS.

THE ONLY ASSUMPTION WE REALLY NEEDED WAS THE DYNAMICAL GENERATION OF THE ρ KINETIC TERM. (THIS HAPPENS ALL THE TIME IN COMPOSITE MODELS)

BACK TO SEIBERG DUALITY: CONCLUDING REMARKS

SQCD IS THE IDEAL TESTING GROUND (SUSY $\rightarrow$ CONTROL)

$\rightarrow$ GIVES STRONG HINT ABOUT NATURE OF $SU(F-N)$ IR GAUGE GROUP

COULD HAVE ASKED: NOT ONLY IS $SU(F-N)$ IR GAUGE GROUP WEIRD, WHAT THE HECK ARE THE MAGNETIC QUARKS? (NO IDENTIFICATION W/ UV DOF ; unlike QCD where, eg, $\pi = q\bar{q}$)

CLAIM: $SU(F-N)$ GAUGE BOSONS $\longleftrightarrow$ ρ MESON
$q, \tilde{q}$ MAGNETIC QUARKS $\longleftrightarrow$ ξ_L, ξ_R NONLINEAR FIELDS

DO NOT APPEAR IN UV THEORY SINCE THEY ARE CHARGED UNDER THE HIDDEN GAUGE REDUNDANCY! UV OBJECTS W/ NO $SU(F-N)$ CHARGE CANNOT BE ARRANGED INTO CHARGED COMPOSITES.

ANALOG OF $\langle q\bar{q} \rangle$ QCD CHIRAL CONDENSATE IS A VEV TO $\dot{q}$ ie WALK OUT SLIGHTLY ALONG BARYONIC DIR OF MAGNETIC MODULI SPACE. DISTANCE FROM ORIGIN CONTROLS ρ MASS.

TURNS OUT THAT SEIBERG MAGNETIC THEORY ALSO EXHIBITS VMD